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NONLINEARITY IN THE FED'S MONETARY POLICY RULE

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SUMMARY

This paper investigates the nature of nonlinearities in the monetary policy rule of the US Federal Reserve (Fed) using the flexible approach to nonlinear inference. We find that while there is significant evidence of nonlinearity for the period to 1979, there is little such evidence for the subsequent period. Possible asymmetries in the Fed's reactions to inflation deviations from target and the output gap in the 1960s and 1970s may tell part of the story, but do not capture the entire nature of the nonlinearity. The inclusion of the interaction between inflation deviations and the output gap, as recently proposed, appears to characterize the nonlinear policy rule more adequately. Copyright © 2005 John Wiley & Sons, Ltd.

1. INTRODUCTION

Since the early 1990s, there has been a great deal of research on monetary policy reaction functions from academic institutes, central bankers and private financial firms. In particular, the so-called Taylor rule (Taylor, 1993) has received considerable attention, in large part because this simple rule describes the actual behaviour of the US Federal Funds rate rather surprisingly well. According to this rule, the Federal Reserve (Fed) sets the Federal Funds rate using current values of real output and inflation in relation to their target values. In a similar context, Clarida *et al.* (1998, 2000) examine a forward-looking monetary policy reaction function in which the central bank proactively adjusts interest rates using expected future gaps in inflation and output compared with target values. The theoretical basis of linear reaction functions of this type rests on two key assumptions, namely that the central bank has a quadratic loss function and that the aggregate supply relation (Phillips curve) is linear.[1]

Recently, however, both of these assumptions have been challenged. In relation to the first, Bec *et al.* (2002), Cukierman (2000), Gerlach (2000), Nobay and Peel (2003), and Ruge-Murcia (2002, 2004) consider nonlinear specifications for the loss function of the central bank, reflecting either asymmetric preferences on inflation or preferences that depend on the state of the business cycle. Dolado *et al.* (2005) and Schaling (1999) relax the second assumption by assuming inflation is a convex function of the output gap, implying a nonlinear Phillips curve. Dolado *et al.* (2002) construct a general model, allowing the joint analysis of both types of departure from the linear-quadratic setup.

This recent literature has provided evidence in favour of nonlinear monetary policy rules. However, all the empirical studies to date assume specific parametric models. In reality, we do not directly observe either the central bank's preferences or the aggregate Phillips curve

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[1] For more general study on monetary policy rule, see Clarida *et al.* (1999).

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in the economy, so that there exists an unbounded universe of possible alternative nonlinear specifications. Because rejection of linearity against a specific nonlinear alternative does not necessarily imply the validity of that nonlinear model, we believe that it is important to investigate the nature of any nonlinearities in the central bank's reaction function while avoiding specific parametric assumptions. To this end, the present paper applies the methodology recently developed by Hamilton (2001) to address this question. This methodology provides a test of the null hypothesis of linearity against a broad range of alternative nonlinear models, consistent estimation of what the nonlinear relation looks like, and formal comparison of alternative nonlinear models. Hamilton (2003) and Kim (2003) show that this methodology is very useful for characterizing the nonlinear relation between oil price changes and GDP growth and nonlinearity in the term structure respectively.

Following Clarida *et al.* (2000) and others, we consider the monetary policy reaction function for the US economy from 1960, both as a single sample and as subsamples before and after Volcker's appointment as the Fed Chairman in 1979. Although we find no evidence of nonlinearity in US monetary policy using the whole period, we find relatively strong evidence of nonlinearity for the pre-Volcker era, but little such evidence in the Volcker–Greenspan era. We also explicitly test whether parametric representations previously suggested for the monetary policy rule capture all the nonlinearity. We find that the specification in which the Fed reacts to the interaction between inflation and the output gap, as proposed by Dolado *et al.* (2005), adequately characterizes the nonlinear policy rule during the 1960s and 1970s.

The plan of the paper is as follows. Section 2 describes the Hamilton (2001) methodology applied in this paper. A flexible version of a nonlinear monetary policy rule is proposed and empirical results, including evaluation of specific nonlinear formulations, are provided in Section 3. Conclusions are offered in Section 4.

2. A FLEXIBLE APPROACH TO NONLINEAR INFERENCE

Hamilton (2001) proposes a new framework that combines the advantages of nonparametric and parametric methods. While the procedure does not assume any specific functional form for the conditional mean function, parameters are used to characterize this function and these parameters are estimated by maximum likelihood or Bayesian methods. Inference is based on classical econometric theory.

Consider the general nonlinear regression model

$$y_t = \mu(\mathbf{x}_t) + \gamma'\mathbf{z}_t + \varepsilon_t \tag{1}$$

where y_t is a scalar dependent variable, $\mathbf{x}_t$ and $\mathbf{z}_t$ are k- and p-dimensional vectors of explanatory variables, and ε_t is an error term with mean zero that is independent of $\mathbf{x}_t$ and $\mathbf{z}_t$ and of lagged values y_{t-j}, $\mathbf{x}_{t-j}$, $\mathbf{z}_{t-j}$ $(j = 1, 2, \ldots)$. Equation (1) allows a subset of variables $\mathbf{z}_t$ for which the researcher is willing to assume linearity, thereby gaining efficiency by imposing this restriction. In our monetary policy application, y_t is the interest rate, with lagged interest rates in $\mathbf{z}_t$. Nonlinearity is explored in relation to inflation and output gap measures in $\mathbf{x}_t$. In contrast to previous analyses of nonlinear monetary policy rules (see Sections 3.1 and 3.4) we treat the form of function $\mu(\cdot)$ as unknown. Following Hamilton (2001), we view this function as the outcome of a random field. Specifically, the value of the function $\mu(\mathbf{x}_t)$ at $\mathbf{x}_t = \boldsymbol{\tau}$ is treated as being a Gaussian random variable with mean equal to the linear component $\alpha_0 + \boldsymbol{\alpha}'\boldsymbol{\tau}$ and variance λ^2, where α_0, $\boldsymbol{\alpha}$, and λ

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are population parameters to be estimated. In the special case of $\lambda = 0$, then $\mu(\mathbf{x}_t)$ is fixed and equation (1) becomes the usual linear regression model. In general, the parameter λ measures the overall extent of nonlinearity.

The basic idea of the method is that nonlinearity implies the values for $\mu(\mathbf{x}_t)$ and $\mu(\mathbf{x}_s)$ will be positively correlated for periods t and s whenever the vectors $\mathbf{x}_t$ and $\mathbf{x}_s$ are close to each other. The key is then parameterizing this correlation based on the distance measure $h_{st} = (1/2)\left[\sum_{i=1}^{k} g_i^2(x_{is} - x_{it})^2\right]^{1/2}$ where x_{it} denotes the ith element of the vector $\mathbf{x}_t$ and $g_1, g_2, \ldots, g_k$ are k additional parameters to be estimated. Hamilton proposes that $\mu(\mathbf{x}_s)$ should be uncorrelated with $\mu(\mathbf{x}_t)$ if $\mathbf{x}_s$ is sufficiently far away from $\mathbf{x}_t$. More precisely:

$$E\{[\mu(\mathbf{x}_s) - \alpha_0 - \boldsymbol{\alpha}'\mathbf{x}_s][\mu(\mathbf{x}_t) - \alpha_0 - \boldsymbol{\alpha}'\mathbf{x}_t]\} = 0 \text{ if } h_{st} > 1 \tag{2}$$

