

# Neoclassical Miracles

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October 2007 - Preliminary

## Abstract

We study the dynamic behavior of a Neoclassical growth model with finite lifetimes and imperfect altruism in which individuals can accumulate both physical and human capital. We use the model to better understand and explain the behavior of economic miracles. Our results suggest that standard Neoclassical forces can account for the performance of the miracle economies and can explain the protracted transition that is inconsistent with the predictions of a Neoclassical set-up with only physical capital. The model is also consistent with the dramatic rise in investments in physical and human capital that these miracle economies experienced.

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\*We thank NSF for financial support. Seshadri thanks the Alfred P Sloan Foundation for financial support.

# 1 Introduction

In any empirical analysis of cross country economic performance it is easy to find a few episodes of fast growth, as well as many instances of economic stagnation. A major challenge for economic theory is to identify what are the driving forces behind the successes and failures. Ultimately, the objective of the theory is to come up with a recipe that a country can use to produce economic miracles. No amount of atheoretical empirical work can discover the engines of growth. Since there are plenty of theoretical models that can, on paper, produce economic miracles, it is necessary to better understand the implications of these models for how an economy responds to shocks, as a prerequisite to finding the growth silver bullet.

To the extent that alternative theoretical models have different implications for the evidence, it seems that a reasonable way to evaluate those models is to quantify their predictions for economic variables that can be measured. The ability of the simple one sector neoclassical growth model to account for growth observations —along a transition path— came into question based on the results of King and Rebelo (1993). Their findings show that if capital share is of the order that we observe in the data, the model implies fast convergence and large changes in the marginal product of capital. On the other hand, if (broadly defined) capital share is large, then convergence is very slow.

Following Lucas (1993), there has been renewed interest in understanding growth miracles. Lucas, tentatively concluded that human capital —including schooling and an unmeasured quality dimension— is the key to understanding episodes of fast growth. At the same time, work by Young (1995) and others quantified the evolution of several economic variables,

investment, years of education in the work force and measures of productivity, among others, for the fast growing countries of East Asia. The ultimate goal was to determine the contribution of each factor to the growth miracles. Since measurement along the lines of Young assumes a particular economic model, alternative specifications of the economy can render the conclusions suspect.

One of the major issues in understanding growth miracles is the identification of the class of models that can be consistent with the observed miracles. In particular, substantial attention has been given to the possibility that the standard one sector neoclassical growth model, that has proved very successful replicating business cycle data, can also account for episodes of fast growth and stagnation. Prescott and Hayashi (2002) argued that low TFP growth explains Japan's poor performance since the early 1990s. More recently, Chen, İmrohoroglu and İmrohoroglu (2006) find that the simple model can also account for the large differences in saving rates between U.S. and Japan. Other studies, e.g. Chang and Hornstein (2007) and Papa-georgiu and Perez-Sebastian (2005), suggest that models that deviate from the standard model are needed to explain the economic performance of East Asian countries.

In this paper, we revisit the neoclassical growth model. We follow Lucas' suggestion (but not his model) that human capital has both a quality and a quantity dimension, and that on-the-job training is an important component of aggregate human capital. Moreover, we take the mortality of human beings seriously and restrict the model so that human (but not physical) capital owned by an individual completely depreciates at death. In order to model human capital in a way that is consistent with observed age-earnings profiles —which we view as the prime evidence on the effect of human

capital over a lifetime— we use a version of the Ben-Porath (1967) model.

We perform several experiments. First, as in King and Rebelo, we study the predictions of the model to one time shocks to exogenous variables. We consider changes in (actual) TFP, fertility and the relative price of capital. Our major finding is that the model produces adjustment paths that are quite different from those identified by King and Rebelo, even though our common parameters are chose to have similar values. In particular, we estimate that the response of investment to a one time shock in TFP is hump shaped, and not decreasing. Thus, the behavior of the saving rate in some growing economies could, potentially, be consistent with simple productivity shocks. We also find that conventionally measured TFP increases by much more than actual TFP, and the increase is distributed over a much longer period of time. Of course, the difference between actual and measured productivity is driven by changes in the quality of human capital, both due to changes in the quality of schooling and in the amount of on-the-job training.

Once and for all decreases in fertility (that correspond to a change in population growth rate from 4% to 2%) have a large impact on output, with just a small fraction of the ultimate change occurring in the first 10 years. As in the case of a TFP shock, the reason lies in the behavior of human capital. A demographic shock ultimately implies a lower effective interest rate and this induces a higher investment in physical and human capital.

In both of our one shock experiments we find that, after the first period in the case of a TFP shock, the response of the growth rate of output per worker is hump shaped: There is a small increase in the 10 years following the shock, and then another period —approximately lasting 10 years as well— in which the growth rate increases. Finally, the growth rate settles into its long run value of 0. We use the model to decompose the growth

experience of the “average” of the fast growing economies of the Far East and we find that demographic change accounts for over 30% of the observed increase in output per worker, while pure TFP shocks explain approximately 44% of the change. The residual is due to the joint effects.

We also use the model to evaluate how well it reproduces the economic performance of the Asian Tigers and some Latin American economies. When we pick TFP to match output per worker and we use observed demographic data, the model is broadly consistent with the evidence from the East Asian economies. Even though the fit is far from perfect, the model is able to reproduce the positive association between investment rates (in physical capital) and schooling.

Finally, we ask whether opening up the economy can lead to large aggregate changes. We find that it can. Opening up the economy in 1960 would lead to large aggregate effects and can account for more than 50% of the change in output per worker between 1960 and 2000.

## 2 Model

The model uses the same technology as in Manuelli and Seshadri (2007). We view the economy as being populated by overlapping generations of individuals who live for  $T$  periods. The time line is the following: After birth, say at time  $t_0$ , an individual remains attached to his parent until he is  $I$  years old (at time  $t_0 + I$ ); at that point he creates his own family and has, at age  $B$  (i.e. at time  $t_0 + B$ ),  $e^{f(t_0+B)}$  children that, at time  $t_0 + B + I$ , leave the parent’s home to become independent.

The utility functional of a parent who has  $h$  units of human capital, and initial wealth (a bequest from his parents) equal to  $b$ , at age  $I$ , in period  $t$

is given by

$$\begin{aligned}
V^P(h, b, t) = & \int_I^T e^{-\rho(a-I)} u[c(a, t + a - I)] da + e^{-\alpha_0 + \alpha_1 f(t+B-I)} \quad (1) \\
& \int_0^I e^{-\rho(a+B-I)} u[c_k(a, t + B - I + a)] da \\
& + e^{-\alpha_0 + \alpha_1 f(t+B-I)} e^{-\rho B} V^k(h_k(I), b_k, g_k, t + B),
\end{aligned}$$

where  $c(a, t)$  [ $c_k(a, t)$ ] is consumption of a parent (child) of age  $a$  at time  $t$ . The term  $f(t)$  denotes the log of the number of children born at time  $t$ .

We assume that parents are imperfectly altruistic: The contribution to the parent's utility of a unit of utility allocated to an  $a$  year old child attached to him is  $e^{-\alpha_0 + \alpha_1 f(t+B-I)} e^{-\rho(a+B-I)}$ , since at that time the parent is  $a + B$  years old. In this formulation,  $e^{-\alpha_0 + \alpha_1 f(t+B-I)}$  captures the degree of altruism. If  $\alpha_0 = 0$ , and  $\alpha_1 = 1$ , the preference structure is similar to that in the infinitely-lived agent model. Positive values of  $\alpha_0$ , and values of  $\alpha_1$  less than 1 capture the degree of imperfect altruism. The term  $V^k(h_k(I), b_k, g_k, t + B)$  stands for the utility of the child at the time he becomes independent.

