INTERNATIONAL ECONOMIC REVIEW
Vol. 41, No. 1, February 2000

PUBLIC EDUCATION, OCCUPATIONAL CHOICE, AND THE GROWTH-INEQUALITY RELATIONSHIP*

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This article develops a dynamic general equilibrium model in which the occupational structure of the economy drives a wedge between the social and private returns to schooling for some workers. I study the impacts of alternative allocations of public resources between basic and higher levels of education on enrollments, income distribution, and growth. In particular, I illustrate how the growth-inequality relationship depends on the tension between two forces: (1) the "trickle-down" effects of expenditures on higher education and (2) the positive impacts on secondary enrollments generated by high-quality basic education and reduced parental inequality.

1. INTRODUCTION

Perhaps the most commonly cited determinant of differences in national economic growth rates is the average level of schooling of the working population (e.g., Benhabib and Spiegel, 1994). While this is often interpreted as reflecting differences in investment levels by representative households, a more accurate interpretation is that in many countries a large fraction of the population does not acquire education beyond the primary level. Since differences in educational attainment are known to be key determinants of income disparities,[2] the incentives provided by the education system are a crucial link between growth and inequality. This article examines the implications of education policy for growth and income distribution when agents face nonconvexities in the private returns to publicly funded education.

A number of recent articles also investigate how the nature of the education system affects the general equilibrium relationship between income distribution and the growth rate. Glomm and Ravikumar (1992) compare the growth-inequality relationship under private and public education regimes. Bénabou (1996a) considers the macroeconomic effects of educational financing at local, state, and national levels. In contrast, this article focuses on the role of heterogeneous incentives to acquire education created by the interaction of two structural features of the economy: (1)

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[1] I am grateful to Dan Bernhardt, Phil Bodman, Burton Hollifield, David Love, Aloysius Siow, Matt Turner, and seminar participants at Queen's, Toronto, Windsor, Victoria, the 1994 Canadian Macroeconomics Study Group meetings, and the 1995 Econometric Society European meetings for their comments. All remaining errors and omissions are my own. Funding from SSHRC is gratefully acknowledged.

[2] Park, et al. (1996), for example, find that 25 percent of the gap in inequality of pay between Brazil and Korea could be explained by differences in the educational attainment of the labor force.

the allocation of limited public resources across different levels of education and (2) asymmetries in the nature of occupational compensation.

Although very few governments charge for their educational services, different income groups do not benefit from them equally. As Jimenez (1986) points out, "Free provision is not free consumption": To benefit from public services, a household has to incur private costs, including transport costs and the opportunity cost of the student's time in terms of home production. Moreover, parents also incur the costs of other inputs to human capital (e.g., nutrition, books, and private tutoring) that complement public spending. For these reasons, even if innate ability is randomly distributed, children from richer backgrounds tend to get further through the education system. These distributional effects are often compounded by the skewed allocation of public resources between basic and higher levels of education. Mingat and Tan (1985) find that for developing countries as a whole, an average of 71 percent of the population in each generation receive primary schooling, but they receive only 22 percent of the resources devoted to education. In contrast, the 6 percent who attain higher education receive 39 percent of the resources.

One justification for allocating more resources to higher education is that this may create important positive spillovers or "trickle-down effects" that generate growth that is beneficial to all workers. An often stated objective of the higher education system is to create "... a class of educated leaders to fill vacancies ... in governmental services, public corporations, private businesses and professions" (Todaro, 1985:346). However, many studies suggest that the returns to public investment in basic education are far higher (see Psacharopoulos, 1985). The recent development experiences of South Korea (high growth, low inequality) and Venezuela (low growth, high inequality) highlight the potential consequences of alternative allocations of public resources across different levels of education. Although both countries spent a similar percentage of their 1985 GDP on education, Korea allocated just 10 percent of its public education budget to higher education, but Venezuela allocated 43 percent (Birdsall, Ross and Sabot, 1995).

To study these issues, this article develops a two-period overlapping generations growth model. When young, agents receive an educational investment from their parents. The marginal impact of parental investments on in-school productivity is greatest for children from poorer backgrounds and declines with household income. The young also receive the minimum level of basic education necessary to undertake any occupation and then choose whether to acquire further education via the secondary school and/or university system. Human capital thus depends on the quality of each level of education as well as the investment made by parents. The quality of education depends, in turn, on public investment financed by direct taxation. When old, agents use their human capital in one of two kinds of occupation: high-skilled, managerial occupations or low-skilled, wage-laboring occupations. The human capital of each high-skilled worker, embodying both formal education and raw ability, is combined with as much wage labor as desired at the equilibrium wage rate. Agents optimally allocate their lifetime earnings between consumption and the human capital of their offspring.

The key distinction between the two occupations is that high-skilled agents can fully appropriate the returns to their education, but low-skilled production workers cannot.

This is so because while the human capital of all workers contributes to total factor productivity, only high-skilled occupations offer salaries that depend on the individual characteristics of the worker. Low-skilled occupations offer wages that depend only on overall productivity.[3] Since the acquisition of higher education requires a discrete investment of time, students who anticipate employment in low-skilled occupations have little incentive to acquire education beyond basic levels.[4] The asymmetry in the nature of occupational compensation thus drives a wedge between the private and social returns to schooling for some workers and results in a nonconvexity that affects enrollment.

Income inequality affects the equilibrium experienced by the next generation in two ways. First, because of decreasing returns in the impact of parental inputs, a (mean-preserving) reduction in household income dispersion raises the average in-school productivity of children. This raises the average productivity of the work force and results in reduced income dispersion. This *productivity effect* is similar to those considered by Glomm and Ravikumar (1992) and Bénabou (1996a). Second, a reduction in income dispersion increases the fraction of agents from low-income households entering higher education by more than it decreases the fraction of agents from high-income households doing so. As a result, enrollments rise, the supply of high-skilled workers expands, and aggregate production increases. The presence of this *occupational effect* plays a crucial role (see below) and is consistent with Figure 1, which depicts the statistically significant negative impact of household income inequality, measured by a Gini coefficient, and contemporaneous enrollment in secondary school across 35 countries.[5] Williamson (1993) documents similar empirical regularities.

The model enables us to detail the immediate, transitional, and long-run impacts of transferring resources between education levels. Because high-skilled and low-skilled workers are complementary factors of production, an equilibrium "trickle-down effect" results from a transfer of resources toward higher education. However, this is offset by the associated reduction in the quality of basic education. Even if the human capital of high-skilled agents rises as a result of the reallocation, marginal agents with relatively low levels of potential human capital drop out of the education system. As a result, enrollments fall, and income inequality grows. If relative inequality increases, the average in-school productivity of the subsequent generation and enrollments decline. This, in turn, raises the level of inequality among that generation so that more of their offspring enroll in higher education, and so on. It follows that

[3] One motivation for this assumption is that higher education provides screening information. For agents with only basic education, there is no such prescreening, so the costs to a firm of sorting are high relative to the benefits. The quantitative importance of education in screening workers has been pointed out by Boissière, et al. (1985).

[4] Birdsall, et al. (1995:500) observe, "...children from low-income households learn from experience that no matter how great their effort, it will be insufficient to compensate them for the low quality of the schools they attend. In contrast, poor children in high-quality schools who see tangible rewards for effort are more likely to make such an effort."

[5] Enrollment data are taken from the World Bank Statistical Tables. Gini coefficients are derived from quintile data reported in the World Development Report (various issues). The significant negative impact remains after controlling for real per capita GDP.

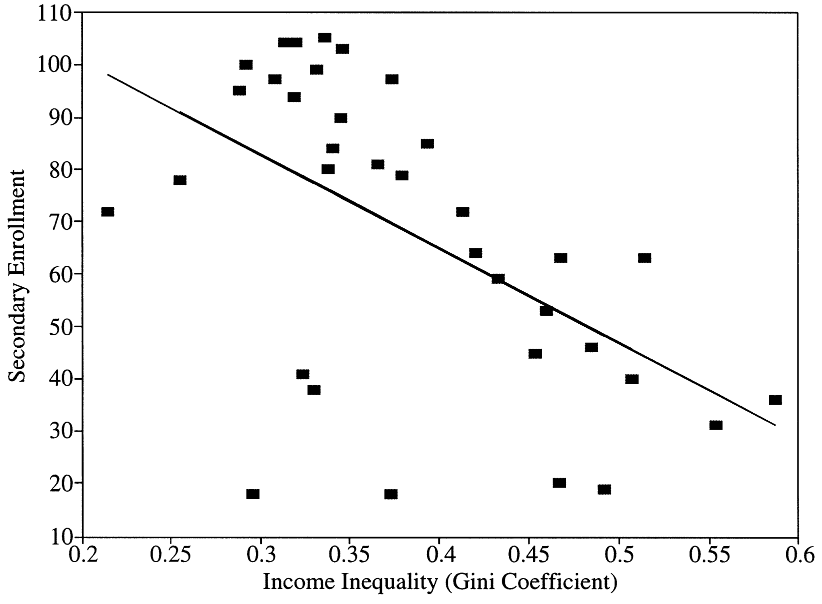

FIGURE 1

INEQUALITY AND SECONDARY ENROLLMENT

reallocations of public resources that improve the quality of higher education at the expense of basic education can result in reduced enrollments in higher education, greater inequality, and lower growth.

With decreasing returns in the impact of parental income, the distribution of *relative* incomes converges to a stationary distribution, and the growth rate converges to a constant function of structural and policy parameters. The long-run relationship between growth and inequality resulting from different allocations of public resources is nonmonotonic. As resources are shifted from higher to basic education, the negative growth effects of the reduction in the quality of higher education are initially outweighed by the positive growth effects of increased enrollments and reduced parental inequality. Eventually, however, the growth benefits of "leveling the playing field" are offset by the costs resulting from the reduced productivity of high-skilled workers. Thus long-run growth initially increases and then declines with the level of inequality.

The model also provides a tractable synthesis of some key aspects of other general equilibrium theories of growth and income distribution. In the models of Glomm and Ravikumar (1992) and Bénabou (1996a), tractability is ensured by the linearity of individual transitions and the assumption of an initial lognormal distribution of human capital. The presence of occupational asymmetries implies that individual transitions are nonlinear and that the Markov process is nonstationary. Such nonlinearities are also present in the work of Galor and Zeira (1993), Aghion and Bolton (1997), Banerjee and Newman (1993), and Lloyd-Ellis and Bernhardt (1995). However, these models do not lend themselves to a tractable analysis of the interaction between inequality and sustained growth. The distribution of income in these models converges to a stationary distribution, and there is no long-run growth. The model considered here offers a tractable analysis of (1) the convergence of the endogenous, nonlinear, nonstationary Markov process describing the evolution of *relative incomes* and (2) the interaction between the endogenous distribution of income and the *growth rate.*

The outline of this article is as follows: Section 2 sets out the structure of the economy, and the optimal behavior of individual agents is derived in Section 3. Section 4 characterizes the macroeconomic equilibrium, and Section 5 analyzes the transitional dynamics of the distribution of income. Section 6 characterizes the implications of alternative allocations of public resources across different levels of education for enrollments, growth, and the distribution of income. All proofs and derivations are contained in the Appendix.