However, when $0 \leq h_{st} \leq 1$, this correlation should increase as h_{st} decreases, with the correlation going to unity as h_{st} goes to zero. In our context, where the nonlinear part of the model includes $k = 2$ explanatory variables, the correlation is assumed to be given by

$$\text{Corr}(\mu(\mathbf{x}_s), \mu(\mathbf{x}_t)) = H_2(h_{st}) \text{ if } 0 \leq h_{st} \leq 1 \tag{3}$$

where

$$H_2(h_{st}) = 1 - (2/\pi)[h_{st}(1 - h_{st}^2)^{1/2} + \sin^{-1}(h_{st})] \tag{4}$$

For the general specification and rationalization of this correlation, see Lemma 2.1 and Theorem 2.2 in Hamilton (2001). It should be emphasized that $H_k(.)$ does not assume any parametric form for the functional relation $\mu(.)$ itself, but rather it parameterizes the correlation between pairs of random outcomes $\mu(\mathbf{x}_s)$ and $\mu(\mathbf{x}_t)$. The coefficient g_i determines the extent to which variation in the ith element of $\mathbf{x}_t$ contributes to nonlinear variation in $\mu(\mathbf{x}_t)$. For g_i small, the value of $\mu(\mathbf{x}_t)$ changes little when the value of the corresponding explanatory changes, with $g_i = 0$ implying linearity of $\mu(\mathbf{x}_t)$ with respect to that variable.

Prior to estimation it is appropriate to determine whether nonlinearity exists by testing $H_0 : \lambda^2 = 0$. As is usual in nonlinear modelling, certain parameters are unidentified under the null of linearity. In the present context, this applies to $g_1, g_2, \ldots, g_k$. For the purpose of the nonlinearity test, Hamilton suggests that the lack of identification can be avoided by setting $g_i = 2\left[k\left(T^{-1}\sum_{t=1}^{T}(x_{it} - \bar{x}_i)^2\right)\right]^{-1/2}$, thereby scaling in terms of the individual sample standard deviations and the number of explanatory variables. Then, for T sample observations, the $(T \times T)$ matrix $\mathbf{H}$ of correlations can be formed, with the row $\boldsymbol{s}$, column $\boldsymbol{t}$ element $H_k\{h_{st}\}$ given in Equation (4) for $k = 2$ and $0 \leq h_{st} \leq 1$, or zero when $h_{st} > 1$. The Lagrange multiplier (LM) test of the null hypothesis can be obtained by using the residuals from an ordinary least-squares (OLS) linear regression of y_t on $(1, \mathbf{x}_t', \mathbf{z}_t')'$. Denoting the OLS residual vector by $\widehat{\boldsymbol{\varepsilon}}$ and the OLS squared standard error as $\widetilde{\sigma}^2 = (T - k - p - 1)^{-1}\widehat{\boldsymbol{\varepsilon}}'\widehat{\boldsymbol{\varepsilon}}$, and the $(T \times T)$ projection matrix $\mathbf{M} = \mathbf{I}_T - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ where $\mathbf{X}$ is a $(T \times (1 + k + p))$ matrix whose tth row is given by $(1, \mathbf{x}_t', \mathbf{z}_t')$ and $\mathbf{I}_T$ is the $(T \times T)$ identity matrix, the test statistic is

$$\nu^2 = \frac{[\widehat{\boldsymbol{\varepsilon}}'\mathbf{H}\widehat{\boldsymbol{\varepsilon}} - \widetilde{\sigma}^2 tr(\mathbf{MHM})]^2}{\widetilde{\sigma}^4(2\ \text{Tr}\{[\mathbf{MHM} - (T - k - p - 1)^{-1}\mathbf{M}tr(\mathbf{MHM})]^2\})} \tag{5}$$

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Under the linearity null hypothesis, ν^2 has an asymptotic $\chi^2(1)$ distribution. Dahl's (2002) Monte Carlo investigations suggest that this test has good size and power properties against a variety of nonlinear alternatives.

In the presence of nonlinearity, Hamilton writes equation (1) as

$$\begin{aligned} y_t &= \alpha_0 + \boldsymbol{\alpha}'\mathbf{x}_t + \boldsymbol{\gamma}'\mathbf{z}_t + \lambda m(\mathbf{x}_t) + \varepsilon_t \\ &= \alpha_0 + \boldsymbol{\alpha}'\mathbf{x}_t + \boldsymbol{\gamma}'\mathbf{z}_t + u_t \end{aligned} \tag{6}$$

where $m(.)$ is the realization of a scalar-valued Gaussian random field with mean zero, unit variance and covariance function given by equations (2) to (4). Assuming that the regression disturbance ε_t is independent and identically distributed (i.i.d.) $N(0, \sigma^2)$, the composite disturbance $u_t = \lambda m(\mathbf{x}_t) + \varepsilon_t$ is also Gaussian. With independence between $(\mathbf{x}_t', \mathbf{z}_t')'$ and ε_t, this specification implies a generalized least-squares (GLS) regression model of the form

$$\mathbf{y}|\mathbf{X} \sim N(\mathbf{X}\boldsymbol{\beta}, \mathbf{P}_0 + \sigma^2\mathbf{I}_T)$$

where $\mathbf{y} = (y_1, y_2, \ldots, y_T)'$, $\boldsymbol{\beta}$ is the $(1 + k + p)$-dimensional vector $(\alpha_0, \boldsymbol{\alpha}', \boldsymbol{\gamma}')'$, and $\mathbf{P}_0$ is a $(T \times T)$ matrix whose row s, column t element is given by $\lambda^2 H_k(h_{st})\delta_{[h_{st}<1]}$ with h_{st} defined above, and the function $H_k(.)$ is specified in equation (4) for the case $k = 2$. The indicator function $\delta_{[.]}$ is unity when the condition [.] holds, and zero otherwise.

In addition to the linear regression parameters $(\alpha_0, \boldsymbol{\alpha}, \boldsymbol{\gamma})$ and σ^2, parameters to be estimated are the variance of the nonlinear regression error λ^2, which governs the overall importance of the nonlinear component, and the parameters $(g_1, g_2, \ldots, g_k)$ determining the variability of the nonlinear component with respect to each explanatory variable in $\mathbf{x}_t$. As the above discussion implies, estimation and inference can be achieved by a GLS Gaussian regression. However, Hamilton (2001) also describes the use of numerical Bayesian methods for the evaluation of the posterior distribution of any statistics of interest. The optimal inference of the value of the unobserved function $\mu(\mathbf{x}^*)$ at an arbitrary point $\mathbf{x}^*$ is given by

$$\widehat{\mu}(\mathbf{x}^*) = \alpha_0 + \alpha'\mathbf{x}^* + \mathbf{q}'(\mathbf{P}_0 + \sigma^2\mathbf{I}_T)^{-1}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) \tag{7}$$

where the $(T \times 1)$ vector $\mathbf{q}$ has tth element $\lambda^2 H_k(h_t^*)\delta_{[h_t^*<1]}$ for $h_t^* = \frac{1}{2}\left[\sum_{i=1}^k g_i^2(x_{it} - x_i^*)^2\right]^{1/2}$, in which x_{it} denotes the ith element of $\mathbf{x}_t$ and x_i^* is the ith element of $\mathbf{x}^*$. Hamilton shows that $\widehat{\mu}(\mathbf{x}^*)$ converges to the true value $\mu(\mathbf{x}^*)$ for any $\mu(.)$ from a broad class of continuous functions. This permits the calculation of confidence intervals, using equation (7) along with its known standard error for each given parameter vector in conjunction with values of $\alpha_0, \boldsymbol{\alpha}, \boldsymbol{\gamma}, \sigma, \lambda$, and $\mathbf{g} = (g_1, g_2, \ldots, g_k)'$ generated from their posterior distributions, and the examination of the resulting distribution of inferences.