Each parent maximizes  $V^P(h, b, t)$  subject to two types of constraints: the budget constraint, and the production function for human capital. The

former is given by

$$\begin{aligned}
& \int_I^T e^{-\int_t^{t+a-I} r(s)ds} c(a, t+a-I) da + \\
& e^{f(t+B-I)} \int_0^I e^{-\int_t^{t+a+B-I} r(s)ds} c_k(a, t+B-I+a) da \\
& + \int_I^R e^{-\int_t^{t+a-I} r(s)ds} x(a, t+a-I) da + \\
& e^{f(t+B-I)} \int_0^I e^{-\int_t^{t+a+B-I} r(s)ds} x_k(a, t+B-I+a) da + \\
& e^{f(t+B-I)} e^{-\int_t^{t+B} r(s)ds} b_k + e^{f(t+B-I)} e^{-\int_t^{t+B+6-I} r(s)ds} x_E \\
\leq & \int_I^R e^{-\int_t^{t+a-I} r(s)ds} w(t+a-I) h(a, t+a-I) (1-n(a, t+a-I)) da \\
& + e^{f(t+B-I)} \int_6^I e^{-\int_t^{t+a+B-I} r(s)ds} [w(t+a+B-I) \\
& h_k(a, t+B-I+a) (1-n_k(a, t+B-I+a))] da + b.
\end{aligned} \tag{2}$$

Since we are interested in understanding transition effects, we allow the interest rate and the wage rate to vary over time.

We adopt Ben-Porath's (1967) formulation of the human capital production technology, augmented with an early childhood period. Specifically, we assume that (ignoring the temporal dependence to simplify notation)

$$\dot{h}(a) = z_h [n(a)h(a)]^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \quad a \in [I, R) \tag{3}$$

$$\dot{h}_k(a) = z_h [n_k(a)h_k(a)]^{\gamma_1} x_k(a)^{\gamma_2} - \delta_h h_k(a), \quad a \in [6, I) \tag{4}$$

$$h_k(6) = h_B x_E^v, \tag{5}$$

$$h(I) \quad \text{given}, \quad 0 < \gamma_i < 1, \quad \gamma = \gamma_1 + \gamma_2 < 1,$$

Even if there is perfect altruism, we assume that when an individual dies, his human capital dies with him. Thus, the depreciation rate is 100% at age  $T$ .

If asset transfers are not constrained, the income maximization and utility maximization problems can be solved independently. In this case, it is optimal for an individual to maximize the present discounted value of net income. We assume that each agent retires at age  $R \leq T$ . The maximization problem for an agent born at time  $t$  is

$$\max_{h,n,x} \int_6^R e^{-\int_{t+6}^{t+a-6} r(s)ds} e^{-r(a-6)} [w(t+a-6)h(a)(1-n(a)) - x(a)] da - x_E \quad (6)$$

subject to

$$\dot{h}(a) = z_h [n(a)h(a)]^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \quad a \in [6, R), \quad (7)$$

and

$$h(6) = h_E = h_B x_E^v \quad (8)$$

with  $h_B$  given. Equations (7) and (8) correspond to the standard human capital accumulation model initially developed by Ben-Porath (1967). This formulation allows for both market goods,  $x(a)$ , and a fraction  $n(a)$  of the individual's human capital, to be inputs in the production of human capital. Investments in early childhood, which we denote by  $x_E$  (e.g. medical care, nutrition and development of learning skills), determine the level of each individual's human capital at age 6,  $h(6)$ , or  $h_E$  for short.<sup>1</sup> This formulation captures the idea that nutrition and health care are important determinants of early levels of human capital, and those inputs are, basically, market goods.<sup>2</sup>

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<sup>1</sup>It should be made clear that market goods ( $x(a)$  and  $x_E$ ) are produced using the same technology as the final goods production function. Hence the production function for human capital is more labor intensive than the final goods technology.

<sup>2</sup>It is clear that parents' time is also important. However, given exogenous fertility, it seems best to ignore this dimension. For a full discussion see Manuelli and Seshadri (2006b).

The solution to the problem is such that  $n(a) = 1$ , for  $a \leq 6 + s(t)$ . Thus, we identify  $s(t)$  as *years of schooling* of the cohort born at time  $t$ . In the stationary case, i.e.  $r(s) = r$  and  $w(s) = w$ , Manuelli and Seshadri (2006)) characterize  $s$  and  $h(s + 6)$ .

An important property of the solution from the point of view of the exercise in this paper is the role played by the real wage. Imagine that technological improvements (or other shocks) results in a higher level of equilibrium wages. This —given  $\gamma_2 - v(1 - \gamma_1) > 0$  which is satisfied in our specification— induces individuals to stay in school longer (i.e.  $s$  increases) and to acquire more human capital per unit of schooling.

In the stationary case, if  $h(s + 6)$  is the amount of human capital that an individual has at age  $6 + s$  (i.e. at the end of the schooling period), it follows that

$$\frac{dh(s + 6)}{dw} = \frac{\partial h(s + 6)}{\partial s} \frac{ds}{dw} + \frac{\partial h(s + 6)}{\partial w}.$$

The first term on the right hand side can be interpreted as the effect of changes in the wage rate on the *quantity* of human capital (years of schooling), while the second term captures the impact on the level of human capital per year of schooling, a measure of *quality*. Direct calculations (see Manuelli and Seshadri (2006)) show that the elasticity of quality with respect to the wage rate is  $\gamma_2/(1 - \gamma)$ , which is fairly large in our preferred parameterization.<sup>3</sup> This result illustrates one of the major implications of the approach that we take in measuring human capital in this paper: differences in years of schooling are not perfect (or even good in some cases) measures of differences in the stock of human capital. Cross-country differences in the quality of schooling can be large, and depend on the level of development. If the

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<sup>3</sup>To be precise, we find that  $\gamma_2 = 0.33$ , and  $\gamma = 0.93$ . Thus the elasticity of the quality of human capital with respect to wages is 4.71.

human capital production technology is ‘close’ to constant returns, then the model will predict large cross country differences in human capital even if TFP differences are small.<sup>4</sup>

It is possible to show that, in the steady state, the interest rate must satisfy

$$r = \rho + [\alpha_0 + (1 - \alpha_1)f]/B.$$

It follows that decreases in fertility result in lower the relevant interest rate. This has three effects. First, it lowers the cost of capital inducing increases in the capital-human capital ratio which, in general, results in higher levels of output per worker. Second, it lowers the opportunity cost of staying in school. As a result, individuals choose to invest more in schooling and to allocate more resources to on the job training. This implies that the effective amount of human capital in the economy increases. Finally, negative fertility shocks have an impact on the age structure of the population. The relevant effect is that the fraction of high human capital individuals —i.e. those in the peak earning years— increases and this, in turn, contributes to an overall increase in the amount of effective labor available in the economy

The last shock that we study is a change in the (relative) price of capital. In the steady state, the condition that pins down the capital-human capital ratio requires that the cost of capital equal its marginal product. In symbols, this corresponds to

$$p_k(t)[r(t) + \delta_k] = z(t)F_k(\kappa(t), 1), \quad (9)$$

where  $\kappa(t)$  is the physical capital - human capital ratio. Thus, a decrease in the price of capital has a direct impact on the physical capital - human

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<sup>4</sup>It can be shown that the elasticity of quality with respect to TFP is  $\gamma_2/[(1-\theta)(1-\gamma)]$ , where  $\theta$  is capital share.

capital ratio. This, in turn, increases the wage rate per unit of human capital and induces more investment in human capital. Even though during the transition the interest rate can respond to the changes in price of capital, in the steady state it is pinned down by demographic factors and, as such, does not add to the effect of  $p_k$

## 2.1 Equilibrium

Given the interest rate, standard profit maximization pins down the equilibrium capital-human capital ratio. However to determine output per worker, it is necessary to compute ‘average’ human capital in the economy. Since we are dealing with finite lifetimes —and full depreciation of human capital— there is no aggregate version of the law of motion of human capital since the amount of human capital supplied to the market depends on an individual’s age (see the expressions in the Appendix). Thus, to compute average ‘effective’ human capital we need to determine the age structure of the population.