2. THE MODEL

The economy consists of a continuum of family lineages whose generations overlap. There is a unit measure of households, each of which consists of an old agent and a young agent. When young, each agent receives a parental investment plus the minimum level of basic education required to undertake any occupation. When old, each agent works and reproduces one agent so that the economy's population is constant over time. The agent allocates his or her lifetime income toward his or her own consumption and to expand the resources of his or her offspring.

Agents are assumed to have identical preferences

$$u(\lambda_t, c_t, e_t) = \log(1 - \lambda_t) + (1 - \beta)\log c_t + \beta \log e_{t+1} \tag{1}$$

where $0 < \beta < 1$, λ_t is the effort exerted in acquiring higher education, c_t is consumption, and e_{t+1} is the parental investment in offspring. Thus parents value the investment in their offspring independently of the impact that it has on income.[6]

Students acquire human capital through the formal education system according to

$$h_t(\lambda_t, e_{t-1}, E_{bt}, E_{ht}) = \left[\theta_b E_{bt}^{\mu} + \lambda_t \theta_h E_{ht}^{\mu}\right]^{\rho/\mu} e_t^{1-\rho} \tag{2}$$

where $0 < \rho < 1$ and $0 < \mu < 1$. Here, E_{bt} represents the basic level of education received by all young agents and E_{ht} represent the higher education acquired by those agents who exert positive effort λ.[7] The productivity of the student at both levels of

[6] It is straightforward to show that having parents value their child's expected human capital does not change any of the results since, in equilibrium, expected human capital is log linear in e_t. Assuming that parents value their child's income or utility significantly complicates the equilibrium analysis. However, provided that parents cannot borrow against their childen's expected income or insure them against idiosyncratic shocks to their income, these alternatives should not change the qualitative results of the article.

[7] One could model the choice of effort in basic education. However, if one unit of effort is the minimum requirement for any occupation, the reduced form would be identical to that considered here.

schooling depends on the investment e_t made by his or her parent. To emphasize the role of different levels of education, I assume that the time spent in school is discrete: Either students leave the education system immediately after the basic level, or they attend both levels of education. The assumed functional form captures key elements of the education system in a simple way. There are decreasing returns to public investments at each level of education, $\mu < 1$, and decreasing returns to parental inputs. However, there are constant returns to scale to all inputs into human capital formation.[8]

Public education is financed via taxes on parental income. If the aggregate income of parents is Y_{t-1} and the income tax rate is τ, then total revenue is τY_{t-1}. We assume that the shares of total revenue allocated to basic and higher education are constant and are given by x_b and x_h respectively. The education received by each student in higher education depends on aggregate public expenditures in the following way:

$$E_{ht} = \frac{x_h \tau Y_{t-1}}{S_t^\alpha} \tag{3}$$

where $0 \leq \alpha \leq 1$ and S_t denotes the total number of students enrolled in higher education. If $\alpha = 1$, then the quality of higher education depends only on spending per student enrolled. If, instead, $\alpha < 1$, then expenditure need not increase in proportion to enrollment to maintain quality. This relationship captures the possibility that there may be public good aspects to some components of public spending on higher education.[9] In the case of basic education, this is a nonissue because everyone always receives it. Thus basic education is simply

$$E_{bt} = x_b \tau Y_{t-1} \tag{4}$$

Combining these assumptions with Equation (2), we get

$$h_t(\lambda_t, e_t, Y_{t-1}) = \left[\theta_b x_b^\mu + \lambda_t \theta_h \left(\frac{x_h}{S_t^\alpha}\right)^\mu\right]^{\rho/\mu} (\tau Y_{t-1})^\rho \, e_t^{1-\rho} \tag{5}$$

When they become economically active, agents are distinguished by two characteristics: the human capital acquired through formal education h and their innate ability z. Ability is drawn from a time-invariant uniform distribution with support [0, 1]. The distribution of human capital is determined endogenously, as described below.

The production side of the economy parallels that described by Lucas (1978) and Jovanovic (1993). There are a continuum of production units each of which requires a single nonproduction worker (a manager) together with l_t production workers in order to produce. I refer to these as *firms*, but they could be thought of as projects, plants, or departments of a firm. Each firm produces a single consumption good according to the common Cobb-Douglas production technology:

$$f(z, h, H_t, l_t) = (zh)^{1-\delta}(H_t l_t)^\delta \tag{6}$$

[8] Although the assumed education system resembles that considered by Eckstein and Zilcha (1994), there are crucial differences. There are two levels of education, students choose their own education beyond the basic level, and parental inputs are complementary.

[9] None of the results that follow depend qualitatively on the value of α.

where l_t is the number of production workers at the firm, H_t is the average human capital of production workers, h is the manager's human capital, and z is his or her innate ability. Thus zh may be regarded as the managerial capital of the firm. There is perfect competition among firms in the market for managers, so they receive all the rents from production.[10] Note that I implicitly assume that firms do not sort low-skilled workers by their human capital levels so that the average human capital is the same across all firms.[11] If $\delta = 0$, only the high-skilled occupation would be necessary for production. This special case provides a useful benchmark for considering the role of occupational asymmetries.

There is no individual or aggregate uncertainty. At the beginning of each period, young agents observe their basic education level and their own innate ability and choose whether or not to acquire the higher education needed to enter the skilled occupation. If his or her managerial capital is sufficient, then an agent becomes a manager. If not, he or she works as a wage laborer at the equilibrium wage w_t. The parental input e, innate ability z, schooling effort λ, and occupational choice of an agent, together with public expenditures, at each education level x_b and x_h determines his or her lifetime income, given the equilibrium obtaining during that period, $y(z, e_{t+1}, \lambda_t, x_b, x_h, w_t)$.

3. OPTIMAL BEHAVIOR

In this section I consider the optimal behavior of individuals, taking the equilibrium low-skilled wage as given. I solve the individual agent's optimization problem in two stages. When old, an agent that earned income y_t and faces a tax rate τ chooses c_t and e_{t+1} to maximize Equation (1) subject to the second-period budget constraint:

$$c_t + e_{t+1} = (1 - \tau)y_t \tag{7}$$

The optimal consumption and parental investment policies are, respectively,

$$c(y_t) = (1 - \beta)(1 - \tau)y_t \qquad e(y_t) = \beta(1 - \tau)y_t \tag{8}$$

Thus each parent expends a constant share of his or her after-tax income on his or her offspring.

When young, given his or her basic schooling and his or her parent's investment and after observing his or her own innate ability, the agent decides whether to continue on to formal higher education. This decision depends on a comparison of the utility received in wage labor and that received in the skilled occupation after having exerted the optimal level of effort. His or her income when old depends on the occupation he or she chooses and on the effort exerted in the education sector.

A firm that hires a manager with formal education h and ability z chooses the quantity of production workers it hires to solve

$$\max_{l_t} \, (zh)^{1-\delta}(H_t l_t)^{\delta} - w_t l_t - v \tag{9}$$

[10] An alternative formulation might interpret the manager as an entrepreneur setting up a firm and hiring labor himself or herself. The resulting equilibrium would be identical.

[11] See footnote 4.

where w_t is the period t equilibrium wage rate and v is the salary paid to the manager. The firm's labor demand at the wage rate w_t is therefore

$$l_t(z, h) = \left(\frac{\delta H_t}{w_t}\right)^{1/(1-\delta)} \frac{zh}{H_t} \tag{10}$$

With perfect competition in the market for managers, the firm earns zero profits, and the manager receives all the rent associated with his or her managerial capital as salary:

$$v_t(z, h) = (1-\delta)\left(\frac{\delta H_t}{w_t}\right)^{\delta/(1-\delta)} zh \tag{11}$$

If an agent becomes a production worker, he or she optimally chooses $\lambda = 0$ and receives utility $\log Bw_t$, where $B = \beta^\beta(1-\beta)^{1-\beta}$. Otherwise, he or she enrolls in higher education and chooses his or her effort, λ, to maximize Equation (1) subject to Equations (5), (8), and (11). Assuming an interior solution, this yields the optimal schooling effort

$$\lambda(S_t) = \frac{\rho}{\rho+\mu} - \left(\frac{\mu}{\rho+\mu}\right)\frac{\theta_b}{\theta_h}\left(\frac{x_b}{x_h}\right)^\mu S_t^{\alpha\mu} \tag{12}$$

Thus all agents who enter higher education exert the same effort. The optimal effort depends on the public spending on each level of education and the aggregate enrollment in higher education. The greater is the quality of higher education, the greater is the marginal benefit of each unit of effort exerted and so the greater is the optimal effort. The greater is the quality of basic education, the less effort is required to achieve any given level of human capital and so the lower is the optimal level of effort.

For allocations of public expenditure such that $\mu\theta_b x_b^\mu > \rho\theta_h x_h^\mu / S_t^{\alpha\mu}$, the optimal schooling effort would be a corner solution where $\lambda(S_t) = 0$. If the marginal productivity of higher education were very low, transferring resources from basic to higher education might reduce the human capital and the utility of *all* agents. The following assumption rules out this possibility.

ASSUMPTION 1.

$$\frac{\theta_h}{\theta_b} > \left(\frac{\mu}{\rho}\right)^{1-\mu} \tag{13}$$

Assumption 1 implies that at the allocation of public expenditures where $\lambda = 0$, a transfer of resources from basic to higher education always makes those who attend higher education better off.

