Standard Intertemporal Choice

Behavioral Economics

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Intertemporal Choice

Many interesting questions in economics involve decisions with outcomes over time:

How should people allocate their wealth between current consumption and future consumption?

When is the optimal time for me to complete a task?

How do people trade off short term benefits and long term costs?

-Alcohol, cigarettes, potato chips

Or short term costs and long term benefits?

-Exercise, grad school

Interest Rate Warm-Up

Suppose you put $1000 in the bank that pays 10% interest per year.

After 1 year, you'll have 1000 * 1.10 =1100

After 2 years, you'll have 1100 * 1.10=1210

After 3 years, you'll have 1210*1.10 = 1331

In general, if you put amount $A at interest rate r for T years, its future value in T years will be: $A*(1+r)^T$

Interest Rate Warm-Up

Suppose you will be paid $1100 one year from today. If the market interest rate is 10%, how much is this future payment worth now?

Answer: It must be $1000, since $1000 today at the 10% interest rate would yield $1100 in a year.

Given per-period interest rate r, the net present value or present discounted value of future amount A is equal to:

$$A / (1+r)^T$$

Interest Rate Warm-Up

Suppose you will be paid $1100 one year from today, $1100 in 2 years, and $1100 in 3 years. If the market interest rate is 10%, how much is this stream of future payments worth now?

Answer: Add up the individual present values.

NPV = 1100 / (1.10) + 1100 / $(1.10)^2$ + 1100 / $(1.10)^3$ = $2736

Interest Rate Warm-Up

Now suppose the interest rate is not constant but is time-varying.

The period t interest rate r_t is the interest rate between period t and t+1. If you have amount A in period t, in period t+1 you will have $A*(1+r_t)$

Hence, if At is the amount in your bank account at time t, and your bank pays per period interest rates of $(r_t, r_{t+1}, \ldots r_T)$, then:

$A_{t+1} = (1+r_t)A_t$

$A_{t+2} = (1+r_{t+1})A_{t+1} = (1+r_t)*(1+r_{t+1})*A_t$

$A_{t+3} = (1+r_{t+2})*A_{t+2} = (1+r_t)*(1+r_{t+1})*(1+r_{t+2})*A_t$

Interest Rate Warm-Up

Given per period interest rates of (r_t, r_{t+1}, …. r_T), the present value of a stream of future amounts (A_{t+1} , A_{t+2} , A_{t+3}) =

$$A_{t+1} /(1 + r_t) + A_{t+2} / (1 + r_t) (1+ r_{t+1}) + A_{t+3} / (1 + r_t) (1+ r_{t+1}) (1+r_{t+2})$$

Example: Suppose you put $1000 in the bank. The interest rate this year is 10%, the interest rate the following year is 6%, and in the 3rd year it's 5%. How much will you have in 3 years?

=1000*(1+.10)(1+.06)(1.05)

Standard Consumer Choice

N goods in the world

x_n denotes the quantity of good n consumed

Preferences are given by utility function $U(x_1, x_2, \ldots x_n)$

I denotes income, and p_n is the price for good n

The consumer chooses $(x_1, x_2, \ldots x_n)$ to maximize $U(x_1, x_2, \ldots x_n)$ subject to $p_1 x_1 + p_2 x_2 + p_n x_n \leq I$

Standard Intertemporal Choice

When modeling intertemporal choice, economists treat one physical good consumed at two different times as two different goods.

Suppose there are two goods, x and y, to be consumed at time periods 1 and 2. If we assume additively separable intertemporal utility:

$$U(x_1, y_1, x_2, y_2) = u(x_1, y_1) + D(t)\, u(x_2, y_2)$$

Here $u(x_t, y_t)$ is the <u>instantaneous utility function</u>

$D(t) < 1$ is the <u>discount function</u> that captures the relative preference for consumption in period 1 versus period 2

Standard Intertemporal Choice

D(t) is the discount function that specifies the amount of discounting that occurs for an outcome t periods away.