Based on a Monte Carlo investigation, Dahl (2002) shows that in many situations Hamilton's random field-based estimator is substantially more accurate than the nonparametric spline smoother. He also finds that the procedure is useful in finite samples for characterizing a wide range of nonlinear time series models.

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3. RESULTS

As noted in Section 1, the linear monetary policy rules used by many authors are based on specific assumptions.[2] Here, we briefly review the grounds on which they have recently been challenged, before turning to the results of our empirical analysis of nonlinearity.

3.1. Monetary Policy Rules

As usual, we assume that monetary policy is conducted by a central bank that chooses the sequence of short-term interest rates in order to minimize the present discounted value of its loss function, which depends on both inflation and output in relation to their target values. Formally, the central bank faces the following problem:

$$\underset{\{i_{t+\tau}\}_{\tau=0}^{\infty}}{\text{Min}} E_t \sum_{\tau=0}^{\infty} \delta^{\tau} L(\widetilde{\pi}_{t+\tau}, \widetilde{y}_{t+\tau}) \tag{8}$$

such that

$$\pi_{t+1} = \pi_t + f(\widetilde{y}_t) + u_{t+1} \tag{9}$$

$$\widetilde{y}_{t+1} = e\widetilde{y}_t + g(r_t) + \eta_{t+1} \tag{10}$$

where δ is the discount factor, $L(.)$ is the unrestricted general loss function of the central banker, $f(.)$ and $g(.)$ are possibly nonlinear functions, π_t is the inflation rate at time t, i_t is the nominal interest rate, $\widetilde{\pi}_{t+\tau}$ is the expected inflation deviation from the inflation target (π^*) at time $t+\tau$, $\widetilde{y}_t$ is the output gap, e is the parameter on the output gap, $r_t = i_t - \pi_t$ is the real interest rate, and u_{t+1} and η_{t+1} are shocks. Equations (9) and (10) describe the supply side (i.e. Phillips curve) and the aggregate demand (AD) of the economy respectively. As general aggregate supply (AS) and AD relations, we assume that $\partial f/\partial \widetilde{y}_t > 0$, $0 \le e < 1$, and $\partial g/\partial r_t < 0$. This is a generalization of the setup of Svensson (1997) that is the basis of many studies. The specific policy rule of a central bank then depends on the functional forms of $L(.)$, $f(.)$, and $g(.)$, with the linear rule being a special case.

Cukierman (2000) and Bec *et al.* (2002) specify the loss function as dependent on the state of the business cycle, with the more general function of Bec *et al.* (2002) being

$$L(\pi_t, \widetilde{y}_t) = \tfrac{1}{2}[\widetilde{\pi}_t^2 + \omega_{\mathrm{e}}\widetilde{y}_t^2]\delta_{[\widetilde{y}_{t-p}>0]} + \tfrac{1}{2}[\widetilde{\pi}_t^2 + \omega_{\mathrm{r}}\widetilde{y}_t^2]\delta_{[\widetilde{y}_{t-p}\le 0]}$$

where ω_{e} and ω_{r} are the relative weights on output stabilization in expansion (e) and recession (r) respectively, and p is the lag.[3] Alternatively, Dolado *et al.* (2002) adopt the linex function in inflation deviations as the loss function, namely

$$L(\pi_t - \pi^*) = \frac{\exp[\theta(\pi_t - \pi^*)] - \theta(\pi_t - \pi^*) - 1}{\theta^2}$$

[2] Examples of the empirical application of these linear rules include Taylor (1993, 1999), Clarida *et al.* (1998, 2000), Judd and Rudebusch (1998), and Gerlach and Schnabel (2000).

[3] For an application of regime switching techniques to the measurement of monetary policy regimes, see Owyang and Ramey (2001).

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where θ is a nonzero parameter.[4] Dolado *et al.* (2002, 2005) allow the Phillips curve, equation (9), to be convex in the inflation–output gap through the functional form $f(\widetilde{y}_t) = a\widetilde{y}_t/(1 - a\phi\widetilde{y}_t)$, $f' > 0$, $f'' > 0$, $a > 0$ and $\phi \geq 0$. Following Schaling (1999), Dolado *et al.* (2005) impose this nonlinearity on the loss function through the specification of equation (8) as $L(.) = \frac{1}{2}(\widetilde{\pi}_{t+k})^2 + \frac{s}{2}f(\widetilde{y}_{t+q})^2$, where s measures the relative importance of stabilizing the output gap.

Although each of the proposed specifications is plausible *a priori* and they may describe certain properties of the nonlinear (asymmetric) relationship between interest rates and inflation–output gap deviations from their targets, these policy rules are driven by the specific assumptions made in each study about the central bank's preferences and/or the AS curve. Perhaps surprisingly, no research appears to have yet allowed for possible nonlinearity in the AD relationship. While embedding nonlinearity in monetary policy rules in specific assumptions about the central bank's loss function or the AS/AD curves is logically attractive, it is also important to take a broader view. One way to avoid potential misspecification problems would be to leave the functions $L(.)$, $f(.)$, and $g(.)$ in equations (8)–(10) unrestricted and allow the data to tell us the form of the nonlinearity that is best supported by the data. Thus, allowing general nonlinearities in the Fed's response to inflation and the output gap, we conjecture only the following flexible monetary policy rule:

$$i_t = \mu(E_t\widetilde{\pi}_{t+k}, E_t\widetilde{y}_{t+q}) + \gamma(L)i_{i-1} + \varepsilon_t \tag{11}$$

where the function $\mu(.)$ is unrestricted, $1 - \gamma(L)$ is a stationary polynomial in the lag operator and ε_t is an error term.

More specifically, the flexible nonlinear monetary policy rule we estimate using the methodology of Hamilton (2001) is

$$i_t = c + \alpha_1 E_t\widetilde{\pi}_{t+k} + \alpha_2 E_t\widetilde{y}_{t+q} + \gamma_1 i_{t-1} + \gamma_2 i_{t-2} + \sigma[\zeta m(g_1 E_t\widetilde{\pi}_{t+k}, g_2 E_t\widetilde{y}_{t+q}) + v_t] \tag{12}$$

where $c, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \sigma, \zeta, g_1$ and g_2 are parameters to be estimated, $v_t \sim N(0, 1)$ and $m(.)$ denotes an unobserved realization from a Gaussian random field with mean zero, unit variance, and correlations given by equations (2) to (4). In comparison with equation (1), the innovation ε_t is written here as σ times v_t, $\mathbf{x}_t = (E_t\widetilde{\pi}_{t+k}, E_t\widetilde{y}_{t+q})'$ is a 2×1 vector, and the vector $\mathbf{z}_t$ contains two lags of interest rates to capture interest rate smoothing by the Fed.[5] Following previous literature, we assume that any nonlinearity in the Fed's reaction function relates only to the output gap and inflation, with lagged interest rates entering in a linear way. This implies $\mu(\mathbf{x}_t) = \alpha_0 + \alpha_1 E_t\widetilde{\pi}_{t+k} + \alpha_2 E_t\widetilde{y}_{t+q} + \sigma[\zeta m(g_1 E_t\widetilde{\pi}_{t+k}, g_2 E_t\widetilde{y}_{t+q})]$, with $\lambda = \sigma\zeta$.

When estimating the central bank reaction function described by equation (12), we consider two different policy rules: (i) a flexible nonlinear forward-looking rule in line with Clarida *et al.* (1998, 2000); (ii) a flexible nonlinear backward-looking rule of the type used by Taylor (1993). The forward-looking specification we employ avoids overlapping forecast intervals, and consequent potential problems with moving-average errors, by assuming that the target horizon in equation (12) is one-quarter for both inflation and the output gap (i.e. $k = q = 1$) and hence requires the use of forecasts.