An agent whose parent's income was $\tilde{y}_{t-1}$ thus attains one of two levels of schooling depending on his or her realized ability z. Either he or she enrolls in higher education and receives human capital

$$\hat{h}_t(\tilde{y}) = \left\{\frac{\rho}{\rho+\mu}\left[\theta_b x_b^\mu + \theta_h\left(\frac{x_h}{S_t^\alpha}\right)^\mu\right]\right\}^{\rho/\mu} (\tau Y_{t-1})^\rho\,[\beta(1-\tau)\tilde{y}_{t-1}]^{1-\rho} \tag{14}$$

or he or she drops out early and receives only a basic education

$$h_t(\tilde{y}) = [\theta_b x_b^{\mu}]^{\rho/\mu} (\tau Y_{t-1})^{\rho} [\beta(1-\tau)\tilde{y}_{t-1}]^{1-\rho} \tag{15}$$

The ratio of the human capital acquired from these alternatives depends only on public spending to each level of education and the level of secondary enrollment:

$$\Theta(S_t) = \frac{h_t(\tilde{y})}{\hat{h}_t(\tilde{y})} = \left\{ \frac{(\rho+\mu)\theta_b x_b^{\mu}}{\rho\left[\theta_b x_b^{\mu} + \theta_h \left(\frac{x_h}{S_t^{\alpha}}\right)^{\mu}\right]} \right\}^{\rho/\mu} \tag{16}$$

I assume that public expenditures on basic and higher education are such that the net utility of a high-skilled worker from entering further education is positive: $(1-\lambda_t)v[z, \hat{h}_t(\tilde{y})] > v[z, h(\tilde{y})]$. This is true for all agents if x_b and x_h satisfy the following assumption:

Assumption 2.

$$1 - \lambda(S) > \Theta(S) \quad \forall\, S \in (0,1) \tag{17}$$

Assumption 2 implies that the allocation of public expenditures are such that some agents always wish to acquire higher education.[12] In an economy with only high-skilled occupations ($\delta = 0$), Equation (17) would imply that *all* agents enroll in levels of education beyond the basic level. However, the presence of the low-skilled occupation introduces a nonlinearity that leads some agents to drop out early. Whether or not a student enters higher education depends on whether his or her realization of ability z yields a utility from nonproduction work that exceeds the utility of being a production worker. The ability of the marginal student whose parent earned $\tilde{y}$ and who is just indifferent between continuing and dropping out is defined by

$$\log(1-\lambda(S_t)) + \log Bv_t(z_t^c, \hat{h}(\tilde{y})) = \log Bw_t \tag{18}$$

Substituting for $v_t(\cdot)$ using Equation (11) and solving for z^c yields the critical ability level:

$$z_t^c(\tilde{y}) = \left[\frac{1}{1-\lambda(S_t)}\right] \left(\frac{w_t}{\Delta H_t}\right)^{1/(1-\delta)} \frac{H_t}{\hat{h}_t(\tilde{y})} \tag{19}$$

The potential salary in the high-skilled occupation is not sufficient to compensate students with $z < z_t^c(\tilde{y})$ for the effort required to attain higher education, and hence they enter the low-skilled occupation. Agents with $z \geq z_t^c(\tilde{y})$ can earn a salary in the high-skilled occupation that compensates them for the additional effort of acquiring higher education.

At this stage it is helpful to normalize the distribution of income. The following preliminary result follows from the uniformity of the ability distribution and considerably simplifies the analysis:

[12] It is straightforward to show that if Assumption 2 holds for $S_t = 1$, it must hold for all $S_t \in (0, 1)$.

LEMMA 1. *The salary function of a high-skilled worker* (11) *can be restated in terms of the level of enrollment in higher education* S_t:

$$v_t(z, h) = \Delta^{1/(1-\delta)} \left(\frac{H_t}{w_t}\right)^{\delta/(1-\delta)} zh = \Delta \left[\frac{\Theta(S_t)}{1-\lambda(S_t)}\right]^{\delta} zh \tag{20}$$

It follows, in particular, that the maximum salary $\bar{v}_t$ evolves according to

$$\bar{v}_t = \Delta \left[\frac{\Theta(S_t)}{1-\lambda(S_t)}\right]^{\delta} \left\{\frac{\rho}{\rho+\mu}\left[\theta_b x_b^{\mu} + \theta_h \left(\frac{x_h}{S_t^{\alpha}}\right)^{\mu}\right]\right\}^{\rho/\mu} \times (\tau Y_{t-1})^{\rho} [\beta(1-\tau)\bar{v}_{t-1}]^{1-\rho} \tag{21}$$

Let η represent the income of an agent relative to $\bar{v}_t$. Then the relative human capital attained by an agent whose parent earned relative income $\tilde{\eta}_{t-1}$ and whose ability is z is given by

$$\frac{h}{\bar{h}_t} = \begin{cases} \tilde{\eta}_{t-1}^{1-\rho} & \text{if } z \geq z_t^c(\tilde{\eta}_{t-1}, w_t) \\ \Theta(S_t)\tilde{\eta}_{t-1}^{1-\rho} & \text{if } z < z_t^c(\tilde{\eta}_{t-1}, w_t) \end{cases} \tag{22}$$

where the ability of the marginal student can be expressed as

$$z_t^c(\tilde{\eta}) = \left[\frac{1}{1-\lambda(S_t)}\right]\left(\frac{w_t}{\bar{v}_t}\right)\frac{1}{\tilde{\eta}^{1-\rho}} \tag{23}$$

Lineage relative income evolves according to

$$\eta_t = \begin{cases} z\tilde{\eta}_{t-1}^{1-\rho} & \text{if } z \geq z_t^c(\tilde{\eta}_{t-1}) \\ \underline{\eta}_{t-1}^{1-\rho} & \text{if } z < z_t^c(\tilde{\eta}_{t-1}) \end{cases} \tag{24}$$

where

$$\underline{\eta}_t = \frac{w_t}{\bar{v}_t} \tag{25}$$

is the relative wage. Observe that because of the presence of occupational choices, the process governing individual transitions is inherently nonlinear. Let $\tilde{z}(\eta, \tilde{\eta})$ be the value of z such that $\eta = z\tilde{\eta}^{1-\rho}$. Then

$$\tilde{z}(\eta, \tilde{\eta}) = \frac{\eta}{\tilde{\eta}^{1-\rho}} \tag{26}$$

is the fraction of agents whose parents earned relative income $\tilde{\eta}$, earning income less than a fraction η of the highest income in the economy. The remaining fraction, $z_t^c(\tilde{\eta})$, become wage laborers and receive relative income $\underline{\eta}_t$. Hence the distribution of relative incomes conditional on relative parental income is given by

$$
P_t(\eta|\tilde{\eta}) = \begin{cases} 0 & \text{if } \eta < \underline{\eta}_t \\ z_t^c(\tilde{\eta}) & \text{if } \underline{\eta}_t \leq \eta < \left[\dfrac{1}{1-\lambda(S_t)}\right]\underline{\eta}_t \\ \tilde{z}(\eta, \tilde{\eta}) & \text{if } \left[\dfrac{1}{1-\lambda(S_t)}\right]\underline{\eta}_t \leq \eta < \tilde{\eta}^{1-\rho} \\ 1 & \text{otherwise} \end{cases} \tag{27}
$$

This distribution is a mixed distribution, consisting of a point mass of wage laborers at $w_t/\bar{v}_t$ and a continuous density of relative incomes on the interval $[w_t/[1 - \lambda(S_t)\bar{v}_t], 1]$. Note that the effort required to attain higher levels of education drives a wedge between the equilibrium production wage and the lowest salary of nonproduction workers.

4. EQUILIBRIUM

Up to now I have considered the behavior of agents given the time t equilibrium wage w_t and the aggregate enrollment in higher education S_t. I now complete the model by determining the time t equilibrium for the economy, given a distribution of relative parental incomes. The time t aggregate supply schedule for wage labor $L_t^S(w\lambda_t)$ is given by

$$
L_t^S(w, \lambda_t) = \int_\eta \int_0^{z_t^c(\eta)} dz F_{t-1}(d\eta) = \left(\frac{1}{1-\lambda_t}\right)\left(\frac{w}{\bar{v}_t}\right)\Psi_t \tag{28}
$$

where the term

$$
\Psi_t = \int_\eta \frac{1}{\eta^{1-\rho}} F_{t-1}(d\eta) \tag{29}
$$

represents an *occupational factor*. A mean-preserving reduction in the dispersion of parental incomes causes the occupational shift factor, and hence the supply of production workers, to fall. For a given wage, transferring income from rich to poor households increases the in-school productivity of children from poorer households by more than it decreases that of children from richer households. It follows that a greater fraction of the young have an incentive to enroll in higher education, thereby reducing the supply of production labor.

The aggregate demand for production labor $L_t^D(w, \lambda_t)$ is given by

$$
L_t^D(w, \lambda_t) = \frac{1}{2}\left(\frac{\delta}{1-\delta}\right)\left(\frac{\bar{v}_t}{w}\right)\Phi_t - \frac{1}{2}\left(\frac{\delta}{1-\delta}\right)\left(\frac{1}{1-\lambda_t}\right)^2\left(\frac{w}{\bar{v}_t}\right)\Psi_t \tag{30}
$$

where

$$
\Phi_t = \int_\eta \eta^{1-\rho} F_{t-1}(d\eta) \tag{31}
$$

is a *productivity factor.* A mean-preserving reduction in the dispersion of parental incomes raises the demand for labor for two reasons. First, it increases the in-school productivity of students from poorer backgrounds by more than it lowers that of those from richer backgrounds. Second, it increases the fraction of agents that become nonproduction workers in equilibrium. Thus the occupational factor has a negative impact on the aggregate demand for low-skilled labor; the greater the supply of low-skilled labor, the lower is the supply of high-skilled workers.

A period *t competitive equilibrium* for an economy with an initial parental distribution of relative incomes $F_{t-1}(\cdot)$ and upper support on incomes $\bar{v}_{t-1}$ is a vector $\{w_t^*, L_t^*, S_t^*, \lambda_t^*\}$ such that

- The government balances its budget: $x_b + x_h = 1$.
- Given the wage w_t^*, firms choose their labor demand to maximize profits Equation (10).
- Firms earn zero profits Equation (11).
- Given the wage w_t^* and the level of secondary enrollment S_t^*, each agent selects his or her occupation and chooses his or her educational effort λ_t^* to maximize utility, Equation (12).
- The market for production workers clears: $L_t^S(w_t^*, \lambda_t^*) = L_t^D(w_t^*, \lambda_t^*) = L_t^*(\lambda_t^*)$.
- All agents that do not become production workers enroll in secondary education: $S_t^* = 1 - L_t^*$.

Given λ_t, the labor market clearing conditions yields the following equilibrium wage, where $\bar{v}_t$ is given in Equation (21)[13]:

$$w_t^* = \left(\frac{A_t \Phi_t}{\Psi_t}\right)^{1/2} \bar{v}_t \tag{32}$$

where

$$A_t = \frac{(1-\lambda_t)^2 \delta}{2(1-\delta)(1-\lambda_t) + \delta} \tag{33}$$

The associated equilibrium fraction of production workers with only basic education is then

$$L_t(\lambda_t) = \left[\frac{\delta \Phi_t \Psi_t}{2(1-\delta)(1-\lambda_t) + \delta}\right]^{1/2} \tag{34}$$

and equilibrium aggregate income is given by

$$Y_t = \frac{w_t L_t}{\delta} = \frac{\Phi_t (1-\lambda_t) \bar{v}_t}{2(1-\delta)(1-\lambda_t) + \delta} \tag{35}$$

Labor market equilibrium is illustrated in Figure 2. The area *OwBL* represents the total wage bill of low-skilled workers; the rest is equal to the total salaries of high-skilled workers. The area *wAB* represents producer surplus, and the area *BCDL* represents the rent accruing to owner/managers. It follows that

[13] See Appendix for derivation.