Properties of discount functions:

D(0) = 1 Something happening right now gets full weight

D(t -> ∞) = 0 Asymptotes to 0

D(t) is monotonically decreasing – A longer delay implies greater impatience

Standard Intertemporal Choice

Why are future events discounted at all? Why is something that happens in the future worth less than something that happens in the present?

1) **Impatience – Consumption today is better than consumption tomorrow**

2) **Uncertainty about the future – I could get hit by a truck, or a meteor, or a truck full of meteors on my way home**

3) **Memory Utility – In addition to consumption, I might also derive utility from memories of previous experiences. Consuming something today versus tomorrow yields one more day of pleasant memories**

Discounting

Consider multiplying future outcomes by a <u>discount factor</u> $\delta < 1$, to capture their reduction in value.

Suppose the yearly $\delta = .75$, implying that something occurring in one year is worth ¾ as much as if it occurred today.

δ is also sometimes written as:

$\delta = 1/(1+\rho)$

$\rho > 0$ is known as the <u>discount rate</u>, or the pure rate of time preference.

ρ is an individual's subjective interest rate

Discounting

The discount factor δ is the marginal rate of substitution between time periods, sometimes directly called the intertemporal marginal rate of substitution

Suppose there are two periods, and we care directly about utilities in each period: $U = u_1 + \delta u_2$

$\delta = 1$ implies that 1 unit of happiness tomorrow is worth 1 unit of happiness today, hence the two are perfect substitutes

Discounting

Suppose δ = .75. One unit of happiness tomorrow is worth ¾ units of happiness today. Alternatively, it would require 1 / .75, or 1.33 units of happiness tomorrow to yield indifference to one unit of happiness today

This implies utility indifference curves of slope with -1.33

Suppose δ = 0. What do the indifference curves look like?

Discounting

More generally, consider the intertemporal utility of a good *x* in time periods 1 or 2

The utility in any one period is known as the instantaneous utility u(x), which has diminishing marginal utility

Thus total lifetime utility is given by $U = u(x_1) + \delta * u(x_2)$

An additional unit of x in period 1 is worth $u'(x_1)$, and an additional until of x in period 2 is worth $\delta u'(x_2)$

Discounting

How should an agent choose how much x to consume in each period?

Natural impatience leads him to consume it all in period 1, since period 2 consumption is only worth δ as much

But diminishing marginal utility implies that eventually it's better to defer some consumption unto period 2

This leads to convex indifferent curves for intertemporal consumption of the good:

Exponential Discounting

The most common discount function employed by economists is the exponential discount function:

$D(t) = \delta^t$

Suppose the yearly $\delta = .75$, implying that something occurring in one year is worth ¾ as much as if it occurred today.

Something happening in 3 years would thus be worth .75^3 = .42 as much as if it happened today

Exponential Discounting

Multiple periods:

$U(c_0, c_1, c_2, \ldots c_T) = u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \ldots + \delta^T u(c_T)$

The MRS between any two periods t and t' is given by D(t') / D (t)

For example, the MRS for exponential discounting between periods 2 and 5 is: $\delta^5 / \delta^2 = \delta^3$

Note that discounting something t periods away to the present is D(t) / D (0) = D(t) / 1 = D(t)

Exponential Discounting

Properties of Exponential Discounting: $D(t) = \delta^t$

It obviously satisfies properties 1-3 for a discount function. In addition:

4) Constant per period discounting: At all time periods, $D(t+1) / D(t) = \delta$

That is, the discounting between any two periods is always with discount factor δ

Exponential Discounting

This represents an even-handedness in how an exponential discounter views time.