The backward-looking model has $k = q = 0$, so that $E_t\widetilde{\pi}_{t+k} = \widetilde{\pi}_t$ and $E_t\widetilde{y}_{t+q} = \widetilde{y}_t$. As in Taylor (1993), in this case the equation is specified in terms of observed inflation π_t and not the deviation

[4] Dolado *et al.* (2002) exclude output stabilization from the loss function in order to obtain a closed-form solution to the central bank's problem.

[5] For a discussion of the interest smoothing behaviour by the Fed, see Amato and Laubach (1999) or Rudebusch (2002a).

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from target $\widetilde{\pi}_t$. The backward-looking model has the advantage over the forward-looking one, in that we do not need to specify the form of the forecasting equations assumed to be used by the Fed for inflation and the output gap, since such equations are implicitly embedded in the nonlinear backward-looking model. Thus, nonlinear forms of the AS and/or AD equations (9)/(10) could be the source of nonlinearity in the backward-looking version of equation (12).

3.2. Data

Our data are quarterly from 1960:I to 2000:IV. The interest rate is the average Federal Fund rate in the first month of each quarter, expressed at annual rates. Inflation is measured as the (annualized) rate of change of the GDP deflator (P_t) between two subsequent quarters: $\pi_t = 400(\ln(P_t) - \ln(P_{t-1}))$. The principal output gap measure we employ is the difference between real GDP and the estimate for potential real GDP constructed by the Congressional Budget Office (CBO). All these data series were downloaded from the Federal Reserve Bank of St Louis (http://research.stlouisfed.org/fred2/).

Forecast values for inflation and the output gap required for the forward-looking rule follow the specifications of Gerlach and Smets (1999) and Aksoy *et al.* (2002), so that we assume that the functions $f(\widetilde{y}_t)$ and $g(r_t)$ in equations (9) and (10) respectively are linear. We also do not impose the unit root for inflation implicitly assumed in the former. More precisely, we assume that inflation is determined by the output gap with a one period lag and past inflation rates, yielding the estimated forecasting equation:[6]

$$\widehat{\pi}_t = \underset{(0.163)}{0.148} + \underset{(0.079)}{0.551}\,\pi_{t-1} + \underset{(0.090)}{0.059}\,\pi_{t-2} + \underset{(0.090)}{0.166}\,\pi_{t-3} + \underset{(0.082)}{0.197}\,\pi_{t-4} + \underset{(0.035)}{0.145}\,\widetilde{y}_{t-1} \tag{13}$$

The output gap is assumed to depend on previous output gaps and the average real interest rate over the year ending in the previous quarter. This yields the one-step ahead forecast for the output gap as

$$\widehat{\widetilde{y}}_t = \underset{(0.100)}{0.204} + \underset{(0.079)}{1.121}\,\widetilde{y}_{t-1} - \underset{(0.119)}{0.054}\,\widetilde{y}_{t-2} - \underset{(0.076)}{0.168}\,\widetilde{y}_{t-3} - \underset{(0.029)}{0.075}\,(\bar{i}_{t-1} - \overline{\pi}_{t-1}) \tag{14}$$

where $\bar{i}_t$ and $\overline{\pi}_t$ denote four-quarter (moving) averages of current and past interest and inflation rates. The lag lengths of inflation and the output gap in equations (13) and (14) respectively were chosen by the Akaike information criterion (AIC).[7] One other word is relevant about timing. As already noted, we use interest rate data for the first month of each quarter t, on the assumption that this captures the reaction of the Fed to the most recent information (relating to quarter $t-1$) about inflation and output. Therefore, following Clarida *et al.* (2000), although we refer to a one-quarter ahead forecast with $k = q = 1$, this forecast is for inflation and the output gap over the quarter t, since this is a whole quarter in advance of the information available when the relevant interest rate decision is made.

The Appendix summarizes the results of robustness checks undertaken in relation to the use of equations (13) and (14). These checks, which are based on both a Monte Carlo analysis and

[6] Recently, some authors have specified current inflation as depending on a weighted average of past and expected inflation; examples include Clark *et al.* (1996, 2001), Laxton *et al.* (1999) and Rudebusch (2002b). However, in terms of using such a specification as a forecasting equation for the Fed, it is unclear how the required predetermined inflation forecasts would be obtained.
[7] The use of a Bayesian information criterion did not qualitatively alter the results.

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the use of alternative inflation and output gap forecasts, verify that potential errors-in-variables problems from the use of these forecasting equations do not substantively affect our results.

We divide the sample into two main subperiods. The first (1960:I–1979:II) encompasses the tenures of William M. Martin, Arthur Burns, and G. William Miller as Federal Reserve chairmen. The second (1979:III–2000:IV) corresponds to the terms of Paul Volcker and Alan Greenspan. Previous analyses, including Clarida *et al.* (2000) and Dolado *et al.* (2002), have indicated substantial differences in US interest rate policy over these subperiods. Since our sample period is similar to Clarida *et al.* (2000), we use their estimates of the inflation target π^*, namely $\pi^* = 4.24$ in the pre-Volcker period and $\pi^* = 3.58$ in the Volcker–Greenspan period.[8]

3.3. Estimated Rules

Table I report the test statistic ν^2 of equation (5) for the null hypothesis of linearity in the forward-looking and backward-looking specifications ($k = q = 1$ and $k = q = 0$ respectively), computed separately for the whole sample and the two subsamples. Over the whole sample, there is effectively no evidence against a linear policy rule. However, linearity is overwhelmingly rejected in the pre-Volcker era, but not in the Volcker–Greenspan period. The distinct result for the two subperiods is compatible with different monetary policies being conducted pre- and post-1979, a finding in line with Clarida *et al.* (2000) and other recent studies. Nevertheless, the lack of evidence of nonlinearity in the later period contrasts with the results of some other studies of this period, including Bec *et al.* (2002) and Dolado *et al.* (2002), but agrees with Dolado *et al.* (2005).

Based on these results, we estimate forward- and backward-looking versions of equation (12) for the first subperiod, but not for either the Volcker–Greenspan period or for the whole sample. Bayesian posterior estimates and their standard errors are reported in Table II.[9] Although the coefficient ($\widehat{\alpha}_1$) on $E_t\widetilde{\pi}_{t+1}$ in the linear part of the forward-looking rule is not statistically significant (indeed, this coefficient is negative and close to zero), the expected output gap and the interest rate of the previous quarter each linearly exert a positive effect on the interest rate. For the backward-looking rule, neither the inflation nor the output gap coefficient is statistically significant in the linear part. In both cases the nonlinear part is significant collectively[10] at 5%, as evidenced by the

Table I. Tests of the linearity null hypothesis

Sample (dates)	Forward-looking rule		Backward looking rule	
	Statistic	*p*-value	Statistic	*p*-value
Whole sample (1960:I–2000:IV)	0.603	0.438	0.036	0.849
Pre-Volcker (1960:I–1979:II)	13.531	0.0002	12.25	0.0005
Volcker–Greenspan (1979:III–2000:IV)	0.499	0.480	0.507	0.476

Notes: The results report the statistic and asymptotic *p*-value for the nonlinearity test of equation (5). The estimate of the Congress Budget Office is used for the output gap, with inflation and output gap estimates obtained for the forward-looking rule from equations (13) and (14) respectively.