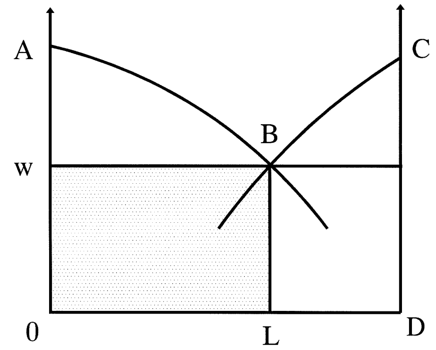

FIGURE 2

LABOR MARKET EQUILIBRIUM

LEMMA 2. *Aggregate current income Y_t is equal to the area below the upper envelope generated by the supply and demand curves.*

Lemma 2 implies that any change that causes the demand and supply curves for labor to shift up will result in an increase in aggregate income as well as the wage.

The equilibrium fraction of nonproduction workers with higher education is

$$S_t(\lambda_t) = 1 - \left[\frac{\delta\Phi_t\Psi_t}{2(1-\delta)(1-\lambda_t)+\delta}\right]^{1/2} \quad (36)$$

Figure 3 illustrates the determination of equilibrium in terms of λ_t and S_t. The *SS* curve represents Equation (36)—the equilibrium level of secondary enrollment as a function of the required effort. Points on this curve represent labor market equilibria. The *EE* curve represents Equation (12)—the optimal effort level as a function of the

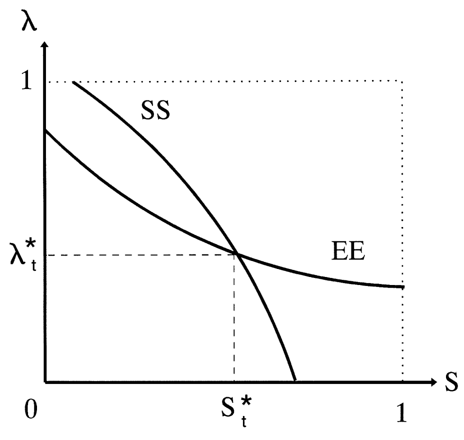

FIGURE 3

SCHOOLING EQUILIBRIUM

anticipated level of secondary enrollment.[14] The period t equilibrium is represented by the intersection of these two curves.

LEMMA 3. *The period t competitive equilibrium exists and is unique.*

The inequality associated with income distributions can be compared in terms of their Lorenz curves. These show the share of total income received by the poorest fraction p of the population as p varies from 0 to 1:

$$\Lambda_t(p) = \frac{\bar{v}_t}{Y_t}\int_0^{\hat{\eta}_t(p)} \eta F_t(d\eta) \tag{37}$$

where $\hat{\eta}_t(p)$ is defined implicitly by $F_t(\hat{\eta}_t) = p$. Distribution $F_1(\cdot)$ *Lorenz dominates* distribution $F_2(\cdot)$ if $\Lambda_1(p) \geq \Lambda_2(p)\ \forall\, p$. That is, the entire Lorenz curve of distribution $F_1(\cdot)$ lies above that of $F_2(\cdot)$. Lorenz dominance is a sufficient condition for a reduction in inequality as measured by most commonly used indices of inequality. The slope of the Lorenz curve at a given point is equal to the ratio of the relative income of the corresponding quantile of the distribution to the mean income of the distribution. Figure 4 illustrates a typical Lorenz curve for this economy. The linear segment OA corresponds to production workers and has slope

$$\underline{\sigma}_t = \frac{w_t}{Y_t} \tag{38}$$

Note that given the Cobb-Douglas technology, the vertical component of this segment is always equal to δ. The convex segment AB corresponds to higher-skilled nonproduction workers. The slope of the Lorenz curve at B is given by

$$\overline{\sigma}_t = \frac{\bar{v}_t}{Y_t} \tag{39}$$

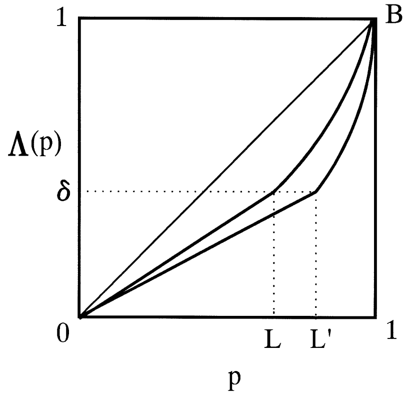

FIGURE 4

LORENZ CURVES

[14] If $\alpha = 0$, the EE curve would be horizontal.

5. TRANSITIONAL DYNAMICS AND CONVERGENCE

Since $P_t(\eta \mid \tilde{\eta})$ is the probability transition function for relative incomes, the distribution of relative income evolves through time according to

$$F_t(\eta) = \int P_t(\eta \mid \tilde{\eta}) F_{t-1}(d\tilde{\eta}) \tag{40}$$

where $F_0(\cdot)$ is given.[15] This section characterizes the evolution of the distribution of income, holding all policy parameters fixed. I assume that the parameters of the model are such that the growth rate is positive (see below).

The transitional dynamics of the economy depend, in part, on the distribution of relative incomes among the initial old generation $F_0(\cdot)$. I focus on the dynamics of an economy for which $F_0(\cdot)$ is such that in the following generation: (1) the distribution of *relative incomes* $F_1(\cdot)$ dominates $F_0(\cdot)$ in the first-order stochastic sense, and (2) the minimum relative income $\underline{\eta}_1$ is not too low. Given these initial conditions, the transitional dynamics of the economy can be summarized as follows:

PROPOSITION 1. *There exists a* $\gamma \in [0, (\delta/(2-\delta))^{1/2}]$ *such that if* $\underline{\eta}_1 > \gamma$ *and if* $F_1(\cdot)$ *dominates* $F_0(\cdot)$ *in the first-order stochastic sense, then in each successive generation*

- (a) *The fraction of the population enrolled in higher education and entering high-skilled occupations increases through time*: $S_{t+1} > S_t$.
- (b) *The minimum relative income increases through time*: $\underline{\eta}_{t+1} > \underline{\eta}_t$
- (c) *The distribution of relative incomes dominates that of the previous generation in the first-order stochastic sense*: $F_{t+1}(\eta) \le F_t(\eta)$ *for all* $\eta \in [0, 1]$.
- (d) *Inequality declines through time, in the Lorenz dominance sense*.

To understand these results, consider any period $t+1$ such that the distribution of relative incomes of the current old generation $F_t(\cdot)$ stochastically dominates that of the previous old generation $F_{t-1}(\cdot)$. Since mean output grows, it follows that $\bar{v}_t > \bar{v}_{t-1}$, so this represents a *second-order stochastic* increase in the distribution of *actual* parental incomes. For a given level of schooling effort λ_t, the implied compression in the distribution of relative incomes generates an increase in the demand for higher education and hence a reduction in the supply of low-skilled workers Equation (28). However, the redistribution also increases the average in-school productivity of high-skilled workers, thereby raising the demand Equation (30) for them (the productivity effect). It follows that the wage and aggregate income must both rise (see Figure 2). Provided that the dispersion of incomes among the $t-1$ old generation (indexed by $\underline{\eta}_{t-1}$) is not too high,[16] the occupational effect always outweighs the productivity effect, so in the equilibrium, levels of low-skilled workers fall and enrollments in higher education rise. This is represented by a rightward shift in the *SS* curve in Figure 3.

[15] Since an individual's relative income depends on the entire distribution of relative incomes via its effect w_t and S_t, it is not possible to analyze the behavior of the economy by studying a single lineage.

[16] Since $\lambda > 0$, the equilibrium relative wage $\underline{\eta}_{t-1}$ must lie in the interval $[0, (\delta/(2-\delta))^{1/2}]$ (see Equation 32).

As enrollments rise, however, the effective quality of higher education falls, so the optimal effort level declines, until the new equilibrium (S^*, λ^*) combination is reached.[17] The reduction in λ causes additional upward shifts in the supply and demand curves for labor demand, so the wage and aggregate income rise still further. The growth in the wage exceeds the growth in the maximum salary, so the minimum relative income $\underline{\eta}_t$ rises.

Since labor moves from low-skilled to high-skilled occupations and the distribution of incomes becomes more compressed, the time $t+1$ distribution of relative incomes $F_{t+1}(\cdot)$ stochastically dominates that at t, $F_t(\cdot)$. The evolution of the distribution of relative income is illustrated in Figure 5. By induction from period 1, as long as η_1 is not too low, the occupational effect always dominates the productivity effect so that $L_{t+1} < L_t$ and parts (a), (b), and (c) of Proposition 1 follow.

Since the distribution of relative income stochastically increases, mean relative income $Y_t/\bar{v}_t$ rises. It follows that the slope of the Lorenz curve at $p = 1$, $\overline{\sigma}_t = \bar{v}_t/Y_t$, falls (see Figure 4). Moreover, since $L_{t+1} < L_t$, the slope of the Lorenz curve segment OA, $\underline{\sigma}_t = w_t/Y_t = \delta/L_t$, must rise. Since the segment AB is strictly convex, these conditions are sufficient to ensure that the entire Lorenz curve at time $t+1$ lies within that at time t.

Since there are decreasing returns to parental inputs to human capital, $\rho \in (0, 1)$, the marginal impact of a poor parent's income on his or her child's human capital exceeds that of a rich parent. In particular, this implies that the lower support on the distribution of relative incomes $\underline{\eta}_t$ increases and eventually converges to a constant value $\underline{\eta}^*$. To see this, note that the lower support on the distribution of relative income is given by

$$\underline{\eta}_t = \frac{w_t}{\bar{v}_t} = \left(\frac{A_t \Phi_t}{\Psi_t}\right)^{1/2} \tag{41}$$

Since $F_t(\cdot)$ dominates $F_{t-1}(\cdot)$ in the first-order stochastic sense, it follows that $\underline{\eta}_t \geq \underline{\eta}_{t-1}$ for all t. Whatever the distribution, it must be the case that $\Phi_t < 1$ and $\Psi_t > 1$.

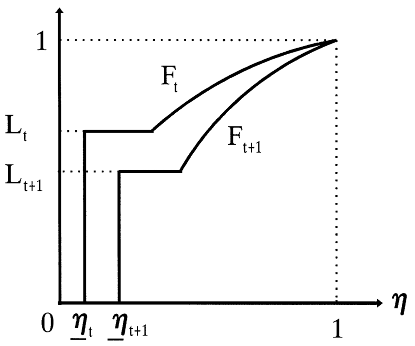

FIGURE 5

EVOLUTION OF THE DISTRIBUTION OF RELATIVE INCOMES

[17] If $\alpha = 0$, λ_t would remain constant.

