Quantitative Political Methodology II (Fall), Course: 582. Homework #3.

1. Calculate the maximum likelihood estimate of the intensity parameter of the Poisson distribution,

$$f(y|\mu) = \frac{e^{-\mu}\mu^y}{y!}, \quad \mu > 0,$$

for the data: $[7, 4, 3, 4, 7, 6, 9, 11, 21, 3]$.

2. Assume, as in the example in class, that $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \boldsymbol{\epsilon}\ \mathcal{N}(0, \sigma^2\mathbf{I})$. Derive the MLE for σ^2. How does this relate to what you learned in your linear regression course?

3. For the simplified Pareto PDF:

$$f(x|\theta) = \theta x^{-\theta-1}, 0 <= x, \theta < \infty,$$

find the maximum likelihood estimate for an iid sample: $X_1, ..., X_n$.

4. The *Fundamental Probability Transformation*, (als called the *The Probability Integral Transformation*) states: Suppose X has an everywhere continuous cdf $F(X)$, and define the new random variable $Y = F(X)$, then Y is uniformly distributed on $(0, 1)$: $P(Y \leq y) = y$ for $0 < y < 1$ and 0 elsewhere. Prove this theorem.

5. Suppose you had a Poisson process with intensity parameter $\lambda = 5$. What is the probability of getting exactly 7 events? What is the probability of getting exactly 3 events? These values are the same distance from the expected value of the Poisson distribution, so why are they different?

6. Read the article: "Alliance Behavior in Balance of Power Systems: Applying a Poisson Model to Nineteenth-Century Europe. by Patrick J. McGowan and Robert M. Rood, *American Political Science Review* 69, (Sep. 1975), 859-870 (see `jstor`).

   (a) Describe the quantitative methodology used.

   (b) What is the authors' main methodological point about model choice?

   (c) What does their empirical analysis show?

   (d) Suppose that you were reviewing this article for a leading political science journal. What specific criticisms would you have? Be specific and defend your claims.

7. Tobit regression (censored regression) deals with an interval-measured outcome variable that is censored such that all values that would have naturally been observed as negative are reported as zero (generalizeable to other values). That is, we can have left censoring and right censoring at any arbitrary value, single and double censoring, mixed truncating and censoring. Common references are : Tobin 1958, Amemiya (1985, Chapter 10), Chibb (1992). If $\mathbf{z}$ is a latent outcome variable in this context with the assumed relation:

$$\mathbf{z} = \mathbf{x}\boldsymbol{\beta} + \boldsymbol{\epsilon} \qquad \text{and} \qquad z_i \sim \mathcal{N}(\mathbf{x}\boldsymbol{\beta}, \sigma^2),$$

then for *left censoring at zero*, the observed outcome variable is produced according to:

$$y_i = \begin{cases} z_i \text{ if } z_i > 0 \\ 0 \text{ if } z_i \leq 0. \end{cases}$$

The resulting likelihood function is:

$$L(\boldsymbol{\beta}, \sigma^2 | \mathbf{y}, \mathbf{x}) = \prod_{y_i=0} \left[1 - \Phi\left(\frac{x_i \boldsymbol{\beta}}{\sigma} \right) \right] \prod_{y_i>0} (\sigma^{-1}) \exp\left[-\frac{1}{2\sigma^2} (y_i - x_i \boldsymbol{\beta})^2 \right].$$

where σ^2 is called the *scale*. For this problem,

(a) Explain the structure of the likelihood function: what parts accomodate observed values and what parts accomodate unobserved values. Be specific.

(b) Replicate Tobin's original analysis in `R` using the `survreg` function in the `survival` package. The data are obtainable from:

```
tobin <- read.table("http://artsci.wustl.edu/~jgill/data/tobin.dat",header=TRUE)
```

and the article is available on `jstor` (Tobin, James. 1958. "Estimation of Relationships for Limited Dependent Variables." *Econometrica* 26, 24-36.)