MLE COURSE: Homework #3.

1. Calculate the maximum likelihood estimate of the intensity parameter of the Poisson distribution,

$$f(y|\mu) = \frac{e^{-\mu}\mu^y}{y!}, \quad \mu > 0,$$

for the data: $[7, 4, 3, 4, 7, 6, 9, 11, 21, 3]$.

2. Assume, as in the example in class, that $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \boldsymbol{\epsilon}\ \mathcal{N}(0, \sigma^2\mathbf{I})$. Derive the MLE for σ^2. How does this relate to what you learned in your linear regression course?

3. For the simplified Pareto PDF:

$$f(x|\theta) = \theta x^{-\theta-1}, 0 <= x, \theta < \infty,$$

find the maximum likelihood estimate for an iid sample: $X_1, ..., X_n$.

4. The *Fundamental Probability Transformation*, (als called the *The Probability Integral Transformation*) states: Suppose X has an everywhere continuous cdf $F(X)$, and define the new random variable $Y = F(X)$, then Y is uniformly distributed on $(0, 1)$: $P(Y \leq y) = y$ for $0 < y < 1$ and 0 elsewhere. Prove this theorem.

5. Suppose you had a Poisson process with intensity parameter $\lambda = 5$. What is the probability of getting exactly 7 events? What is the probability of getting exactly 3 events? These values are the same distance from the expected value of the Poisson distribution, so why are they different?

6. Read the article: "Alliance Behavior in Balance of Power Systems: Applying a Poisson Model to Nineteenth-Century Europe. by Patrick J. McGowan and Robert M. Rood, *American Political Science Review* 69, (Sep. 1975), 859-870 (see `jstor`).

   (a) Describe the quantitative methodology used.

   (b) What is the authors' main methodological point about model choice?

   (c) What does their empirical analysis show?

   (d) Suppose that you were reviewing this article for a leading political science journal. What specific criticisms would you have? Be specific and defend your claims.