Since $A_t < \delta/(2-\delta)$, it follows from Equation (41) that $\underline{\eta}_t < [\delta/(2-\delta)]^{1/2}$. Since it is increasing and bounded from above, the sequence $\underline{\eta}_t$ must converge to some constant value $\underline{\eta}^*$. From Equation (27) the process governing the evolution of the distribution of relative incomes therefore converges to a Markov process with a stationary transition function given by

$$(42) \qquad P^*(\eta|\tilde{\eta}) = \begin{cases} 0 & \text{if } \eta < \underline{\eta}^* \\ z^c(\tilde{\eta}) & \text{if } \underline{\eta}^* \leq \eta < \left(\dfrac{1}{1-\lambda}\right)\underline{\eta}^* \\ \tilde{z}(\eta, \tilde{\eta}) & \text{if } \left(\dfrac{1}{1-\lambda}\right)\underline{\eta}^* \leq \eta < \tilde{\eta}^{1-\rho} \\ 1 & \text{otherwise} \end{cases}$$

The process is monotone because the distribution of relative incomes among the offspring of agents with high relative income dominates that among the offspring of agents with lower relative incomes in the first-order stochastic sense. Moreover, the process satisfies the monotone mixing condition (Hopenhayn and Prescott, 1992). That is, for any $\tilde{\eta} \in (\underline{\eta}^*, 1)$, after a sufficient number of generations, there is a positive probability that the descendant of an agent with η in any neighborhood of 1 earns relative income below $\tilde{\eta}$ and a positive probability that the eventual descendant of an agent with η in any neighborhood of $\underline{\eta}^*$ receives relative income above $\tilde{\eta}$. As a result, after a sufficient number of generations, the distributions of relative incomes among the descendants of the richest and poorest agents living at time t converge to identical distributions. By monotonicity, the distribution of relative human capital among the descendants of all agents living at time t must converge to the same distribution, and so

PROPOSITION 2. *The distribution of relative incomes converges to a unique time-invariant limiting distribution* $F^*(\cdot)$, *which is independent of initial conditions.*

Since the distribution of relative incomes converges to a stationary distribution, the extent of relative inequality as measured by the Lorenz curve converges. In particular, the slopes of the Lorenz curve at $p = 0$ and $p = 1$ converge to steady-state values $\underline{\sigma}$ and $\overline{\sigma}$. The distribution of actual incomes is not stationary in the long run. As mean income rises, the distributions of actual incomes grows in the first-order stochastic sense.

6. EDUCATION POLICY

This section details the effects of public subsidies at different levels of education on growth, the distribution of income, and the relationship between them. I trace out the immediate, transitional, and long-run consequences of a permanent reallocation of public resources.

6.1. *Impact Effects.* An increase in the share of revenue going to higher education x_h increases the incomes of those agents which acquire both levels of education and enter high-skilled occupations. However, it also increases the incomes of agents that acquire only basic education. There are two reasons for this. First, because high-skilled and low-skilled workers are complementary factors of production, the increased productivity of high-skilled workers increases the demand for low-skilled workers. Second, for any given low-skilled wage, the increased spending on higher education also creates a greater incentive for marginal students to acquire higher education. It follows that the supply curve of low-skilled workers shifts in. Since the demand curve shifts out and the supply curve shifts in, the equilibrium wage w_t unambiguously increases. Thus there is an equilibrium "trickle-down effect" from increased spending on higher education that raises *all* agents' incomes.

If, however, the increased spending on higher education is at the expense of resources allocated to basic education x_b, the trickle-down effect is offset by the associated reduction in the quality of basic education. This makes it more difficult for students from poorer backgrounds and/or relatively low ability levels to acquire sufficient levels of human capital to make higher education worthwhile. As a result, marginal students drop out early and enrollments decline, even though the quality of higher education has increased. The overall effect can be represented in Figure 3 by an upward shift in the *EE* curve.

Depending on the initial balance of resources, the human capital acquired by high-skilled workers could either increase or decrease as a result of such a reallocation. However, *even if the human capital acquired by high-skilled workers rises overall*, secondary enrollments decline, and public resources become concentrated among even fewer students. Thus the increased inequality resulting from the reallocation is compounded by the associated decline in enrollments. The following proposition summarizes these effects:

PROPOSITION 3. *A reallocation of public resources from basic* (*higher*) *to higher* (*basic*) *education*

(a) *Reduces* (*increases*) *enrollments in higher education*.
(b) *Increases* (*reduces*) *inequality in the Lorenz dominance sense*.

Note that the result that enrollments *always* decline as resources are shifted from basic to higher education stems from the assumption of a Cobb-Douglas production function, which implies that the shares of total income going to low-skilled labor is fixed, $wL = \delta Y$. As a result, an increase in inequality (a reduction in w/Y) always must be associated with a decline in enrollments (an increase in L). Although with a different production function enrollments might rise over some ranges, this need not change the impact on inequality.

Although enrollments in higher education decline, the impact of the reallocation on aggregate output and average income is, in general, ambiguous. On the one hand, it raises the productivity of those agents which acquire higher education; on the other hand, it reduces the number that acquire that level of education and lowers the productivity of those who do not. The impact on the subsequent generation is therefore nontrivial. The increase in inequality reduces enrollments and the average productivity of the following generation. If aggregate income declines, then so also do tax revenues and aggregate parental investments, thereby reinforcing the impact on subsequent generations. If, instead, aggregate income rises, the impact on the next generation is ambiguous. Note, however, that since the evolution of the distribution of relative incomes is independent of the level of aggregate income, this ambiguity does not affect the convergence of the distribution

6.2. *Steady-State Effects*. As $F_t(\cdot)$ converges to $F^*(\cdot)$, both $\overline{\sigma}_t$ and S_t converge to their long-run values $\overline{\sigma}$ and S, respectively. Using Equations (21) and (35), the steady-state gross growth rate is given by

$$\frac{Y_t}{Y_{t-1}} = \frac{\bar{v}_t}{\bar{v}_{t-1}} = \tau^{\rho}\left[\frac{\beta(1-\tau)}{\overline{\sigma}}\right]^{1-\rho}\left(\frac{\rho}{\rho+\mu}\right) \times \left[\theta_b x_b^{\mu} + \theta_h\left(\frac{x_h}{S^{\alpha}}\right)^{\mu}\right]^{\rho/\mu} \Delta\left[\frac{\Theta(S)}{1-\lambda(S)}\right]^{\delta} \tag{43}$$

Taking logarithms yields the long-run growth rate:

$$\Gamma = \log \tau^{\rho}\left[\beta(1-\tau)\right]^{1-\rho} + \log\left(\frac{\rho}{\rho+\mu}\right)\left[\theta_b x_b^{\mu} + \theta_h\left(\frac{x_h}{S^{\alpha}}\right)^{\mu}\right]^{\rho/\mu} - (1-\rho)\log\overline{\sigma} - \log\Delta\left[\frac{1-\lambda(S)}{\Theta(S)}\right]^{\delta} \tag{44}$$

This expression provides a convenient decomposition of the various avenues through which the education system affects the growth rate. The first two components represent the growth rate that would obtain in the absence of long-run inequality (no random ability factor) and no nonconvexities due to occupational asymmetries ($\delta = 0$, $S = 1$). In this special case, the growth rate depends only on the quality of the education system and the total fraction of GNP allocated to educational expenditures τ.

Since from Equation (35)

$$\overline{\sigma} = \frac{2(1-\delta)(1-\lambda)+\delta}{(1-\lambda)\Phi} \tag{45}$$

the third term in Equation (44) measures the drag on growth imposed by the impact of long-run inequality on productivity Φ. This is similar to the effect emphasized by Glomm and Ravikumar (1992) and Bénabou (1996a) and arises due to decreasing returns to parental inputs into human capital formation. In contrast to these articles, however, the absence of random ability here does *not* imply that this effect converges to zero ($\overline{\sigma} \to 1$). Occupational asymmetries always ensure that the distribution of income is nondegenerate.

The final term measures the loss in growth due to the occupational effect. Lack of appropriability results in underinvestment in human capital by low-skilled agents. In the absence of occupational asymmetries ($\delta = 0$), this term would be zero. The presence of occupational asymmetries also generates a long-term loss due to the reduced productivity caused by the extra inequality it induces.

A change in the fraction of aggregate income allocated to education τ, holding the relative shares going to the basic and higher education x_b and x_h constant, does not affect relative income inequality in the current period or in the long run. However, since it represents the share of investment in human capital formation coming from the public sector relative to private sources, a change in the tax rate does affect the growth rate.

PROPOSITION 4. *Growth increases with the share of aggregate income allocated to public education if $\tau < \rho$ and decreases if $\tau > \rho$.*

This result is similar to Barro's (1990) proposition that the growth rate first rises with the ratio of government expenditures to GNP but declines with excessive government spending. In my context, government expenditure goes to the production of human capital, not output. The main implication here is that once education expenditures are such that $\tau = \rho$, there is a tradeoff, in terms of growth, between expenditures at different levels of education.

Changing the relative expenditures on basic and higher education (while keeping the share of income τ constant) has several impacts. It is convenient to let X denote the relative expenditure shares:

$$X = \frac{x_b}{x_h} \tag{46}$$

To begin with, suppose we ignore the last two terms of Equation (44) and focus on the second. Maximizing this component of overall growth is equivalent to maximizing the incomes of high-skilled workers who attend higher education. If we were to hold enrollments S_t constant, then the ratio of basic to higher education expenditures that maximizes this component would be given by

$$X(S_t) = \left(\frac{\theta_b}{\theta_h}\right)^{1/1-\mu} S_t^{\alpha\mu/1-\mu} \tag{47}$$

If expenditures on secondary education represented a pure public good so that $\alpha = 0$, then this would be the optimum. However, enrollments vary with relative expenditures so that in general, with $\alpha > 0$, we must account for the equilibrium response of S_t. An increase in X causes the EE curve to shift down so that equilibrium secondary enrollment increases (see Figure 3). The curve denoted EQ_0 in Figure 6 represents this positive equilibrium relationship between X and S_t. It may be derived by substituting out λ from Equations (12) and (14) to get

$$f(S, X) = (1 - S)^2\left[1 + 2\left(\frac{1-\delta}{\delta}\right)\left(\frac{\mu}{\rho + \mu}\right)\left(1 + \frac{\theta_b}{\theta_h} X^{\mu} S^{\alpha\mu}\right)\right] = \Phi\Psi \tag{48}$$

Combining this with the XX_0 curve representing Equation (47) yields the equilibrium expenditure ratio that maximizes the aggregate income of high-skilled workers X^*. As the curves are drawn, this optimum is unique. The following lemma shows that this is in fact generally the case:

LEMMA 4. *There exists a unique ratio of public spending on basic education to that on higher education X^* that maximizes the aggregate income of high-skilled workers.*

The change in the expenditure ratio X also operates in the short run through the occupational effect [the last term in Equation (44)]. Given enrollments S_t, the transfer increases the productivity of low-skilled workers by more than it decreases that of high-skilled workers. As a result, the current income-maximizing expenditure ratio for any S_t must exceed X^*. This is represented by the upward shift in the XX curve to XX_1 shown in Figure 6, which results in a higher optimal expenditure share X^{**} and a consequent increase in secondary enrollments.