**Demographics** We assume that, at time  $t$ , each  $B$  year old individual has  $e^{f(t)}$  children at age  $B$ . Thus, the total mass of individuals of age  $a$  at time  $t$  satisfies

$$\begin{aligned} N(a; t) &= e^{f(t-a)} N(B; t-a), \\ N(t', t) &= 0, \quad t' > T. \end{aligned}$$

If the economy converges to a steady state (as we assume), the birth rate,  $f(t)$ , converges to  $f$ . In this case, the steady state measure of the populations satisfies

$$N(a, t) = \phi(a)e^{\eta t}, \tag{10}$$

where

$$\phi(a) = \eta \frac{e^{-\eta a}}{1 - e^{-\eta T}}, \quad (11)$$

and  $\eta = f/B$  is the (long run) growth rate of population.

**Aggregation** To compute total output it is necessary to estimate the aggregate amount of human capital effectively supplied to the market, and the physical capital - human capital ratio. Effective human capital,  $H^e(t)$  is

$$H^e(t) = \int_{\delta+s}^R h(a, t)(1 - n(a, t))dN(a; t).$$

This formulation shows that, even if  $R$  —the retirement age— is constant, changes in the fertility rate can have an impact on the average stock of human capital.

**Equilibrium** Optimization on the part of firms implies that

$$p_k(r(t) + \delta_k) = z(t)F_k(\kappa(t), 1), \quad (12)$$

where  $\kappa(t)$  is the physical capital - human capital ratio. The wage rate per unit of human capital,  $w$ , is,

$$w(t) = z(t)F_h(\kappa(t), 1). \quad (13)$$

Then, feasibility requires

$$\int_0^T [c(a, t) + x(a, t)]dN(a; t) + \dot{K}(t) \leq [z(t)F_h(\kappa(t), 1) - \delta_k]H^e(t),$$

where, given the specification of age, it is no longer necessary to distinguish between parent and children variables.

### 3 Calibration

We use standard functional forms. The production function is assumed to be Cobb-Douglas

$$F(k, h) = zk^\theta h^{1-\theta},$$

and the utility function is given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

Our calibration strategy involves choosing the parameters so that the steady state implications of the model economy presented above are *consistent with observations for the United States* (circa 2000). When we apply the model to the study of other economies we only vary  $z$ , which we identify as TFP.

Following Cooley and Prescott (1995), the depreciation rate is set at  $\delta_k = .06$ . We set  $\sigma = 3$ . Not much information is available on the fraction of job training expenditures that are not reflected in wages. There are several reasons why earnings ought not to be equated with  $wh(1-n) - x$ . First, some part of the training is off the job and directly paid for by the individual. Second, firms typically obtain a tax break on the expenditures incurred on training. Consequently, the government (and indirectly, the individual through higher taxes) pays for the training and this component is not reflected in wages. Third, some of the training may be firm specific, in which case the employer is likely to bear the cost of the training, since the employer benefits more than the individual does through the incidence of such training. Finally, there is probably some smoothing of wage receipts in the data and consequently, the individual's marginal productivity profile is likely to be steeper than the individual's wage profile. For all these reasons,

we set  $\pi = 0.5$ .<sup>5</sup> We also assume that the same fraction  $\pi$  is not measured in GDP.

Our theory implies that it is only the ratio  $h_B^{1-\gamma}/(z_h^{1-v}w^{\gamma_2-v(1-\gamma_1)})$  that matters for all the moments of interest. Consequently, we can choose  $z$ ,  $p_k$  (which determine  $w$ ) and  $h_B$  arbitrarily and calibrate  $z_h$  to match a desired moment. The calibrated values of  $z_h$  and  $h_B$  are common to all countries. Thus, the model does not assume any cross-country differences in an individual's ‘ability to learn,’ or initial endowment of human capital. We set  $B = 25$  and  $R = \min\{64, T\}$ . We also assume that  $\rho = 0.04$ . This leaves us with 9 parameters,  $\theta, r, \delta_h, z_h, \gamma_1, \gamma_2, v, \alpha_0$  and  $\alpha_1$ . The moments we seek in order to pin down these parameters are:

1. Capital’s share of income of 0.33. Source: NIPA
2. Capital output ratio of 2.52. Source: NIPA
3. Earnings at age R/Earnings at age 55 of 0.8. Source: SSA
4. Earnings at age 50/Earnings at age 25 of 2.17. Source: SSA
5. Years of schooling of 12.08. Source: Barro and Lee
6. Schooling expenditures as a fraction of GDP of 3.77. Source: OECD, Education at a Glance.
7. Pre-primary expenditures per pupil relative to GDP per capita of 0.14. Source: OECD, Education at a Glance.
8. Interest rate of 7%.

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<sup>5</sup>If we were to take the view that  $\pi = 1$ , our estimate of the returns to scale,  $\gamma = \gamma_1 + \gamma_2$  increases to 0.96 thereby further increasing the elasticity of output with respect to TFP. In a sense, choosing  $\pi = 0.5$  understates our case.

9. Lifetime Intergenerational Transfers/GDP of 4.5%. Gale and Scholz, 1994

The previous equations correspond to moments of the model when evaluated at the steady state. This, calibration requires us to solve a system of 9 equations in 9 unknowns. The resulting parameter values are

Parameter	$\theta$	$r$	$\delta_h$	$z_h$	$\gamma_1$	$\gamma_2$	$\nu$	$\alpha_0$	$\alpha_1$
Value	0.315	0.07	0.018	0.361	0.63	0.3	0.55	0.75	0.55

## 4 One Time Shocks and Growth

In this section we describe the impact of a one time shock to different exogenous variables on the equilibrium level of output per worker, investment in both schooling and capital relative to output, and years of schooling. We assume that, initially, the economy is in the steady state—which we associate with 1960—and we shock it with a once and for all change in the relevant variable. We then trace the dynamic effects over the following 60 years.

In order to concentrate on economies that have experienced significant growth, we take our base economy to be the average of the fast growing East Asian countries: Malaysia, South Korea, Singapore and Hong Kong. That is, we use the U.S. calibration and pick  $z$ —our measure of TFP—so that it implies an output level that is, relative to the U.S., similar to that of those countries when we use their average demographic characteristics.

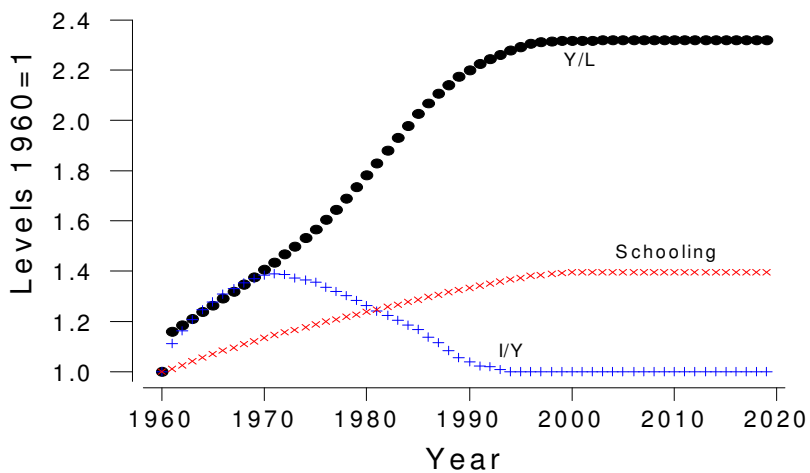


Figure 1: Transitional Dynamics: TFP shock

#### 4.1 A Shock to TFP

We study the impact of a once and for all TFP shock that, in the long run, is such that output per worker increases by 132%. For our calibrated economy, this requires a “true” increase in TFP of the order of 17%. The graph below presents the results for output per worker ( $Y/L$ ), years of schooling ( $s$ ), and the investment-output ratio ( $I/Y$ ). All values have been normalized to one in the initial period (as per our convention this is 1960)