[8] Dolado *et al.* (2005) and Bec *et al.* (2002) assume that the inflation target is time varying and use the index published in the reports of the Council of Economic Advisors as the inflation target measures for the US. However, as in Clarida *et al.* (2000), we assume that the inflation target is constant over the tenure of these Fed chairmen.

[9] Based on 10,000 draws from the importance sampling density described in Hamilton (2001).

[10] Significance is judged in Table II using two-sided tests. However, a one-sided alternative could be used for $\widehat{\zeta}$, since Hamilton's method is seeking evidence of nonlinearity through positive correlation between certain residuals. We are

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t-statistics for $\zeta = 0$, whereas inflation (but not the output gap) is significant at the 10% level, as reflected by the t-statistics on the individual g_i coefficients.

It is noteworthy that the nonlinearity overall is substantially more significant than either individual g_i coefficient, indicating that it is not straightforward to associate the nonlinearity with inflation or the output gap alone. Therefore, we graphically examine the nonlinear function $\mu(.)$, by fixing the value of inflation or the output gap at its sample mean and examining the consequences for the estimated reaction functions of changing the value of the other variable. This is achieved using the posterior distribution whose mean and standard deviation for each parameter are reported in Table II. Each value of the two variables gives an $\mathbf{x}^*$ of interest, with equation (7) used to compute the corresponding estimated conditional mean $\widehat{\mu}(\mathbf{x}^*)$. By generating a range of estimates of $\mu(\mathbf{x}^*)$, as explained in Section 2, 95% probability regions are also computed. However, these probability regions are often relatively broad at extreme values for the variables, where little sample information is available, so that inferences on the shapes of these functions at the extremes must be treated with some caution.

Initially we fix $E_t\widetilde{y}_{t+1}$ (or $\widetilde{y}_t$) at its sample mean while $E_t\widetilde{\pi}_{t+1}(\pi_t)$ is allowed to vary. That is, for the forward-looking case, we set $\mathbf{x}^* = (E_t\widetilde{\pi}_{t+1}, \overline{E_t\widetilde{y}_{t+1}})$ and evaluate the Bayesian posterior expectation of equation (7) for various values of $E_t\widetilde{\pi}_{t+1}$. In the corresponding backward-looking case, $\mathbf{x}^* = (\pi_t, \overline{\widetilde{y}_t})$, with π_t allowed to vary. Figure 1(a) and (b) plots the results, including 95%

Table II. Bayesian estimates of the flexible nonlinear policy rule in the pre-Volcker subsample

	Forward-looking rule	Backward-looking rule
$\widehat{c}$	1.822**	1.797**
	(0.574)	(0.830)
$\widehat{\alpha}_1$	−0.021	−0.018
	(0.222)	(0.165)
$\widehat{\alpha}_2$	0.180*	0.098
	(0.106)	(0.110)
$\widehat{\gamma}_1$	0.823***	0.899***
	(0.111)	(0.116)
$\widehat{\gamma}_2$	−0.169	−0.225**
	(0.112)	(0.110)
$\widehat{\sigma}$	0.620***	0.630***
	(0.084)	(0.083)
$\widehat{\zeta}$	1.811**	1.766**
	(0.804)	(0.808)
$\widehat{g}_1$	0.370*	0.260*
	(0.225)	(0.152)
$\widehat{g}_2$	0.160	0.199
	(0.147)	(0.161)

Notes:
1. The values in parentheses are the standard errors of Bayesian posterior estimates with $N = 10,000$ Monte Carlo simulations.
2. ***, ** and * denote statistical significance at the 1%, 5% and 10% level respectively, in a two-tailed t-test.

grateful to James Hamilton for helpful discussion on this point. Based on a one-sided test, $\widehat{\zeta}$ is close to 1% significance in Table II for the forward-looking specification.

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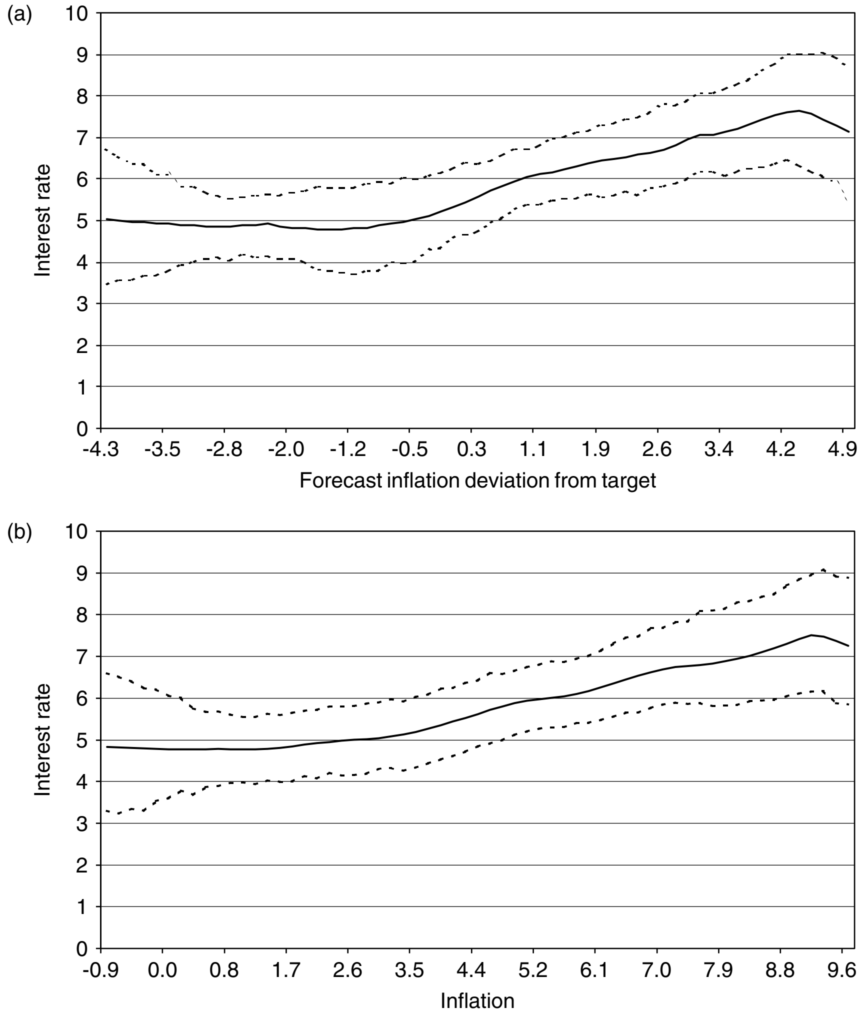

Figure 1. Effect of inflation on the interest rate: (a) forward-looking model; (b) backward-looking model. Note: based on estimated models of Table II for the pre-Volcker period, solid line plots the posterior expectation of the function $\alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t + \boldsymbol{\alpha}_2'\mathbf{z}_t + \lambda m(\mathbf{x}_t)$ evaluated at $\mathbf{x}_t = (x_1, \overline{E_t\tilde{y}_{t+1}})'$ for the forward-looking model and $\mathbf{x}_t = (x_1, \overline{\tilde{y}_t})'$ for the backward-looking model and $\mathbf{z}_t = (\bar{i}_{t-1}, \bar{i}_{t-2})'$ as a function of x_1, where $\bar{z}_{t-j} = T^{-1}\sum_{t=1}^{T} z_{t-j}$ and where the expectation is with respect to the posterior distribution of α_0, $\boldsymbol{\alpha}_1'$, $\boldsymbol{\alpha}_2'$, λ, and $m(\mathbf{x}_t)$ conditional on observation of $\{y_t, \mathbf{x}_t, \mathbf{z}_t\}_{t=1}^{T}$, with this posterior distribution estimated by Monte Carlo importance sampling with 10,000 simulations. Dotted lines give 95% probability regions

probability regions, as a function of $E_t\tilde{\pi}_{t+1}(\pi_t)$, for the forward- and backward-looking cases respectively. In both cases, the implied function is nonlinear, suggesting a more aggressive reaction by the Fed to expected inflation above than below the target. Indeed, for inflation beyond about 0.5% under target, the slope in Figure 1(a) is essentially flat, implying the same reaction by the Fed to any value of inflation below this threshold. In Figure 1(b), there is little response from the Fed to observed inflation (at an annual rate) of around 2% or less. Thereafter, the response is effectively linear, until it flattens off at approximately 9%. Nevertheless, since the confidence interval gets wider as the rate of inflation increases, the estimated response at these higher rates is less reliable.

