Learning Asymmetries in Real Business Cycles

Stijn Van Nieuwerburgh and Laura Veldkamp

Motivation

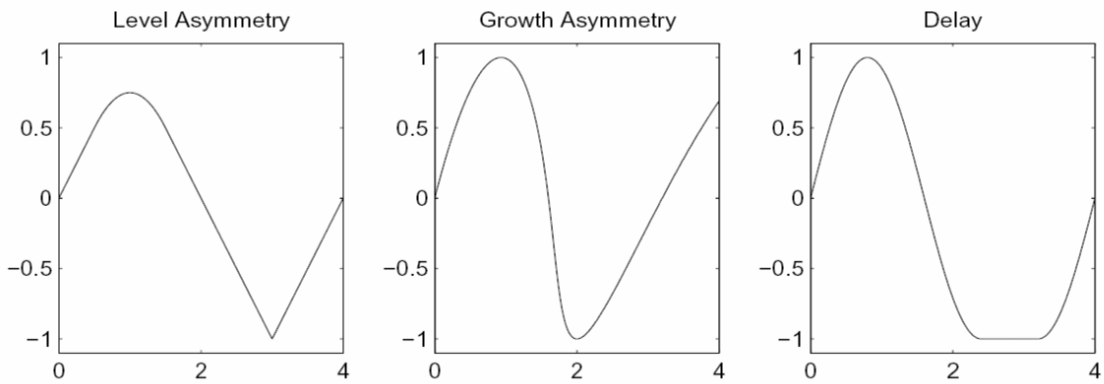
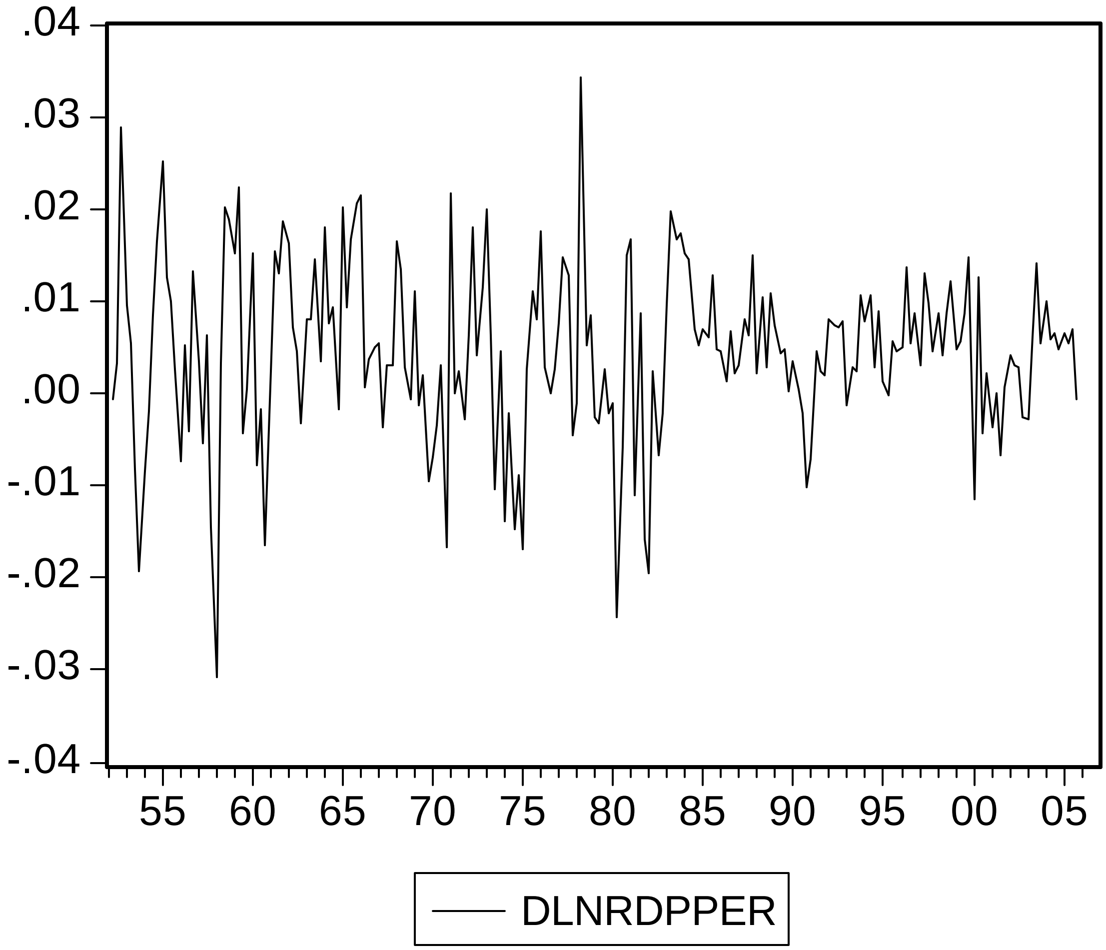


Figure 1: Types of Asymmetry.