There are a couple of noteworthy deviations from the standard growth model. First, it takes a long time (more than 30 years) for output per worker to reach the new steady state. Second, the response of investment is not decreasing —as we expect from the one sector neoclassical growth model—

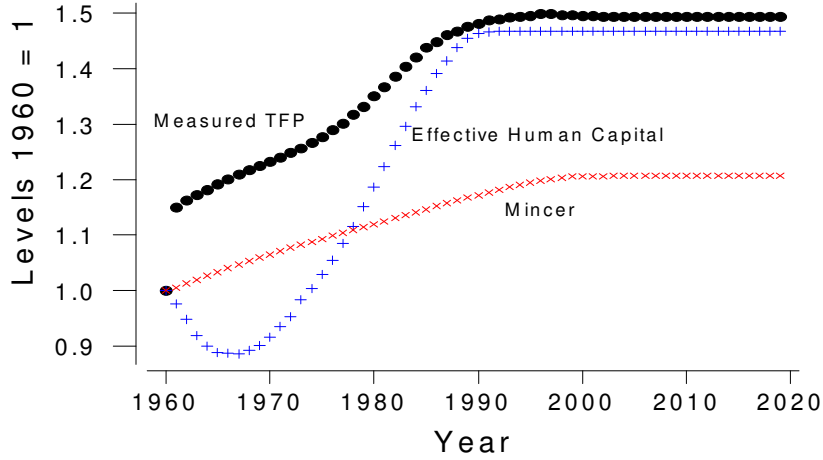


Figure 2: TFP Shock. Effective Human Capital, Mincerian Human Capital, and Measured TFP

but it increases for a period of 10 years, and then it goes back to its pre-shock level over the next 20 years. The reason for this non-monotonicity lies in the equilibrium response of effective human capital to a TFP shock. Recall that, in the standard model, investment rises to induce the economy to reach the new capital-labor ratio. This increase is moderated by higher interest rates, but convergence is monotone. In the context of this model, there is another force that induces the capital labor ratio to rise: a decrease in the effective amount of human capital allocated to market production. The behavior of effective human capital,  $h^e(t) = H^e(t)/N(6 + s(t); t)$ , is displayed in Figure 2.

The somewhat surprising finding is that effective human capital initially decreases. This is due to two forces. First, there is a change in labor force participation due to individuals choosing to acquire more schooling. Quantitatively this effect is a small (especially in the first 10 years). The second force is that the increase, contemporaneous and expected, in real wages per unit of human capital, induces more on the job training, especially among younger individuals. Of course, the decrease in effective human capital reduces the capital-labor ratio and, consequently, requires a smaller level of investment in physical capital.<sup>6</sup>

In Figure 2 we also report an alternative measure of human capital which we label “Mincerian.” This notion,  $h^m$ , is computed according to

$$h^m(t) = h_0^m e^{\phi s(t)},$$

where  $\phi$  is often associated with the return to education as estimated in a Mincer regression. For this calculation we chose  $\phi = 0.10$ , but the results are very similar for slightly higher and lower values. We take  $s$  to be the average years of schooling of the individuals in the work force. The interesting observation is that Mincerian human capital is increasing from the very beginning. Thus, an observer that studies the first 10 years after the shock, would conclude that the return to education is very low.

Finally, we computed “measured TFP.” This is the productivity measure that one would recover had one used the Mincerian measure of human capital without adjusting for on the job training and quality changes (i.e. changes in  $h(6 + s)$ ). Following the initial jump —which corresponds to the actual

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<sup>6</sup>The result is reminiscent of the effect of technological change described in Greenwood and Yorokoglu (1997) even though, in our setting, there is no complementarity between technology and skill: workers are simply responding to higher returns to human capital.

change in TFP— this measure exhibits an upward slow trend. Using this estimate one would conclude that productivity increased close to 50%, and that it takes 25 years to reach 90% of its final value. Given the difference between this measure and the actual path of TFP—which, of course, is constant following an initial jump—we are hesitant to use existing measures of productivity as driving shocks in the model.

It is easy to see how mismeasurement of human capital can account for this behavior. Let  $\eta_x$  denote a growth rate of any variable  $x$ . Let effective human capital per worker,  $h^e(t)$ , be decomposed as

$$h^e(t) = \bar{h}(t)e^{\phi s(t)}.$$

In this context,  $\bar{h}(t)$  can be viewed as a measure of quality of human capital. Simple algebra shows that the growth rate of measured TFP,  $\hat{z}$ , is given by

$$\eta_{\hat{z}} = \eta_z + (1 - \theta)\eta_{\bar{h}}.$$

In our experiment,  $\eta_z = 0$  after the first period. Consequently, standard growth accounting would identify the change in quality as a productivity improvement. This, of course, tends to overstate the quantitative impact of any given productivity change.

Given the one time shock that we study, the model predicts an instantaneous (and fairly large) jump in the growth rate. However, unlike the standard model, it also implies a growth acceleration that occurs approximately 10 years into the experiment. For our parameterized economy the relevant growth rate (omitting the first term) is displayed in Figure 3.

The growth rate remains constant (around 2%) for the first 10 years. It increases up to 2.8% over the following 10 years, and then it decreases to

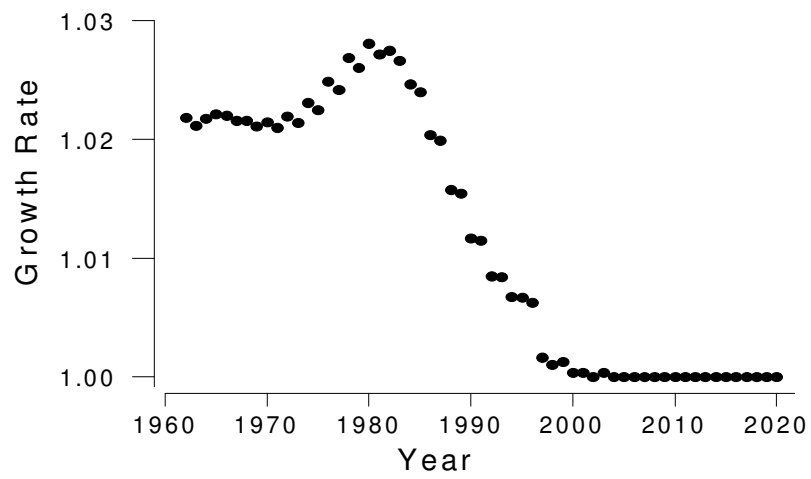


Figure 3: TFP Shock. Transitional Dynamics: Growth Rate of Output per Worker

its long run level of 0. The acceleration is being driven by the increase in effective human capital (see Figure 2), corresponding to the higher quality as a result of the previous investment in on-the-job training, and that the new generations acquire more human capital per year of schooling.

Since changes in TFP cannot induce any changes in the capital-output ratio, we now explore the role of demographic shocks that can, through their impact on interest rates, generate such a change.

## 4.2 A Decrease in Population Growth

In this section we study a once and for all decrease in the log of the number of children per person,  $f$ . We change  $f$  —the log of the number of children per person— so that the population growth,  $\eta = f/B$ , which we take to be initially 4%, decreases to 2%. This corresponds to a decrease in the fertility rate ( $2e^f$ ) from 5.44 to 3.3. In this exercise we assume that the average age of conception,  $B$ , is unchanged. This, is not neutral and in future work we plan to explore the effect of changing  $B$ .