When $E_t\widetilde{\pi}_{t+1}(\pi_t)$ is fixed at its sample mean and $E_t\widetilde{y}_{t+1}(\widetilde{y}_t)$ is varied, Figure 2(a) and (b) indicates a steeper slope in reaction to a negative output gap than for a positive one, suggesting that prior to 1979 the Fed may have reacted more strongly to output below than above potential.

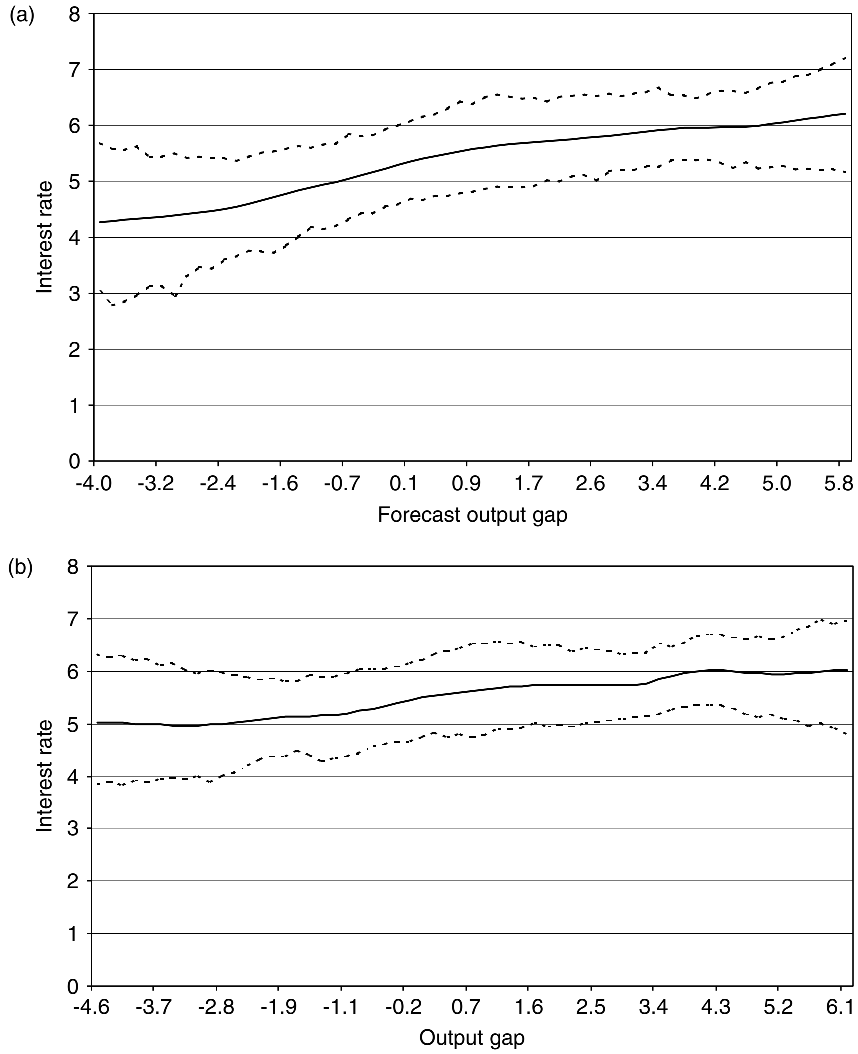

Figure 2. Effect of output gap on the interest rate: (a) forward-looking model; (b) backward-looking model. Note: based on estimated models of Table II for the pre-Volcker period, solid line plots the posterior expectation of the function $\alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t + \boldsymbol{\alpha}_2'\mathbf{z}_t + \lambda m(\mathbf{x}_t)$ evaluated at $\mathbf{x}_t = (\overline{E_t\widetilde{\pi}_{t+1}}, x_2)'$ for the forward-looking model and $\mathbf{x}_t = (\overline{\pi_t}, x_2)'$ for the backward-looking model and $\mathbf{z}_t = (\bar{i}_{t-1}, \bar{i}_{t-2})'$ as a function of x_2. Dotted lines give 95% probability regions

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However, the shape is less smooth in the backward-looking rule (Figure 2(b)) than in the forward-looking rule (Figure 2(a)). Nevertheless, within the central range of ±2% it appears that the Fed reacted more aggressively to a negative than a positive output gap. These results are compatible with the notion that the Fed was more concerned about recession than expansion in the 1960s and 1970s, as found by Gerlach (2000).

In sum, our investigation finds little evidence against the hypothesis that the Fed operated a linear monetary policy rule when data over the whole sample or post-1979 are used. In particular, this implies that the Fed has reacted in an essentially linear way to pin inflation down to its target in the Volcker–Greenspan period. However, we find substantial support for a nonlinear policy rule in the pre-Volcker period, with graphical evidence in the forward-looking case suggesting that the Fed reacted more vigorously to expected inflation above than below the inflation target and more strongly to expected output below than above potential. The latter is consistent with a recession aversion story in which policy makers care more about falls than increases in output.

The Appendix reports results from the nonlinearity test of equation (5) applied with different measures of inflation expectations and the output gap, and also for the Volcker–Greenspan period from 1983 onwards. These results verify the robustness of our conclusions as to the presence of nonlinearity in US monetary policy. Indeed, adopting nonlinear forms of equations (9) and (10) to forecast inflation and the output gap does not remove evidence of nonlinearity in the forward-looking monetary policy rule during the pre-Volcker period. To the extent that these nonlinear forecasts fully capture any nonlinearity in the AS/AD relations, these results suggest that the nonlinearity indicated for the pre-Volcker period is in the interest rate reaction function itself. However, since our analysis in the Appendix does not fully explore the potential nonlinearity in equations (9) and (10), we do not draw any firm conclusions as to the underlying source of nonlinearity.

3.4. Alternative Nonlinear Specifications

To examine whether the parametric models suggested in previous literature adequately capture the nonlinearity in monetary policy in the pre-Volcker era, in this section we adopt a formal statistical basis for comparing these alternative specifications with the nonlinearity revealed by the data through the flexible inference procedure. Table III summarizes four alternative nonlinear specifications for the forward- and backward-looking rules. Models 1F and 1B are threshold-type models in which the Fed is allowed to react differently to positive and negative deviations of inflation from target and of the output gap. Models 2F and 2B are versions of the business-cycle-dependent model of Bec *et al.* (2002), where the Fed's reaction to the deviation of inflation from target and the output gap depends on the stage of the business cycle. Models 3F and 3B are Dolado *et al.*'s (2002) specification in which asymmetric preferences lead to prudent behaviour whereby the Fed responds to the conditional variance of inflation. Following those authors, conditional volatility in inflation is parameterized through a GARCH(1,1) model, here applied to the residuals of our inflation forecasting equation (13). Finally, Models 4F and 4B are Dolado *et al.*'s (2005) interaction model of inflation and the output gap.