Shifting resources toward basic education also has two dynamic effects via the distribution of income. First, by compressing the distribution of incomes, it raises the aggregate productivity of future generations by raising their average in-school productivity (i.e., Φ increases). Second, it generates future occupational effects by making it easier for future generations to enter higher education. Both these effects cause the *SS* curve to shift to the right and hence the *EQ* curve to shift to EQ_1, as illustrated in Figure 6. It follows that growth is unambiguously increased by allocating greater resources to basic education, so the growth-maximizing ratio is $\hat{X}$.

PROPOSITION 5. *The growth-maximizing ratio of basic to higher education expenditures $\hat{X}$ exceeds that which maximizes the aggregate income of high-skilled workers and satisfies*

$$X^* < \hat{X} \leq \left(\frac{\rho\theta_h}{\mu\theta_b}\right)^{1/\mu} \tag{49}$$

Proposition 5 implies that when occupational asymmetries are present, a nonmonotonic relationship between growth and inequality arises from the reallocation of public resources across different levels of education. As X is increased beyond the level

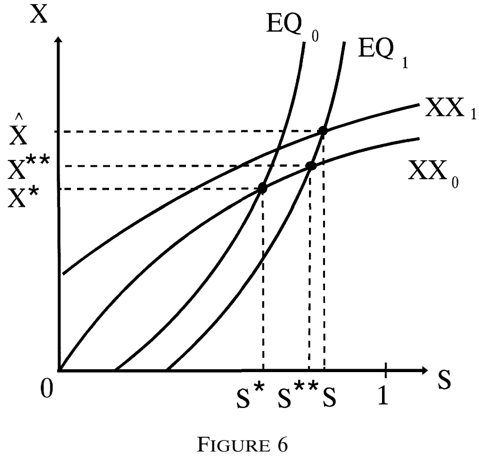

FIGURE 6

DETERMINATION OF THE GROWTH-MAXIMIZING EXPENDITURE RATIO in Equation (47), the negative growth effects of the reduction in the quality of higher education are outweighed by the positive growth effects of increased enrollments and reduced parental inequality. Thus, over this range, growth rises as inequality declines. Eventually, however, the effects of the reduced quality of higher education dominate, so growth declines as inequality falls. The nonmonotonicity results from the tradeoff between the quantity and quality of nonproduction workers. Eventually, the growth benefits of "leveling the playing field" resulting from expanded enrollments are offset by the costs resulting from the reduced human capital acquired by each student.[18]

7. CONCLUDING REMARKS

This article constructed a dynamic general equilibrium model in which the allocation of educational resources affected the distribution of income because of the presence of asymmetries in the nature of occupational compensation. The model was used to illustrate the immediate, transitional, and long-run implications of the resulting nonconvexity in the returns to education for growth and income distribution and to demonstrate the impacts of alternative allocations of educational resources on the equilibrium growth-inequality relationship. In particular, it was shown that the impact of the allocation of public resources on growth reflects a tension between the trickle-down effects of higher education and the positive enrollment effects of high-quality basic education and reduced parental income inequality.

This article provides a tractable framework that could be extended in a number of directions. One strand of the recent literature on growth and inequality emphasizes the endogenous role of voting and lobbying in determining outcomes (see Bénabou, 1996b). As Birdsall et al. (1995) observe, "The allocation of limited fiscal resources for tertiary education, so common in Latin America, is an example of a fiscal policy that reflects pressure for public spending on favored groups." Another application of the basic framework outlined here is the current debate concerning the causes of the recent rise in the wage premium to nonproduction relative to production workers (Wood, 1995). The model developed in this article generates a general equilibrium theory in which the supply of low- and high-skilled workers is partly determined by education policy.

APPENDIX

PROOF OF LEMMA 1. The aggregate human capital of low-skilled workers whose parents earned $\tilde{y}$ is

$$z_t^c(\tilde{y}, w_t)h(\tilde{y}) = \left(\frac{1}{1-\lambda_t}\right)\left(\frac{w_t}{\Delta H_t}\right)^{1/(1-\delta)} \Theta_t H_t \tag{A.1}$$

Observe that this cohort aggregate is independent of parental income—those cohorts with greater human capital contain proportionately fewer production workers.

[18] The upper bound on X in Propostion 5 represents the allocation at which no agent will acquire secondary education (see Assumption 2). At this point, growth drops to zero.

Since the population is normalized to one, the aggregate human capital of production workers must satisfy

$$H_t = \left(\frac{1}{1-\lambda_t}\right)\left(\frac{w_t}{\Delta H_t}\right)^{1/1-\delta}\Theta_t H_t \tag{A.2}$$

so that

$$\left(\frac{\Delta H_t}{w_t}\right)^{1/1-\delta} = \frac{\Theta_t}{1-\lambda_t} \tag{A.3}$$

Substituting this into Equation (11) yields Equation (20). □

DERIVATION OF THE EQUILIBRIUM WAGE RATE. The aggregate supply of low-skilled labor is given by

$$L_t^S(w) = \int_\eta \int_0^{z_t^c(\eta,w)} dz F_{t-1}(d\eta) = \int_\eta z_t^c(\eta,w)F_{t-1}(d\eta) = \left(\frac{1}{1-\lambda}\right)\left(\frac{w}{\bar{v}_t}\right)\Psi_t \tag{A.4}$$

where $\Psi_t = \int_\eta \eta^{\rho-1}F_{t-1}(d\eta)$. The aggregate demand for labor $L_t^D(w)$ is

$$\begin{aligned} L_t^D(w) &= \int_\eta \int_{z_t^c(\eta,w)}^1 l_t(z,\eta,w)\, dz F_{t-1}(d\eta) \\ &= \left(\frac{\delta}{1-\delta}\right)\frac{\bar{v}_t}{w}\int_\eta \eta^{1-\rho}\int_{z_t^c(\eta,w)}^1 z\, dz F_{t-1}(d\eta) \end{aligned} \tag{A.5}$$

Integrating over z's gives

$$L_t^D(w) = \frac{1}{2}\left(\frac{\delta}{1-\delta}\right)\frac{\bar{v}_t}{w}\int_\eta \eta^{1-\rho}[1 - z_t^c(\eta,w)^2]F_{t-1}(d\nu) \tag{A.6}$$

Substituting the expression for $z_t^c(\eta, w)$ and rearranging yields

$$L_t^D(w) = \frac{1}{2}\left(\frac{\delta}{1-\delta}\right)\left(\frac{\bar{v}_t}{w}\right)\Phi_t - \frac{1}{2}\left(\frac{\delta}{1-\delta}\right)\left(\frac{1}{1-\lambda}\right)^2\left(\frac{w}{\bar{v}_t}\right)\Psi_t \tag{A.7}$$

The equilibrium condition is $L_t^S(w_t) = L_t^D(w_t)$, so

$$\begin{aligned} \left(\frac{1}{1-\lambda}\right)\left(\frac{w}{\bar{v}_t}\right)\Psi_t &= \frac{1}{2}\left(\frac{\delta}{1-\delta}\right)\left(\frac{\bar{v}_t}{w}\right)\Phi_t \\ &\quad - \frac{1}{2}\left(\frac{\delta}{1-\delta}\right)\left(\frac{1}{1-\lambda}\right)^2\left(\frac{w}{\bar{v}_t}\right)\Psi_t \end{aligned} \tag{A.8}$$

Rearranging and solving for the time t equilibrium wage yields Equation (32).

PROOF OF LEMMA 2. Aggregate income is the sum of total high- and low-skilled wages:

$$Y_t = \int_\eta \int_{z_c(\eta,w_t)}^1 v(\eta,z,w_t)\, dz F_{t-1}(d\eta) + w_t L_t \tag{A.9}$$

Since both $v(\cdot)$ and $z^c(\cdot)$ are continuous and differentiable with respect to the wage,

$$\text{(A.10)}\quad Y_t = \int_{w_t}^{\infty} \int_{\eta} \left[-\int_{z^c(\eta,w)}^{1} v_w(\eta, z, w)\, dz - w z_w^c(\eta, w) \right] F_{t-1}(d\eta)\, dw + w_t L_t$$

By the envelope theorem, $v_w(\eta, z, w) = -l(\eta, z, w)$. Hence

$$\text{(A.11)}\qquad Y_t = \int_{w_t}^{\infty} \left[L_t^D(w) - \frac{\int_{\nu} w z_w^c(\eta, w)}{F_{t-1}}(d\eta) \right] dw + w_t L_t$$

But

$$\text{(A.12)}\qquad \frac{dwz^c(\eta, w)}{dw} = w z_w^c(\eta, w) + z(\eta, w)$$

so that

$$\text{(A.13)}\quad Y_t = \int_{w_t}^{\infty} \left\{ L_t^D(w) - \int_{\eta} \left[\frac{dw\, z^c(\eta, w)}{dw} - z^c(\eta, w) \right] G_t(d\eta) \right\} dw + w_t L_t$$

$$= \int_{w_t}^{\infty} \left[L_t^D(w) + S_t(w) \right] dw + w_t L_t + w_t S_t$$

which is the area under the upper envelope created by the supply and demand curves. □

PROOF OF LEMMA 3. The *EE* curve Equation (12) has the following properties:

$$\text{(A.14)}\qquad \left.\frac{d\lambda_t}{dS_t}\right|_{EE} = -\left(\frac{\mu}{\rho + \mu} \right) \frac{\theta_b}{\theta_h} \left(\frac{x_b}{x_h} \right)^{\mu} \alpha\mu S_t^{\alpha\mu - 1} < 0$$

$$\text{(A.15)}\qquad \left.\frac{d^2\lambda_t}{dS_t^2}\right|_{EE} = \left(\frac{\mu}{\rho + \mu} \right) \frac{\theta_b}{\theta_h} \left(\frac{x_b}{x_h} \right)^{\mu} \alpha\mu(1 - \alpha\mu) S_t^{\alpha\mu - 2} > 0$$

$$\text{(A.16)}\qquad \lambda_t(0) = \frac{\rho}{\rho + \mu} < 1 \qquad \text{and}$$

$$\lambda_t(1) = \frac{\rho}{\rho + \mu} - \left(\frac{\mu}{\rho + \mu} \right) \frac{\theta_b}{\theta_h} \left(\frac{x_b}{x_h} \right)^{\mu} > 0$$

Inverting Equation (36), the *SS* curve can be expressed as

$$\text{(A.17)}\qquad \lambda_t = 1 + \frac{\delta}{2(1 - \delta)} - \frac{\delta \Phi_t \Psi_t}{2(1 - \delta)} (1 - S_t)^{-2}$$