The results of this experiment for the base case are presented in Figure 4.

Unlike a TFP shock, a decrease in the number of children has a relatively small effect in the short run: After 9 years output per worker has increased only 10%. Investment in capital goods and years of schooling increase in the same proportion. However, schooling expenditures relative to output (not reported in the Figure) increases by 20 %, reflecting the anticipation of a needed increase in quality.

After 20 years, the investment-output ratio settles into its new, long run level which, for our parameterization, is 21% (the initial level was 17%).

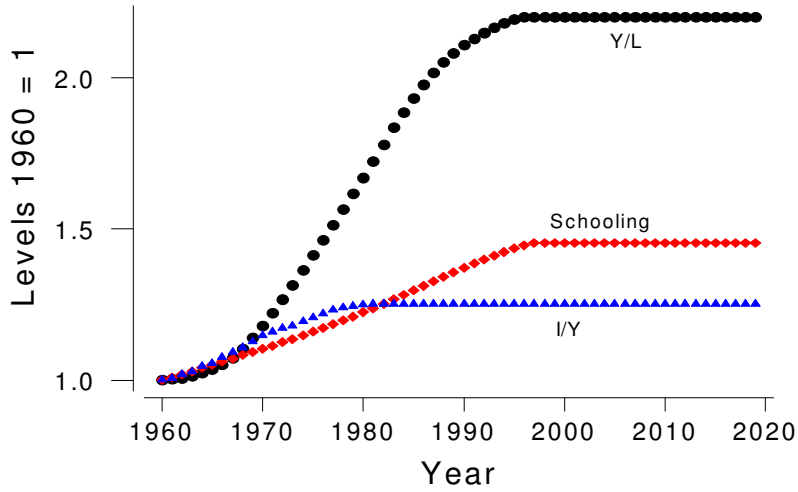


Figure 4: Demographic Shock: Transitional Dynamics

This corresponds to a 56% increase in the capital output ratio, a result that is consistent with the evidence from fast growing economies.<sup>7</sup>

In Figure 5 we report the predictions of the model for effective human capital per worker, its Mincerian counterpart and measured TFP.

The pattern that emerges is similar to the case of a TFP shock. The only significant difference is that the anticipated decrease in interest rates is not strong enough to result in a decrease in  $h^e(t)$ .

As in the case of a TFP shock, the response of the growth rate of output is not monotonic. In Figure 6 we present the results for our base parameterization. The growth rate—which we interpret as the deviation over the long

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<sup>7</sup>This corresponds to the capital-output ratio measured in domestic prices. For this concept, changes in the price of capital have no effect on the ratio.

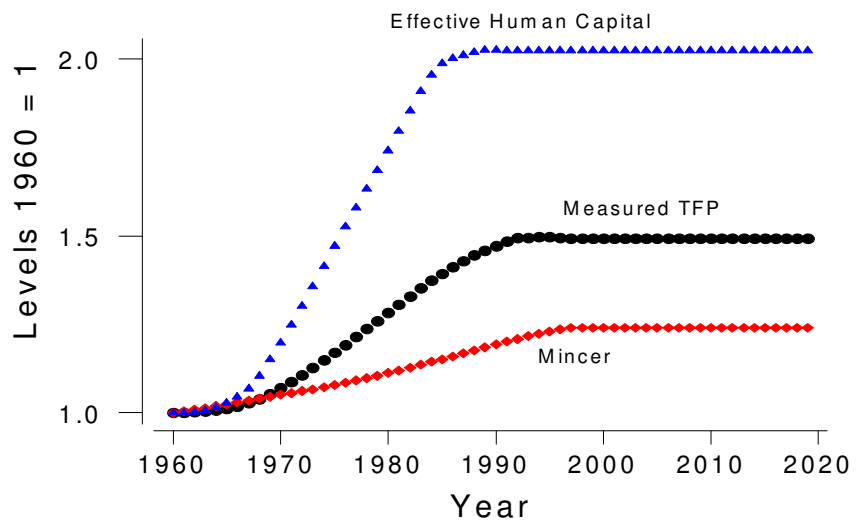


Figure 5: Demographic Shock: Measured TFP, Mincerian and Effective Human Capital

run growth rate— is fairly low for the first 7 years, but it accelerates in the following 7 years. It peaks, at 3.5%, in year 14. The behavior of investment and schooling (see Figure 4) cannot account for this. As in the previous case, growth is influenced by changes in human capital quality. That is, growth accelerates when the higher level of effective human capital is supplied to the market sector.

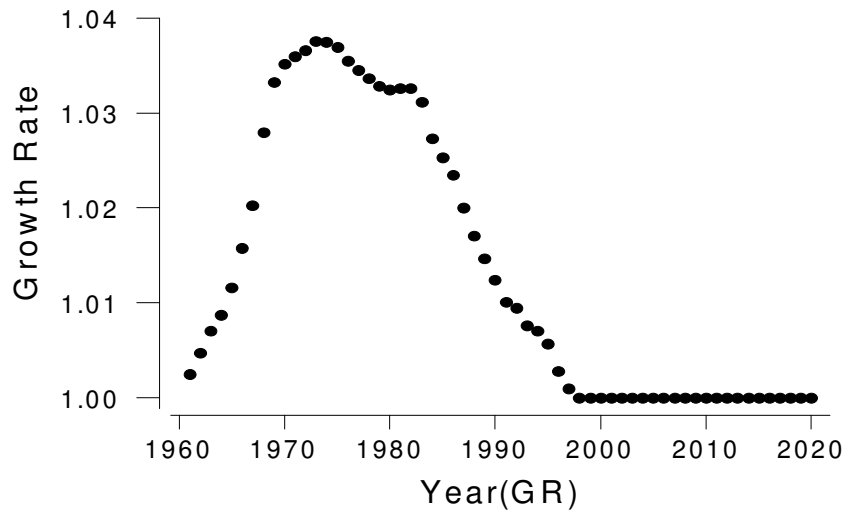


Figure 6: Demographic Shock: Growth Rate of Output per Worker

### 4.3 A Decrease in the Price of Capital

In this section we report the effect of a one time decrease in the price of capital,  $p_k$ , such that, in the long run, output per worker increases by 160%. The results are displayed in Figure 7

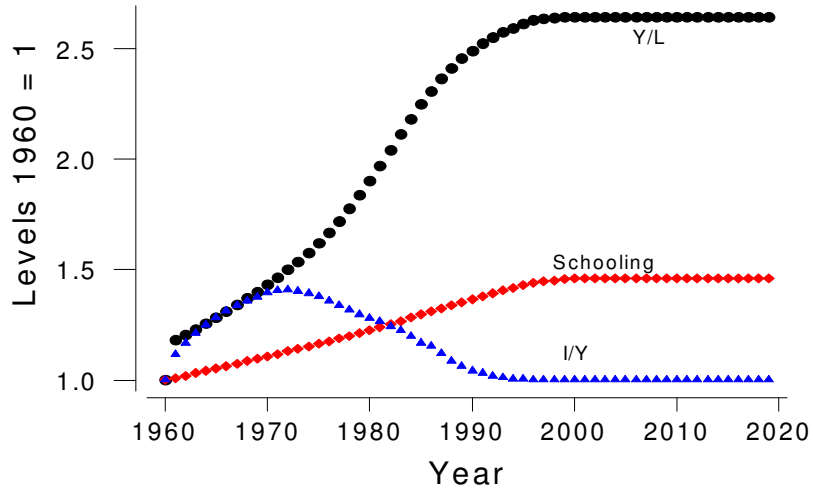


Figure 7: Price of Capital Shock: Transitional Dynamics

Qualitatively, the predictions of the model are similar to those obtained in the case of a TFP shock, both for levels and growth rate (See Figure 8)