Note that although they are nonlinear functions of the inflation deviation (or inflation) and the output gap, the alternative specifications of Table III are linear functions of the parameters. Therefore, each of these models can be described by a linear regression model of the form

$$y_t = \alpha_0 + \boldsymbol{\alpha}'\mathbf{z}_t + \varepsilon_t \qquad (15)$$

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Table III. Alternative nonlinear specifications

Model	Specification
1F	$i_t = c + \alpha_1 E_t\widetilde{\pi}_{t+1} + \alpha_2 E_t\widetilde{\pi}_{t+1}\delta_{[E_t\widetilde{\pi}_{t+1}>0]} + \beta_1 E_t\widetilde{y}_{t+1} + \beta_2 E_t\widetilde{y}_{t+1}\delta_{[\widetilde{y}_{t+1}>0]} + \gamma i_{t-1} + \varepsilon_t$
2F	$i_t = c + \alpha_1 E_t\widetilde{\pi}_{t+1} + \alpha_2 E_t\widetilde{\pi}_{t+1}\delta_{[\widetilde{y}_{t-1}>0]} + \beta_1 E_t\widetilde{y}_{t+1} + \beta_2 E_t\widetilde{y}_{t+1}\delta_{[\widetilde{y}_{t-1}>0]} + \gamma i_{t-1} + \varepsilon_t$
3F	$i_t = c + \alpha E_t\widetilde{\pi}_{t+1} + \beta E_t\widetilde{y}_{t+1} + \omega\sigma^2_{\pi_t} + \gamma i_{t-1} + \varepsilon_t$
4F	$i_t = c + \alpha E_t\widetilde{\pi}_{t+1} + \beta\widetilde{y}_t + \psi(E_t\widetilde{\pi}_{t+1}\widetilde{y}_t) + \gamma i_{t-1} + \varepsilon_t$
1B	$i_t = c + \alpha_1\pi_t + \alpha_2\pi_t\delta_{[\pi_t>\pi^*]} + \beta_1\widetilde{y}_t + \beta_2\widetilde{y}_t\delta_{[\widetilde{y}_t>0]} + \gamma i_{t-1} + \varepsilon_t$
2B	$i_t = c + \alpha_1\pi_t + \alpha_2\pi_t\delta_{[\widetilde{y}_{t-1}>0]} + \beta_1\widetilde{y}_t + \beta_2\widetilde{y}_t\delta_{[\widetilde{y}_{t-1}>0]} + \gamma i_{t-1} + \varepsilon_t$
3B	$i_t = c + \alpha\pi_t + \beta\widetilde{y}_t + \omega\sigma^2_{\pi_t} + \gamma i_{t-1} + \varepsilon_t$
4B	$i_t = c + \alpha\pi_t + \beta\widetilde{y}_t + \psi(\pi_t\widetilde{y}_t) + \gamma i_{t-1} + \varepsilon_t$

Notes: F and B denote forward- and backward-looking models respectively. $\delta_{[.]}$ is unity if the statement [.] is true and 0 otherwise. $\pi^* = 4.24$, the assumed inflation target, as in Clarida *et al.* (2000).

for suitable specifications $\mathbf{z}_t$. For example, with two lags of interest rates, Model 4F is a special case of equation (15) with $\mathbf{z}_t = (i_{t-1}, i_{t-2}, \widetilde{\pi}_{t+1}, \widetilde{y}_t, \widetilde{\pi}_{t+1}\widetilde{y}_t)'$. As such, we can test directly whether this particular specification for $\mathbf{z}_t$ adequately captures any nonlinearity in the data by comparing equation (15) with the more general model

$$y_t = \alpha_0 + \boldsymbol{\alpha}'\mathbf{z}_t + \lambda m(\mathbf{x}_t) + \varepsilon_t \tag{16}$$

for $\mathbf{x}_t = (\widetilde{\pi}_{t+1}, \widetilde{y}_t)'$ and $m(.)$ a realization of the random field whose correlations are characterized by equations (2)–(4). A test of $H_0 : \lambda = 0$ is now a test of whether the definition of $\mathbf{x}_t$ adequately captures the nonlinear dependence of y_t on $\widetilde{\pi}_{t+1}, \widetilde{y}_t$. Thus, the test is adapted from testing the null hypothesis of linearity to testing the null that the nonlinearity takes a particular known parametric form.

Table IV reports the ν^2 test statistics for these alternative nonlinear specifications in the pre-Volcker era. Given the very limited evidence we have found of nonlinearity in the monetary policy function over the Volcker–Greenspan period, we do not consider this later period. The nonlinearity evidenced by our test statistics for the pre-Volcker period in Table I are effectively undiminished in both versions of Model 3, implying that the introduction of inflation volatility does not account for this nonlinearity. We do not argue against the results of Dolado *et al.* (2002) that such volatility may play a significant role in the Fed's monetary policy, but rather our conclusion is that it is not sufficient to characterize the nonlinearity during this period. The threshold-type Models 1 and 2 are generally more successful. Model 2F reduces the nonlinearity test statistic in the forward-looking model from 13.5 in Table I to 4.22 in Table IV, with the latter having a p-value of 0.04. Thus, business cycle dependence in the model of Bec *et al.* (2002) may be part of the underlying source of the nonlinearity. Similarly, the backward-looking Model 1B, which also includes a business cycle dependence, is reasonably successful.

The most successful model overall, however, is the interaction model of inflation with the output gap (Models 4F, 4B). In this case, the ν^2 test statistic is not significant at any conventional level in the backward-looking specification, and is at the margin of significance at the 5% level for the forward-looking version. Therefore, this interaction apparently summarizes the nonlinearity in the monetary policy rule adequately, especially in the backward-looking specification.

This formal comparison suggests that the nature of nonlinearity in the monetary policy rule prior to 1979 might result partly from asymmetric reactions by the Fed to inflation deviations

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Table IV. Tests of the linearity null hypothesis $\mu(\mathbf{x}_t) = \alpha_0 + \alpha_1'\mathbf{x}_t + \alpha_2'\mathbf{z}_t$ for alternative nonlinear specifications in the pre-Volcker period

Model	Forward-looking rule		Backward-looking rule	
	Statistic	*p*-value	Statistic	*p*-value
1	9.77	0.002	4.88	0.027
2	4.22	0.040	10.63	0.001
3	12.35	0.0004	19.46	1.0×10^{-5}
4	3.85	0.050	0.98	0.32

Notes: The models here refer to those of Table III. See also notes of Table I.

and (especially in the forward-looking case) the output gap, but more importantly from the Fed reacting to the interaction between inflation and the output gap as in the model of Dolado *et al.* (2005). As these authors point out, the intuition behind this interaction is plausible. If inflation is above target, then the real interest rate will be below its equilibrium, causing an increase in the output gap next period through the AD relation in equation (10). Since in their model the Phillips curve, equation (9), is convex, that anticipated future increase in the output gap will lead to greater anticipated inflationary pressure than in the linear case. To offset this latent inflationary pressure, the Fed increases the interest rate at t by more than in the linear model.[11] In line with Dolado *et al.* (2005), our evidence in Table I does not find nonlinearity of this type in the post-1979 period for the US. It is plausible that the Fed might have placed more weight on the inflation target than the output gap during this recent period, and hence not reacted in a significant way to the interaction between these.