Asymmetric growth rates I

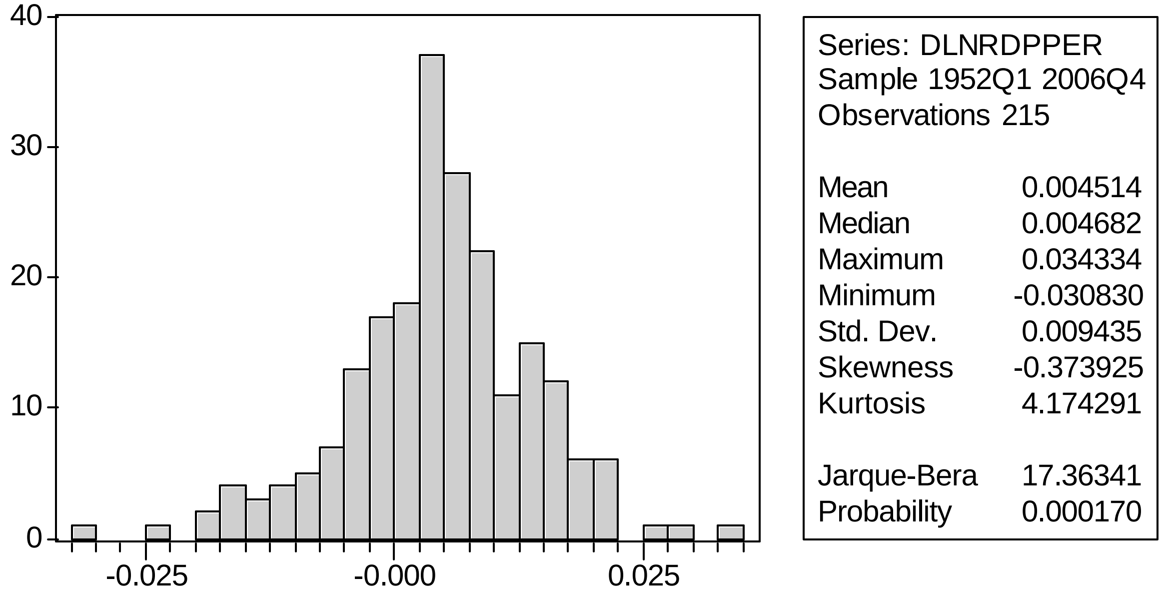
First difference of log per capita real GDP.



.04
.03
.02
.01
.00
-.01
-.02
-.03
-.04
55 60 65 70 75 80 85 90 95 00 05
DLNRDPPER

Asymmetric growth rates II

- Third central moment is defined as $\mu_3 = E(X - \mu)^3$.
- Skewness is the third standardized moment (μ_3/σ^3). It is a measure of asymmetry.


40
30
20
10
0
-0.025
-0.000
0.025
Series: DLNRDPPER
Sample 1952Q1 2006Q4
Observations 215
Mean 0.004514
Median 0.004682
Maximum 0.034334
Minimum -0.030830
Std. Dev. 0.009435
Skewness -0.373925
Kurtosis 4.174291
Jarque-Bera 17.36341
Probability 0.000170

Asymmetric learning

Explain growth rate asymmetry through an asymmetric learning process

- Meaning of asymmetric learning: The process of learning differs in booms and recessions.
- Cause of asymmetric learning: Endogenously varying rate of information.
- Intuition: More output, more information, precise estimates of z_t during booms $\longrightarrow$ Detect negative shocks in quickly. Faster learning. $\longrightarrow$ Firms abruptly reduce production. Downturn is sudden (short and sharp) $\longrightarrow$ Low output, less information, less precise estimates of z_t $\longrightarrow$ Slower learning. Booms are gradual (long and slow).

Evidence of business cycle learning

- Asymmetric learning argument predicts lower precision of forecasts in recessions than in booms.
- Regress detrended GDP on median forecast error and they find *negative correlation*.

Approach

- Embed asymmetric learning model in DSGE model.
- Show how much of the skewness observed in data can be explained by a model of asymmetric learning.

Key function

- Production technology faces 2 shocks, both unobservable, z_t and η_t^i

$$y_t^i = z_t f(k_t^i, n_t^i) + \eta_t^i$$

- Let $f(k_t^i, n_t^i) = 1$. Aggregating across N_t firms we get $y_t = z_t N_t + \sum_{i=1}^{N_t} \eta_t^i$.
- Signal-to-noise ratio is defined as $Var(z_t N_t)/Var(\sum_{i=1}^{N_t} \eta_t^i)$.
  - Assume η_t^i are i.i.d.. Signal-to-noise ratio is increasing in N_t.
  - Implication: Signal to noise ratio is *procyclical*. Easier to extract signal during booms than in recessions.

Preferences

- Infinitely lived representative consumer derives utility from consumption and leisure

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t^s)$$

- Agents are endowed with initial capital and 1 unit of time in each period.

Technology

- Competitive firms have the following technology

$$y_t = z_t f(k_t, n_t) + \eta_t,$$

where

  - $$\eta_t \sim i.i.d.N(0, \sigma_\eta^2)$$
  - z_t is a two-state Markov switching process with the transition matrix Π.