#### 4.4 Fast Growers: A First Pass

In order to obtain a more precise picture of the relative importance of TFP and demographic shocks, we now study a version of their combined impact. As a first pass to study the fast growing countries of East Asia, we calibrate the level of TFP ( $z$ ) so that the predictions of the model for output per worker relative to the U.S. matches the *average* of the East Asian fast growing countries. We then analyze the effects of two shocks: a once and for all TFP shock —as in the previous section— coupled with the actual

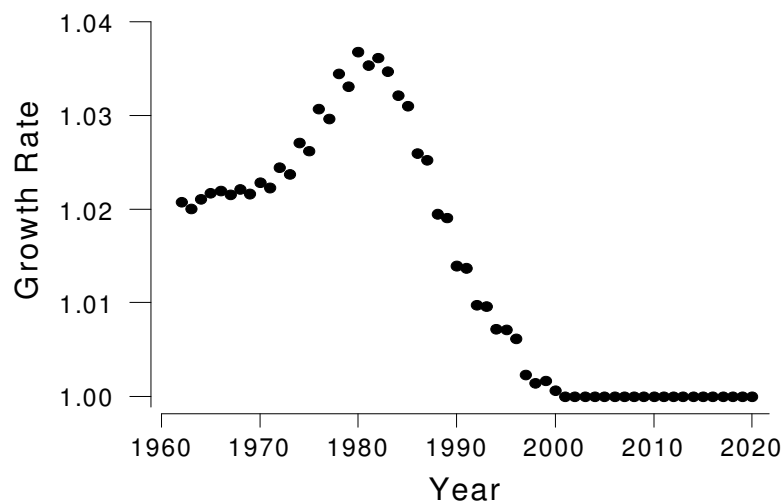


Figure 8: Price of Capital Shock: Growth Rate of Output per Worker

changes in fertility. We pick the size of the TFP shock so that, in the long run, output per worker displays a 7.7 fold increase (which corresponds to the average increase in output per worker across the miracle economies). Our demographic shock is such that the population growth rate decreases from 3.83% in 1960 to 1.68% in 2000, which is an average of the years surrounding the beginning and the end of the period under study.

The results of this experiment are presented in Figure 9. Output per worker evolves along an S-shaped path, which implies a delayed response of the growth rate to the shocks. The reason is, as before, that it takes time to accumulate human capital: After the economy is hit with a TFP shock, agents find it optimal to increase their stock of human capital. New cohorts

go to school longer than did their older counterparts. Individuals who are already working now engage in more on the job training. All this implies that the stock of human capital takes time to respond. This slows down the transition to the new steady state. The model is able to capture the rise in schooling, as well as the dramatic rise in the investment to GDP ratio.

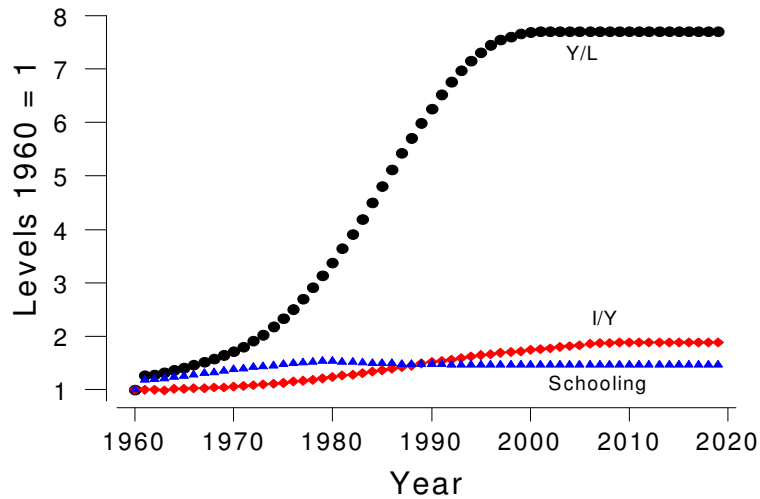


Figure 9: TFP and Fertility Shocks: Transitional Dynamics

As in the case of the one time shocks, we use the model to estimate TFP assuming a Mincerian human capital production function. The results are in Figure 10. For this experiment, the increase in TFP induces a decrease in effective human capital. It takes more than 10 years for  $h^e$  to get back to its pre-shock level. In the mean time, the increase in output is due to the TFP shock and capital accumulation.

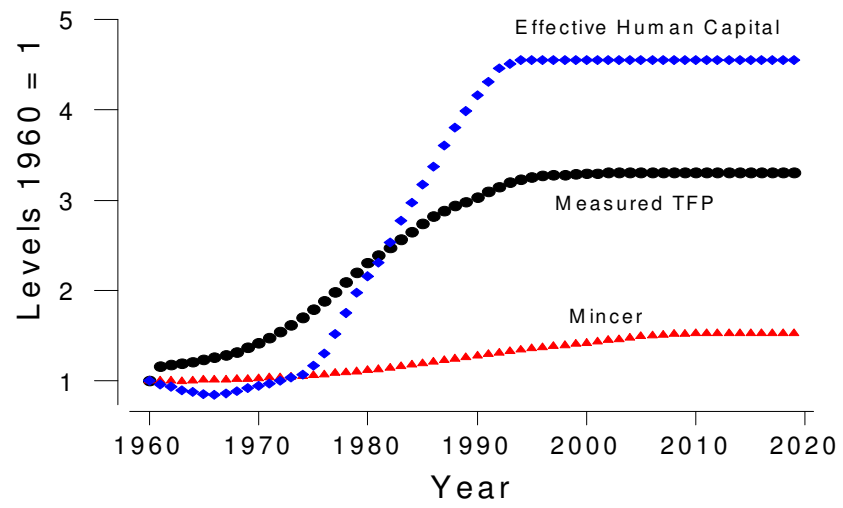


Figure 10: Fast Growers: Measured TFP, Mincerian and Effective Human Capital

What is the relative importance of the two shocks? In get a sense of the relative effects of the two shocks, we present the transitional dynamics for this economy when it is hit “one shock at a time.” The results are in Figures 11 and 12. The total effect is not the sum of the individual shocks as these exercises ignore interactions. For our base case, approximately 30% of the increase in output per worker is due to the change in population, 44% to the change in TFP, and the rest to the interaction between those two shocks.

The response of schooling to both shocks is similar, quantitatively and qualitatively. The investment-output ratio responds quite differently, exhibiting a temporary increase in the case of a pure TFP shock and a permanent increase associated with a decrease in fertility.