How he feels about this year versus next year is the same (δ /1) as how he feels about 5 years from now versus 6 years from now (δ^6 / δ^5)

How he feels about tomorrow versus the day after tomorrow is that same as between 112 days from now and 113 days from now

Exponential Discounting

5) Constant per-period discounting implies <u>time consistency</u> – the property whereby the optimal decision at one point in time remains the optimal decision with the passage of time

Suppose a decision maker must choose between $100 in 5 years or $125 in 6 years. A decision maker will be willing to wait for the larger amount if:

$\delta^5 u(100) < \delta^6 u(125)$ which reduces to $u(100) < \delta u(125)$

Now suppose 5 years has gone by and he can choose again. The choice is now between $100 right now or $125 in 1 year. He will be willing to wait for the larger amount if:

$u(100) < \delta u(125)$

which is the same condition as above

Intertemporal Choice

Further assumptions of intertemporal choice:

1) Integration – An individual evaluates new alternatives by integrating them with existing plans

Consider someone with consumption plan (c_0, c_1, c_2, …c_T) who is given some choice X, for example, gving up $5000 today to get $10,000 in 5 years.

Integration implies that the prospect X is not evaluated in isolation, but against how it would change consumption in ALL periods.

In other words, choices affect a person's intertemporal budget set, and people optimize subject to this.

Intertemporal Choice

2) Utility Independence – Decision makers maximize $U(c_0, c_1, c_2, \ldots c_T)$ which is the sum of the per-period utilities:

$$u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \ldots + \delta^T u(c_T)$$

All that matters is maximizing this sum. Decision makers are assumed to have no preference for the distribution of utilities.

This rules out preferences for any kind of patterns for utility over time , such as a preference for a flat utility profile over a "roller coaster" profile, or a preference for an increasing sequence of utilities

Intertemporal Choice

3) Consumption Independence - Utility of the form:

$u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \ldots + \delta^T u(c_T)$

implies that utility in period t +k is independent of consumption in any other period – all that matters is consumption in t+k

That is, an outcome's utility is unaffected by outcomes in prior or future periods.

For example, my preference for Mexican versus Vietnamese food tonight is unaffected by the fact that I ate Vietnamese food last night

Or that the amount of beer I drank last night and the amount of beer I will drink tomorrow night has no impact on my optimal choice to drink tonight

Intertemporal Choice

4) Stationary Intertemporal Choice – It is often assumed that the cardinal utility function $u(c_t)$ is constant across time, so that the well-being generated by any activity is the same at different periods

However, tastes often do change over time in predictable ways, or in particular situations (emotions)

This is different from consumption independence. Things like habit formation and reference dependence are examples of consumption independence

Example

One Good and Two Periods:

Let c_1 be consumption in period 1, and c_2 consumption in period 2. Thus, the consumer chooses consumption bundle (c_1, c_2) to maximize U(c_1, c_2)

Let r be the market interest rate, and let δ be the consumer's subjective discount factor.

Y_1 denotes income in period 1, Y_2 denotes income in period 2.

The consumer thus chooses (c_1, c_2) to maximize U(c_1, c_2) subject to:

$$c_1 + \frac{c_2}{1+r} \leq Y_1 + \frac{Y_2}{1+r} \equiv W$$

Example

Suppose U(c1, c2) = ln c1 + δ ln c2

Then: $$c_1^* = \frac{W}{1+\delta} \quad \text{and} \quad \mathrm{c}_2^* = \frac{\delta(1+r)W}{1+\delta}$$

Notice there are two forces in opposing directions:

1) **Impatience: δ < 1 implies that that $c_1 > c_2$**

2) **Consumption smoothing: Diminishing marginal utility implies that a feast on Monday and eating nothing on Tuesday isn't as desirable as a normal lunch on both days.**

So, rather than stacking all consumption in period 1, it's better to smooth consumption towards period 2 as well.

Example

Another important implication is that consumption is independent of the timing of income.

Holding lifetime wealth constant, the optimal (c_1, c_2) is the same whether all income comes in period 1, period 2, or some mixture of the two.

If Y1 > c_1^* , then the consumer saves enough to reach desired consumption in both periods.

If Y2 < c_1^* , then the consumer borrows to reach the desired consumption.

Example

That is, consumption should not “track” income.

A higher income in period 1 should lead to higher consumption in both periods.

Further, it should yield the exact same desired consumption as if the (present-value adjusted) income increase happened in period 2 instead.

This is the permanent income / life-cycle hypothesis – people consume out of expectations of “lifetime income”, and hence transitory (expected) income shocks should have no effect