Hence the *SS* curve has the following properties:

$$\text{(A.18)}\qquad \left.\frac{d\lambda_t}{dS_t}\right|_{SS} = -\frac{\delta \Phi_t \Psi_t}{(1 - \delta)} (1 - S_t)^{-3} < 0$$

$$\text{(A.19)}\qquad \left.\frac{d^2\lambda_t}{dS_t^2}\right|_{SS} = -3 \frac{\delta \Phi_t \Psi_t}{(1 - \delta)} (1 - S_t)^{-4} < 0$$

$$\text{(A.20)}\qquad \lim_{S_t \to 1} \lambda_t = -\infty \qquad \text{and} \qquad \lambda_t(0) = 1 + \frac{\delta(1 - \Phi_t \Psi_t)}{2(1 - \delta)} > 1$$

Given these properties, the curves must intersect once and only once in the relevant range. □

PROOF OF PROPOSITION 1. (a) Reorganizing Equation (36), we can write

$$f(S_t) = (1 - S_t)^2 \left\{ 1 + 2\left[\frac{1-\delta}{\delta}\right][1 - \lambda(S_t)] \right\} = \Phi_t \Psi_t \tag{A.21}$$

Lemma 3 implies that, in equilibrium, $f'(S_t) < 0$ and that $f(\cdot)$ represents a one-to-one correspondence. The change in enrollments between periods t and $t+1$ therefore can be decomposed implicitly as follows:

$$\begin{aligned} f(S_{t+1}) - f(S_t) &= \left[\Phi_{t+1}\Psi_{t+1} - \Phi_t\Psi_t\right] \\ &= \left[\Psi_{t+1}(\Phi_{t+1} - \Phi_t) + \Phi_t(\Psi_{t+1} - \Psi_t)\right] \end{aligned} \tag{A.22}$$

But the change in the productivity effect is

$$\Phi_{t+1} - \Phi_t = \int_{\underline{\eta}_t}^{1} \eta^{1-\rho} F_t(d\eta) - \int_{\underline{\eta}_{t-1}}^{1} \eta^{1-\rho} F_{t-1}(d\eta) \tag{A.23}$$

Partial integration yields

$$\Phi_{t+1} - \Phi_t = (1-\rho) \int_{\underline{\eta}_{t-1}}^{1} \eta^{-\rho}\left[F_{t-1}(\eta) - F_t(\eta)\right] d\eta \tag{A.24}$$

Similarly, the change in the occupational effect can be written as

$$\Psi_{t+1} - \Psi_t = -(1-\rho) \int_{\underline{\eta}_{t-1}}^{1} \eta^{\rho-2}\left[F_{t-1}(\eta) - F_t(\eta)\right] d\eta \tag{A.25}$$

Substituting into the preceding expression yields

$$f(S_{t+1}) - f(S_t) = (1-\rho) \int_{\underline{\eta}_{t-1}}^{1} (\Psi_{t+1}\eta^{-\rho} - \Phi_t \eta^{\rho-2})\left[F_{t-1}(\eta) - F_t(\eta)\right] d\eta \tag{A.26}$$

But for any value of η, it must be the case that $F_{t-1}(\eta) - F_t(\eta) < 1$. Hence it follows that

$$\begin{aligned} f(S_{t+1}) - f(S_t) &< (1-\rho)\left[\int_{\underline{\eta}_{t-1}}^{1} \left(\Psi_{t+1}\eta^{-\rho} - \Phi_t\eta^{\rho-2}\right) d\eta\right] \\ &= \left[\Psi_{t+1}\left(1 - \underline{\eta}_{t-1}^{1-\rho}\right) + \Phi_t\left(1 - \underline{\eta}_{t-1}^{\rho-1}\right)\right] \\ &= \left(1 - \underline{\eta}_{t-1}^{1-\rho}\right)\left(\Psi_{t+1} - \frac{\Phi_t}{\underline{\eta}_{t-1}}^{1-\rho}\right) \\ &< \left(1 - \underline{\eta}_{t-1}^{1-\rho}\right)\left(\Psi_t - \frac{\Phi_t}{\underline{\eta}_{t-1}}^{1-\rho}\right) \end{aligned} \tag{A.27}$$

Thus a sufficient condition for $f(S_{t+1}) < f(S_t)$ and hence for $S_{t+1} > S_t$ is that

(A.28) $$\frac{\Phi_t}{\Psi_t} > \underline{\eta}_{t-1}^{1-\rho}$$

which, using Equation (32), is equivalent to

(A.29) $$\underline{\eta}_t^2 > A_t \underline{\eta}_{t-1}^{1-\rho}$$

Since, by assumption, $\underline{\eta}_t > \underline{\eta}_{t-1}$ and $A_t < \delta/(2-\delta)$, this condition is satisfied if

(A.30) $$\underline{\eta}_1 > \left(\frac{\delta}{2-\delta}\right)^{1/1+\rho}$$

Note that $S_{t+1} > S_t$ implies that $L_{t+1} < L_t$.

(b) Recall that the lower support on the distribution of relative income is given by

(A.31) $$\underline{\eta}_t = \frac{w_t}{\bar{v}_t} = \left(\frac{A_t \Phi_t}{\Psi_t}\right)^{1/2}$$

Since $F_t(\cdot)$ dominates $F_{t-1}(\cdot)$ in the first-order stochastic sense, it follows immediately that $\Phi_{t+1} > \Phi_t$ and $\Psi_{t+1} < \Psi_t$. In equilibrium, if $S_{t+1} > S_t$, as implied by part (a), then $\lambda_{t+1} < \lambda_t$ (see Lemma 2). Since

(A.32) $$\frac{dA_t}{d\lambda_t} = -\frac{\delta(1-\delta)(1-\lambda_t)^2 + \delta^2(1-\lambda_t)}{2[(1-\delta)(1-\lambda_t)+\delta]^2} < 0$$

it follows that $A_{t+1} > A_t$. Thus it must be the case that

(A.33) $$\frac{w_{t+1}}{\bar{v}_{t+1}} > \frac{w_t}{\bar{v}_t}$$

(c) Since $L_{t+1} < L_t$ and $w_{t+1}/\bar{v}_{t+1} > w_t/\bar{v}_t$, it must be the case that $F_{t+1}(\eta) < F_t(\eta)\ \forall\ \eta < w_{t+1}/(1-\lambda_{t+1})\bar{v}_{t+1}$ (see Figure 5). For relative incomes $\eta > w_{t+1}/(1-\lambda_{t+1})\bar{v}_{t+1}$, the change in the distribution of relative incomes is given by

(A.34) $$F_{t+1}(\eta) - F_t(\eta) = \int_{\eta^{1/1-\rho}}^{1} \frac{\eta}{\tilde{\eta}^{1-\rho}}[F_t(d\tilde{\eta}) - F_{t-1}(d\tilde{\eta})]$$

Partial integration yields

(A.35) $$F_{t+1}(\eta) - F_t(\eta) = (1-\rho)\int_{\eta^{1/1-\rho}}^{1} \left(\frac{\eta}{\tilde{\eta}^{2-\rho}}\right)[F_t(\tilde{\eta}) - F_{t-1}(\tilde{\eta})]d\tilde{\eta}$$

Since $F_t(\cdot)$ dominates $F_{t-1}(\cdot)$ in the first-order stochastic sense, this expression is negative. It follows that $F_{t+1}(\cdot)$ dominates $F_t(\cdot)$ in the first-order stochastic sense.

(d) With Cobb-Douglas technology, the fact that $L_{t+1} < L_t$ implies that

(A.36) $$\frac{w_{t+1}}{Y_{t+1}} > \frac{w_t}{Y_t}$$

Hence the linear segment OA of the time $t+1$ Lorenz curve (see Figure 4) must lie everywhere above that at time t. Moreover, since it is equal to $(1-\lambda)^{-1}$ times the slope of OA, the slope of the convex segment of the time $t+1$ Lorenz curve at A must exceed that at time t. Since $\Phi_{t+1} > \Phi_t$, it is immediate from Equation (35) that

$$\frac{\bar{v}_{t+1}}{Y_{t+1}} < \frac{\bar{v}_t}{Y_t} \tag{A.37}$$

Hence the slope of the time $t+1$ Lorenz curve at B must be less than that at time t. Since both Lorenz curves are strictly convex over this range, the segment AB of the time $t+1$ Lorenz curve must lie everywhere above that at time t. It follows that the distribution of incomes at time $t+1$ exhibits less inequality than that at time t in the Lorenz dominance sense.

Given that $F_1(\cdot)$ dominates $F_0(\cdot)$ in the first-order stochastic sense, parts (a), (b), (c), and (d) of Proposition 1 must hold for all t by induction. □

PROOF OF PROPOSITION 2. A sufficient condition for the distribution to converge is that $P^*(\cdot \mid \eta)$ satisfies the monotone mixing condition (see Hopenhayn and Prescott, 1992). The MMC requires that $\exists\ \hat{\eta} \in (\underline{\eta}^*, 1)$ such that $\forall\ \epsilon > 0\ \exists N$ such that

$$P^N([\underline{\eta}, \hat{\eta}]|1) > \epsilon \qquad \text{and} \qquad P^N([\hat{\eta}, 1] \mid \underline{\eta}) > \epsilon \tag{A.38}$$

where $P^N([\eta_1, \eta_2]|\eta)$ denotes the probability that the Nth descendant of an agent with relative income η earns a relative income in the interval $[\eta_1, \eta_2]$.

To verify the first condition, consider the offspring of an agent with the highest possible level of relative income $\eta = 1$. The probability that such an agent becomes a production worker is

$$z_c(1, w_t) = \left(\frac{1}{1-\lambda}\right)\frac{w_t}{\bar{v}_t} > 0 \tag{A.39}$$

Since there is a strictly positive probability that the offspring of the richest agent becomes a wage laborer, it follows immediately that the first condition holds. To verify that the second condition holds, consider the child of a production worker who earns the lowest possible relative income $\underline{\eta}^* < A^{1/2}$. The probability that such a child becomes a nonproduction worker is

$$\begin{aligned} 1 - z_c(\underline{\eta}^*, w_t) &= 1 - \left(\frac{1}{1-\lambda}\right)\frac{\underline{\eta}^*}{(\underline{\eta}^*)^{1-\rho}} \\ &> 1 - \frac{A^{\rho/2}}{1-\lambda} = 1 - \left[\frac{\delta}{\delta + 2(1-\delta)(1-\lambda)}\right]^{\rho} > 0 \end{aligned} \tag{A.40}$$

Thus there is always a positive probability of an agent becoming skilled, even at the lowest possible level of income. For any $\varpi > 0$, there is positive probability the child of a wage laborer receives a quality draw $z \in [Z - \varpi, Z]$. Similarly, with positive probability, his or her grandchild receives a quality draw $z \in [Z - \varpi, Z]$, and so on. For a sufficiently small ϖ and after a sufficiently large number of generations, say,

N, there is positive probability that the Nth descendant of the wage laborer receives a relative human capital level in any neighborhood of 1. □

PROOF OF PROPOSITION 3. (a) From Equation (12), $\frac{d\lambda_t}{dX}|_{S_t=\bar{S}} < 0$, so the EE curve in Figure 3 shifts down when X is increased. It follows that S_t increases and λ_t decreases with X.