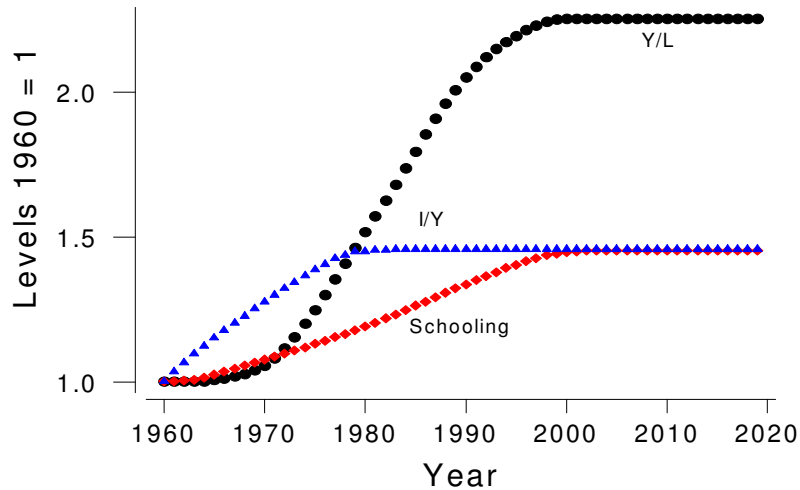


Figure 11: Fertility Shock: Transitional Dynamics

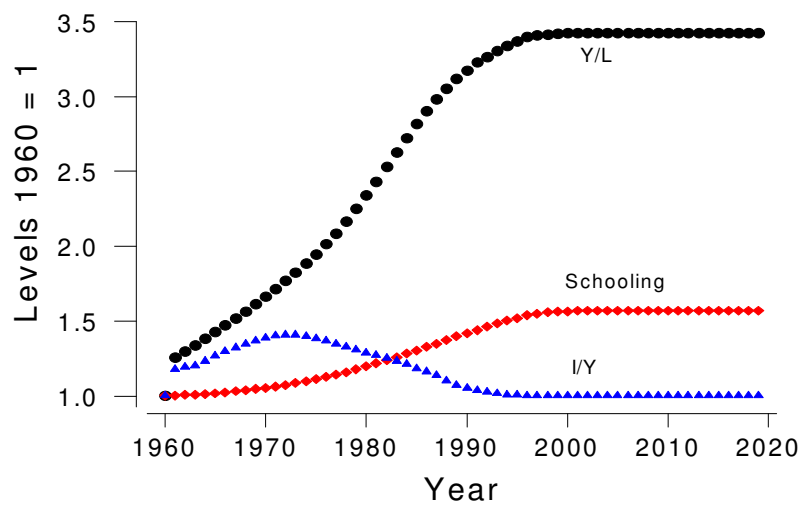


Figure 12: TFP Shock: Transitional Dynamics

If we interpret the effect of TFP as capturing transitional dynamics, then we must conclude that transitional dynamics cannot be ruled out as explanations for the growth performance of this group of countries.

The implications of these two shocks (plus the combined effect) on the growth rate (omitting the first term corresponding to a jump) follow a familiar pattern (see Figure 13), with the demographic shock being mostly responsible for the hump shape.

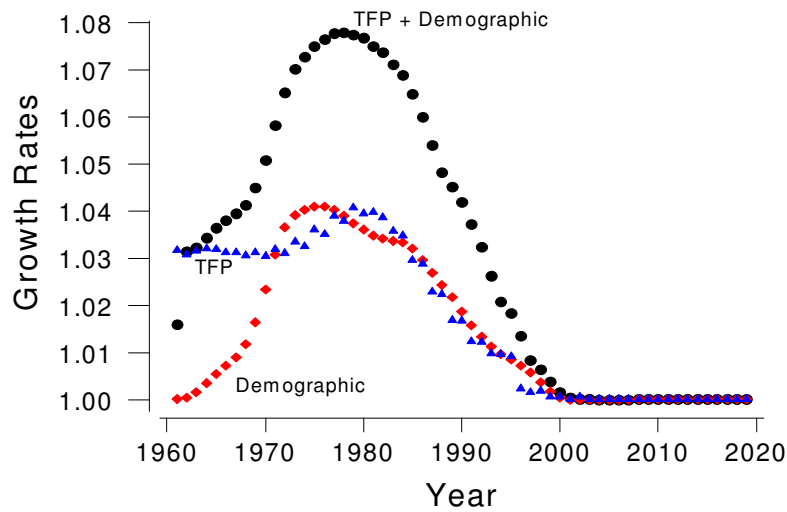


Figure 13: TFP and Demographic Shocks: Dynamics of the Growth Rate of Output per Worker

## 5 Growth Episodes

In this section we study how well the growth model can account for growth episodes driven by technology shocks while taking into account demographic changes. To this end, we choose the path of TFP for each country from 1960 to 2000 so that *output per worker in the model matches a seven year moving average in the data*. We assume that TFP remains constant after 2000 (to compute a steady state). Moreover, we change the demographic variables to match each country's data. We consider the closed economy case.

### 5.1 East Asia

We first consider the fast growing economies of East Asia. In particular, we examine the implications of the model for Singapore, Hong Kong, Malaysia, Taiwan and Korea. In all cases, we assume 1960 is a steady state (this turns out not to be crucial).

The following table presents the results of the experiment. We report the predictions for the total change in output per worker (which in this experiment is perfectly matched by the model), the change in the investment-output ratio, the change in actual ( $z$ ) and measured ( $\hat{z}$ )TFP (using the methodology of the previous section), as well as the predictions for years of schooling, when  $f$  is chosen to match measured fertility.

	$\Delta(Y/L)$	Years of Schooling				$\Delta(I/Y)$		$\Delta(z)$	$\Delta(\hat{z})$
	Data	Data		Model		Data	Model	Model	Model
Country		1960	2000	1960	2000				
Sing.	6.6	3.14	8.12	3.32	8.74	1.65	1.83	1.15	2.08
H.K.	9.09	4.74	9.47	4.42	8.95	0.89	1.91	1.20	2.71
Mal.	4.49	2.34	7.88	3.02	6.32	1.62	1.49	1.13	1.97
Taiwan	10.14	3.32	8.53	2.58	7.93	1.68	1.72	1.22	2.86
Korea	8.05	3.23	10.46	2.99	8.93	2.67	2.13	1.17	2.19

As can be seen, the model's predictions for years of schooling in 2000 line up reasonably well with the data. The predictions for the investment-output ratio are also quite reasonable with one exception: Hong Kong. The reason, in the context of the model, is simple: Lower fertility induces a decrease in the interest rate and this, in turn, results in a higher investment-output ratio (in the steady state). However, as is well known (e.g. Young (1992)), the investment ratio in Hong Kong has not shown any trend.

An alternative approach to matching demographic change is to pick  $f$  so that the model's implication for the growth rate of population match the data. This, it turns out, makes a significant difference. In Figures ?? and ?? we present for Hong Kong and Taiwan the time series for schooling and the investment ratio implied by the model as well as actual schooling (taken from Barro-Lee) for this case. It is clear that the model is consistent with a substantial increase in schooling but it tends to overpredict schooling in 1960. Thus, relative to the level of human capital, output was too low. Since this crucially depends on the specification of demographic change we will analyze this more carefully in the next version.

## 5.2 The Latin American Experience

Is the model consistent with the Latin American experience? To analyze the predictions of the model for each of the Latin American economies, we conduct the same experiment between 1960 and 2000 and examine the predictions for years of schooling and investment for both 1960 and 2000.

The results are in the following table.

	GDP per worker		Years of Schooling			
			Data		Model	
Country	1960	2000	1960	2000	1960	2000
Argentina	18732.67	25670.27	4.99	8.49	4.53	7.92
Bolivia	6811.40	6829.04	4.22	5.54	4.32	4.76
Brazil	7376.20	19220.33	2.83	4.56	2.55	5.31
Chile	11693.66	25083.56	4.99	7.89	4.01	7.24
Colombia	8240.58	11477.08	2.97	5.01	3.25	4.23
Costa Rica	11326.14	14826.86	3.86	6.01	3.45	5.14
Ecuador	6115.14	10903.19	2.95	6.52	2.67	5.26
Mexico	13357.17	24588.29	2.41	6.73	2.54	5.83
Paraguay	7366.88	10438.59	3.35	5.74	3.52	4.79
Peru	10087.53	10094.90	3.02	7.33	2.96	3.64
Uruguay	14484.33	21149.76	5.03	7.25	5.24	7.57
Venezuela	25292.39	17754.19	2.53	5.61	2.34	1.60

In most countries, the predictions of the model and the data are roughly consistent. There are two notable exceptions: Peru and Venezuela. In Venezuela, output per worker went down while schooling rose, while in Peru output per worker stayed virtually constant while schooling rose.

On the other hand, in countries that did not experience extreme shocks the results are reasonable, but by no means perfect. In particular, it seems that the model tends to underpredict the increase in the years of schooling in almost all the countries. This suggests that there may be other forces at work.