Information structure

- $F_t = \{y^{t-1}, c^{t-1}, d^{t-1}, k^t, i^t, n^{d,t}, n^{s,t}, \theta^t, w^t, p^t)$ is information available at time t.
- $P(z^i_{t-1}|F_t)$: Use Bayes law to forecast z_{t-1} given F_t.
- $P(z^i_t|F_t)$: Use Π to adjust for the possibility of state change at date t.
- Calculated the expected value of the unobserved technology

$$\hat{z}_t = P(z^H_t|F_t)z^H + P(z^L_t|F_t)z^L$$

Household problem

- At date t, households
  - have beliefs about technology $\hat{z}_t$,
  - have stocks that they purchased last period θ_t,
  - take w_t and p_t as given and
  - choose n_t^s and θ_{t+1} to maximize expected lifetime utility subject to budget constraint

$$c_t + p_t\theta_{t+1} \leq w_t n_t^s + p_t\theta_t + d_t\theta_{t+1}$$

- z_t and η_t is realized. Output y_t is produced and dividends are paid.
- Consumption is residual and absorbs unexpected shocks.

Firm problem

At date t firms maximize lifetime expected shareholders value subject to

- dividend determination equation

$$d_t = z_t f(k_t, n_t) + \eta_t - w_t n_t^d - i_t,$$

- capital accumulation equation

$$k_{t+1} = (1 - \delta)k_t + i_t.$$

Competitive equilibrium

Given k_0 and initial distribution of z_0, equilibrium is a sequence of quantities $\{y^t, c^t, i_t, k_t, n_t^d, n_t^s, \theta^t\}_{t=0}^{\infty}$ and prices $\{w^t, p^t\}_{t=0}^{\infty}$ such that

- Households solve household problem.
- Firms solve firm problem.
- Markets for goods, labor, and firms shares clear.

$$y_t = c_t + i_t; n_t^s = n_t^d; \theta_t = 1$$

Planner Problem

Recursive formulation

$$V(k_t, \hat{z}_t) = max_{i_t,n_t}[u(c_t, 1 - n_t) + \beta E[V(k_{t+1}, \hat{z_{t+1}})|F_t]]$$

subject to

- $$k_{t+1} = (1 - \delta)k_t + i_t,$$
- $$c_t = z_t f(k_t, n_t) + \eta_t - i_t,$$
- updating rules for unobserved technology z_t.

Simplifying assumption: No "active" learning.

Data, Calibration and Computation

- Data: 1952:1-2002:1.
- Detrending: Compute variance of percentage deviations from H-P trend. Skewness is of first-differenced log series.
- Calibration
  - Basic parameters are $\delta = 0.0186$, $\beta = 0.98$, $\sigma = 0.386$, $\alpha = 0.34$, and $\phi = 4$.
  - Parameters related with learning are
    - Parameters in Π are chosen such that the implied autocorrelation matches 0.95 (Hansen and Prescott (1995))
    - $(z^H, z^L) = (1 + 0.032, 1 - 0.032)$
    - $\sigma_\eta = 0.02$

	standard deviation	relative std deviation	first-order autocorrelation	correlation with y	skewness
	Panel A: Data				
y	1.69	1.00	0.85	1.00	-0.40
inv	7.41	4.39	0.79	0.89	-0.72
n	1.55	0.92	0.89	0.85	-0.16
k	0.27	0.16	0.96	0.31	-0.11
c	1.26	0.74	0.84	0.89	-0.33
	Panel B: No Learning Model				
$\hat{y}$	1.63	1.00	0.71	1.00	-0.23
	(0.028)	(0.000)	(0.004)	(0.000)	(0.177)
inv	6.34	3.88	0.70	0.97	-0.24
	(0.109)	(0.012)	(0.004)	(0.013)	(0.177)
n	0.57	0.35	0.70	0.97	-0.16
	(0.010)	(0.002)	(0.004)	(0.009)	(0.176)
k	0.41	0.30	0.95	0.04	-0.01
	(0.007)	(0.026)	(0.001)	(0.009)	(0.052)
$\hat{c}$	0.80	0.50	0.72	0.99	-0.18
	(0.014)	(0.008)	(0.004)	(0.000)	(0.167)
	Panel C: Learning Model				
$\hat{y}$	1.52	1.00	0.79	1.00	-0.41
	(0.022)	(0.000)	(0.004)	(0.000)	(0.084)
inv	6.15	3.98	0.70	0.82	-0.42
	(0.088)	(0.007)	(0.006)	(0.004)	(0.096)
n	0.55	0.36	0.70	0.82	-0.33
	(0.008)	(0.001)	(0.006)	(0.004)	(0.097)
k	0.40	0.26	0.96	-0.15	-0.04
	(0.006)	(0.001)	(0.001)	(0.004)	(0.055)
$\hat{c}$	1.09	0.72	0.29	0.82	-0.22
	(0.014)	(0.003)	(0.009)	(0.001)	(0.079)