(b) It is immediate that the slope of the linear segment of the Lorenz curve $\underline{\sigma}_t$ increases. Rewriting Equation (35) yields

$$\overline{\sigma}_t = \frac{\bar{v}_t}{Y_t} = \frac{2(1-\delta)}{\Phi_t} + \frac{\delta}{\Phi_t(1-\lambda_t)} \tag{A.41}$$

Thus, since λ_t declines, the slope of the convex segment of the Lorenz curve at B decreases. Since the segment AB is strictly convex, it follows that the entire Lorenz curve must shift up.

PROOF OF LEMMA 4. The EQ curve is given by Equation (48). From Lemma 3, we know that, in equilibrium, $f_S < 0$. We also can obtain the following derivatives:

$$f_X = (1-S)^2 C \frac{\theta_b}{\theta_h} \mu X^{\mu-1} S^{\alpha\mu} = \frac{\mu}{X}\left[f(S,X) - (1-S)^2(1+C)\right] > 0 \tag{A.42}$$

$$f_{XX} = -\left(\frac{1-\mu}{X}\right) f_X < 0 \quad \text{and} \quad f_{XS} = \frac{\mu}{X}\left[f_S + 2(1-S)(1+C)\right] \tag{A.43}$$

where

$$C = 2\left(\frac{1-\delta}{\delta}\right)\left(\frac{\mu}{\rho+\mu}\right) \tag{A.44}$$

The EQ curve is therefore positively sloped and strictly convex:

$$\left.\frac{dS}{dX}\right|_{EQ} = -\frac{f_X}{f_S} > 0 \tag{A.45}$$

$$\left.\frac{d^2S}{dX^2}\right|_{EQ} = -\frac{f_X}{f_S}\left(\frac{f_S f_{XX} - f_X f_{SX}}{f_S f_X}\right) = \frac{f_X}{f_S}\left[\frac{1+2\mu(1-S)(1+C)}{X}\right] < 0 \tag{A.46}$$

Also,

$$\lim_{x\to\infty} S = 1 \quad \text{and} \quad S > 0 \quad \text{if } X = 0 \tag{A.47}$$

The XX curve is positively sloped and either strictly concave or strictly convex:

$$\left.\frac{dS}{dX}\right|_{XX} = \frac{\alpha\mu}{1-\mu}\left(\frac{\theta_b}{\theta_h}\right)^{1/(1-\mu)} S^{\alpha\mu/(1-\mu)-1} > 0 \tag{A.48}$$

$$\left.\frac{d^2S}{dX^2}\right|_{XX} = \left(\frac{\alpha\mu}{1-\mu}\right)\left(\frac{\alpha\mu}{1-\mu} - 1\right)\left(\frac{\theta_b}{\theta_h}\right)^{1/(1-\mu)} S^{\alpha\mu/(1-\mu)-2} \times \begin{cases} < 0 & \text{if } \alpha\mu < 1-\mu \\ > 0 & \text{if } \alpha\mu > 1-\mu \end{cases} \tag{A.49}$$

Also,

$$X = \left(\frac{\theta_b}{\theta_h}\right)^{1/(1-\mu)} > 0 \quad \text{if } S = 1 \qquad \text{and} \qquad X = 0 \quad \text{if } S = 0 \tag{A.50}$$

Given these properties, the curves must intersect once and only once on the relevant range.

PROOF OF PROPOSITION 5. Let $X = x_b/x_h$ represent the allocation of public resources. Then the growth rate in Equation (44) may be expressed as

$$\Gamma = k - (1-\rho)\log\overline{\sigma} + \rho\delta\log XS^{\alpha\mu} - \rho\log(1 + XS^{\alpha\mu}) + \left(\frac{\rho - \rho\delta - \mu\delta}{\mu}\right)\log(\theta_b X^{\mu} S^{\alpha\mu} + \theta_h) \tag{A.51}$$

where k represents terms that do not depend on the allocation of public resources. To begin with, ignore the effects on growth resulting from increased parental inequality. Then, differentiating with respect to x yields

$$\frac{d\Gamma}{dx} = \frac{\rho\delta}{X} - \frac{\rho S^{\alpha\mu}}{1 + XS^{\alpha\mu}} + (\rho - \rho\delta - \mu\delta)\left(\frac{\theta_b X^{\mu-1} S^{\alpha\mu}}{\theta_b X^{\mu} S^{\alpha\mu} + \theta_h}\right) \tag{A.52}$$

Rearranging yields

$$\frac{d\Gamma}{dx} = \frac{\rho\delta\theta_h - \mu\delta\theta_b X^{\mu} S^{\alpha\mu}}{X(\theta_b X^{\mu} S^{\alpha\mu} + \theta_h)} + \rho S^{\alpha\mu}\left[\frac{\theta_b X^{\mu-1} S^{\alpha\mu} - \theta_h}{(1 + XS^{\alpha\mu})(\theta_b X^{\mu} S^{\alpha\mu} + \theta_h)}\right] \tag{A.53}$$

Suppose that the allocation of resources X_1 is such that the quality of the education received by high-skilled workers is maximized. Equation (47) holds, so

$$X_1 = \left(\frac{\theta_b}{\theta_h} S^{\alpha\mu}\right)^{1/(1-\mu)} \tag{A.54}$$

and the second term in Equation (A.53) is zero. At allocation X_1,

$$\text{Sign}\left(\frac{d\Gamma}{dX}\right) = \text{sign}\left(\rho\theta_h^{1/(1-\mu)} - \mu\theta_b^{1/(1-\mu)} S^{\alpha\mu/(1-\mu)}\right) \tag{A.55}$$

From Equation (13) this expression must be positive for all $S \in [0,1]$, so growth could be increased by transferring resources toward basic education (increasing X). Now suppose that the allocation of resources X_2 is such that $\lambda^* = 0$, so the first term of Equation (A.53) would be zero and

$$X_2 = \left(\frac{\rho\theta_h}{\mu\theta_b}\right)^{1/\mu} \frac{1}{S^{\alpha}} \tag{A.56}$$

It follows that at this allocation

$$\text{Sign}\left(\frac{d\Gamma}{dX}\right) = \text{sign}\left(\mu^{(1-\mu)/\mu}\theta_b^{1/\mu} S^{\alpha} - \rho^{(1-\mu)/\mu}\theta_h^{1/\mu}\right) \tag{A.57}$$

From Equation (13) this expression must be negative for all $S \in [0, 1]$, so growth could be increased by transferring resources toward higher education (reducing X).

A transfer of resources from higher to basic education unambiguously lowers the value of $\overline{\sigma}$ and generates a first-order stochastic increase in the distribution of *relative* incomes that raises future enrollments S [as shown in Proposition 1(a)]. Both these effects raise growth. However, if $X > X_2$, no agent would enter higher education, and growth would be zero. It follows that the growth maximizing allocation of resources $\hat{X}$ must satisfy Equation (49). □

REFERENCES

Aghion, P., and P. Bolton, "A Theory of Trickle-Down Growth and Development," *Review of Economic Studies* 64 (1997), 151–172.

Banerjee, A.V., and A.F. Newman, "Occupational Choice and the Process of Development," *Journal of Political Economy* 101 (1993), 274–298.

Barro, R.J., "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98 (1990), 103–125.

Bénabou, R., "Unequal Societies," National Bureau of Economic Research Working Paper No. 5583, 1995.

———, "Heterogeneity, Stratification and Growth: Macroeconomic Implications of Community Structure and School Finance," *American Economic Review* 86 (1996a), 584–609.

———, "Inequality and Growth," *NBER Macroeconomics Annual* (1996b).

Benhabib J., and M. Spiegel, "The Role of Human Capital in Economic Development: Evidence from Aggregate Cross-Country Data," *Journal of Monetary Economics* 24 (1994), 143–173.

Birdsall N., D. Ross, and R. Sabot, "Inequality and Growth Reconsidered: Lessons from East Asia," *World Bank Economic Review* 9 (1995) 477–508.

Boissière, M., J.B. Knight, and R.H. Sabot (1985), "Earnings, Schooling, Ability and Cognitive Skills," *American Economic Review* 75 (1985) 1016–1030.

Eckstein, Z., and I. Zilcha, "The Effects of Compulsory Schooling on Growth, Income Distribution and Welfare," *Journal of Public Economics* 54 (1994), 339–359.

Galor, O., and J. Zeira, "Income Distribution and Macroeconomics," *Review of Economic Studies* 60 (1993) 35–52.

Glomm, G., and B. Ravikumar, "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality" *Journal of Political Economy* 100 (1992), 818–834.

Hopenhayn, H., and E.C. Prescott, "Stochastic Monotonicity for Dynamic Economies," *Econometrica* 60 (1992) 1387–1406.

Jimenez, E., "The Public Subsidization of Education and Health in Developing Countries: A Review of Equity and Efficiency," *Research Observer* 1 (1986), 141–159.

Jovanovic, B., "The Diversification of Production," *Brookings Papers on Economic Activity* (1993) 197–220.

Lloyd–Ellis, H., and D. Bernhardt, "Enterprise, Inequality and Economic Development," *Review of Economic Studies*, forthcoming, 2000.

Lucas, R.E., Jr., "On the size distribution of business firms," *Bell Journal of Economics* 9 (1978) 508–523.

Mingat, A., and J.P. Tan, "On Equity in Education Again: An International Comparison," *Journal of Human Resources* 20 (1985), 298–308.

Psacharopoulos, G., "Returns to Education: A Further International Update and Implications," *Journal of Human Resources* 20 (1985) 590–604.

Park, Y., D. Ross, and R. Sabot, "Educational Expansion and the Inequality of Pay in Brazil and Korea," in N. Birdsall and R. Sabot, eds., *Opportunities Foregone: Education, Growth and Inequality in Brazil* (Washington, D.C.: Inter-American Development Bank, 1996).

Todaro, M.P., *Economic Development in the Third World* (London: Longmans, 1985).

WILLIAMSON J., "Human Capital Deepening, Inequality and Demographic Events Along the Asia-Pacific Rim," in N. Ogawa, G. W. Jones, and J. Williamson, eds., *Human Resources in Development Along the Asia-Pacific Rim* (Singapore: Oxford University Press, 1993).

WOOD, A., *North-South Trade, Employment and Inequality* (Oxford: Clarendon Press, 1994).

WORLD BANK, *World Development Report*, various issues.