We now present detailed results for two countries: Argentina and Chile.

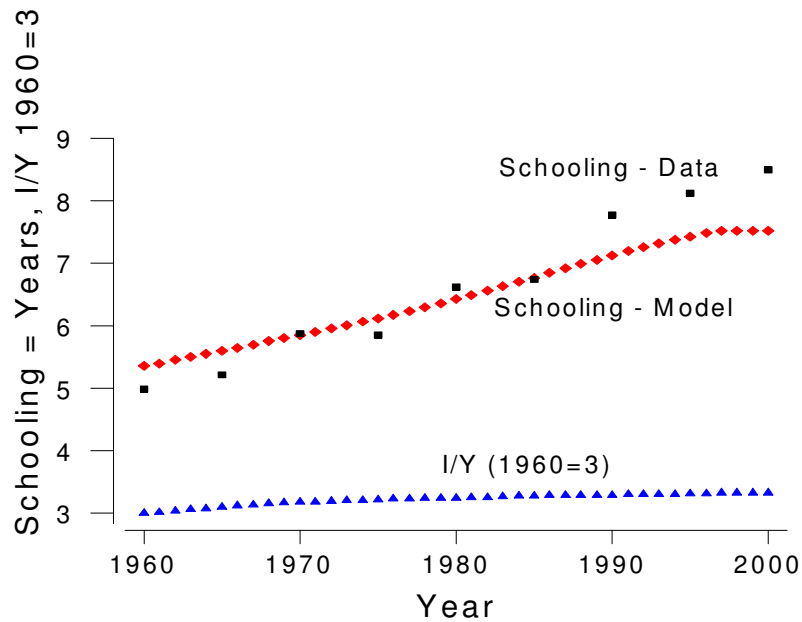


Figure 14: Argentina: Schooling and Investment

In both cases the model seems to do a reasonable job of tracking the changes in schooling, but it fails to reproduce the path of the investment-output ratio. Unlike the fast growing countries of East Asia, TFP changes play a minor role in the cases of Chile and Argentina. To be precise, the total change (over a 40 year period) of actual ( $z$ ) TFP is essentially 0 in

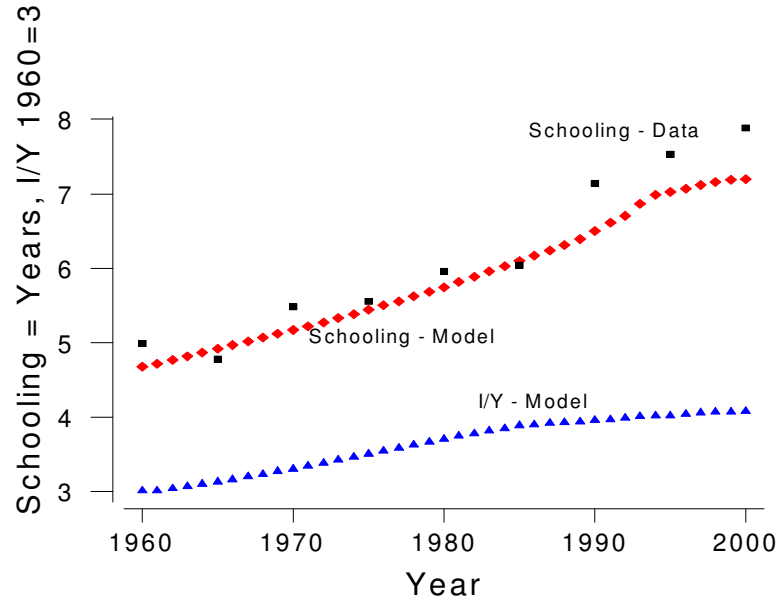


Figure 15: Chile: Schooling and Investment

both Chile and Argentina. Total, over a 40 year period, growth in measured TFP ( $\hat{z}$ ) 3.6% in Argentina and 2% in Chile. This absence of productivity improvements coupled with a negative (albeit small) fertility shock results in increasing investment-output ratios. The data show no trend in the case of Argentina and a U-shape relationship in the case of Chile. In both cases, investment is measured in domestic prices.

## 6 Opening up the economy

Up until now, we have considered the effects of exogenous changes in productivity and demographics on the evolution of macroeconomic aggregates. This, of course begs the question, what caused the miracle? What is behind

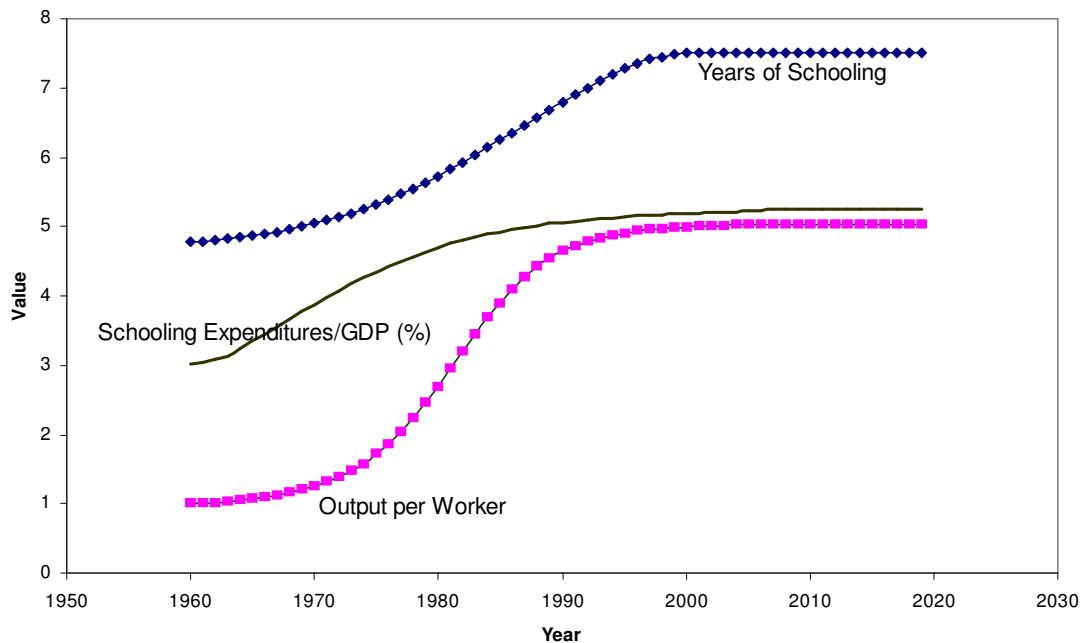


Figure 16: The Effect of Opening up the Economy

the TFP shock? In what follows, we examine the implications of opening up the economy to world capital markets. In particular, we assume that in 1960, the economy is closed. Suddenly, the economy is opened up and the real interest rates now correspond to that of the American economy. We then analyze the transitional path as the economy approaches the new steady state.

There are a few noteworthy features of the effects of opening up the economy. First, note that the aggregate consequences are large. Output per worker increases by almost a factor of five. Second, the gains are not immediate even though the economy is opened up all of a sudden. Finally,

this exogenous change increases investment in physical and human capital, exactly the experience of the East Asian tigers. This experiment demonstrates that a substantial part of the performance of the East Asian tigers could well be attributed to the opening up of the economy.

## **7 Concluding Comments**

In this paper we ask whether the Neoclassical paradigm is consistent with the performance of the miracle economies. We find that it is. In particular, we find that a standard Neoclassical set-up with finitely-lived individuals who accumulate human capital is consistent with protracted transitions and a rise in investments in human and physical capital. Unlike in the model without human capital, a one time shock to productivity can generate ‘slow’ convergence to the new steady state. We also make some progress toward understanding what caused the miracle. Preliminary results indicate that a substantial part of the increase in output per worker could well have been the consequence of opening up the economy.

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