4. CONCLUDING REMARKS

The linear monetary policy rule proposed by Taylor (1993) has since been widely used, including in the influential work of Clarida *et al.* (1998, 2000). However, this has recently been challenged on two grounds. Firstly, the central bank may have asymmetric responses to inflation deviations from target and/or to the output gap and, secondly, the underlying Phillips curve may be convex. Pursuing these two routes, contributors to the literature have specified sets of nonlinear relationships based on parametric models and provided some evidence in favour of nonlinearity in the policy rule. Our view is that because neither the preferences of the central bank nor the Phillips curve are directly observed, any inferences drawn from specific parameterizations should be confirmed against a flexible nonlinear specification. Detecting nonlinearity in a particular parametric form could otherwise lead to incorrect conclusions about the validity of the particular model.

The contribution of this paper is to address this question using the framework of Hamilton (2001), which parameterizes the nonlinear relations in a flexible way and takes into account uncertainty about the functional from in conducting hypothesis tests. We find that while there is quite strong evidence that the Fed operated a nonlinear monetary policy rule during the pre-Volcker period (1960–1979), the evidence is relatively weak in the Volcker–Greenspan era. Our results are robust to whether the policy rule is forward or backward looking. As shown in the Appendix,

[11] An alternative interpretation of the interaction is that the Fed is using a nominal growth target. However, we do not pursue this issue, as it is beyond the scope of the present paper.

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our results are also generally robust to the use of a Hodrick–Prescott (HP) filter-based output gap measure, the adoption of nonlinear inflation and output gap forecasts and also the use of actual inflation forecasts.

In the context of the flexible, unrestricted framework, we also examine particular parametric specifications proposed in recent work. The notion that the Fed reacted more vigorously to inflation deviations above than below target and more strongly to output below than above potential in the 1960s and 1970s characterizes the nature of the nonlinearity fairly successfully, but still leaves some unexplained nonlinearity. More promisingly, the interaction between inflation and the output gap, specified by Dolado *et al.* (2005) as arising through a convex Phillips curve, does rather well. Hence, we suggest that future structural models might build on this work to allow interactions of the dynamics of inflation and the output gap to influence the nonlinear monetary policy rule.

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APPENDIX: ROBUSTNESS CHECKS FOR THE FORWARD-LOOKING RULE

The use of inflation and output gap forecasts generated from equations (13) and (14) respectively imply that the results for the forward-looking monetary policy rule may be subject to errors-in-variables problems. This appendix explores the potential importance of these, verifying that our results are not subject to substantial problems in this respect.

The values used for $E_t\widetilde{\pi}_{t+1}$ and $E_t\widetilde{y}_{t+1}$ are 'generated regressors', and Pagan (1984) shows that such regressors can result in invalid inferences. In particular, significance may be overstated through the use of generated predictions rather than actual expectations. In order to examine the importance of this potential problem for our analysis, we undertook a small-scale Monte Carlo analysis. This uses a linear trivariate system with data-generating process given by equations (9), (10)[12] and the forward-looking linear monetary policy rule

$$i_t = 1.668 + 0.269E_t(\pi_{t+1}) + 0.149E_t(\widetilde{y}_{t+1}) + 1.015i_{t-1} - 0.345i_{t-2} + u_t,$$

where this rule is estimated from the pre-Volcker subsample (1960 : I–1979 : II). The disturbances in each equation are assumed to be mutually and temporally independent normal variables, with variances equal to those estimated in the respective equations.

[12] One modification is made to the output gap equation, by replacing $\bar{i}_t - \bar{\pi}_t$ by $\bar{i}_{t-1} - \bar{\pi}_{t-1}$ in equation (10) to avoid simultaneity with the current interest rate . However, since $\bar{i}_t$ and $\bar{\pi}_t$ are four- quarter moving averages, we anticipate that the impact of this will be minor.

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Using 5000 replications of data generated for this system corresponding to the pre-Volcker subsample, the nonlinearity test statistic in equation (5) was computed using both generated linear predictions for inflation and the output gap, and also for true expected values $E_t\widetilde{\pi}_{t+1}$ and $E_t\widetilde{y}_{t+1}$. At the nominal 5% level of significance, the rejection rate of the linearity null hypothesis was 4.9% when using the true expected values and 4.8% for the linear predictions. Inference on nonlinearity was also investigated by estimating the flexible nonlinear specification of equation (11). The simulated distributions for the *t*-ratios for $\widehat{\zeta}$ are also very similar, with mean values of 1.686 for the true expectations and 1.683 for the generated forecasts. Although both the true and generated forecasts lead to some over-rejection of the true linearity null hypothesis (with rejection rates of 7.0% and 5.9% respectively at the nominal 5% level), it should be noted that we estimate this equation only when evidence of nonlinearity is uncovered in the prior test using equation (5). Our conclusion from this Monte Carlo evidence is that the nonlinearity uncovered in our analysis of Section 3 is not a spurious consequence of using predicted values rather than true expectations in a linear data-generating process.

As a further check on the robustness of our conclusions regarding nonlinearity in the monetary policy rule, Table A.I reports the test statistic ν^2 of the null hypothesis of linearity and its *p*-value when alternative measures of inflation expectations and of the output gap are employed in the forward-looking specification. Here, firstly, we use the actual one-quarter ahead forecasts made in real time by the *Survey of Professional Forecasters* and the Greenbook of the Federal Reserve Board of Governors.[13] These are combined with our one-quarter ahead output gap forecasts obtained from equation (14). Results are presented in this case for the whole sample and the

Table A.I. Tests of the linearity null hypothesis: robustness analysis for forward-looking model

Sample	Inflation	Output gap	Statistic	*p*-value
Whole sample	*SPF* (68:IV–00:IV)	CBO linear	0.074	0.785
(1960:I–2000:IV)	*Greenbook* (65:IV–96:IV)	CBO linear	2.333	0.127
	Linear	HP linear	1.409	0.235
	Nonlinear	CBO nonlinear	0.899	0.343
	Nonlinear	HP nonlinear	1.024	0.312
Pre-Volcker	Linear	HP linear	3.451	0.063
(1960:I–1979:II)	Nonlinear	CBO nonlinear	13.189	0.0003
	Nonlinear	HP nonlinear	2.752	0.097
Volcker–Greenspan	*SPF* (79:III–00:IV)	CBO linear	0.472	0.492
(1979:III–2000:IV)	*Greenbook* (79:III–96:IV)	CBO linear	4.944	0.026
	Linear	HP linear	1.829	0.176
	Nonlinear	CBO nonlinear	0.389	0.533
	Nonlinear	HP nonlinear	2.530	0.112
1983:I–2000:IV	Linear	CBO linear	0.255	0.613
	Linear	HP linear	0.374	0.541
	SPF	CBO linear	1.109	0.292

Notes: *SPF* indicates the median one-quarter ahead forecasts of the *Survey of Professional Forecasters; Greenbook* is the one-quarter ahead forecast obtained from the Greenbook. Linear forecasts for inflation and the CBO output gap use equations (13) and (14) respectively. Linear forecasts for HP-detrended output are based on three lags of the output gap and the lagged real interest rate. Nonlinear forecasts of inflation and the output gap are generated from equations (17) and (18) respectively.

[13] The *Survey of Professional Forecasters* is the oldest quarterly survey of macroeconomic forecasts in the US. The survey began in 1968 and was conducted by the American Statistical Association and the National Bureau of Economic Research

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Volcker–Greenspan subsamples, but not for the pre-Volcker subsample, as neither inflation forecast series is available for the early part of this period.

Another robustness check replaces the CBO output gap by real GDP detrended by the Hodrick and Prescott (1997) filter.[14] Further, we also allow the possibility that the Fed adopts nonlinear specifications of the AD/AS functions when forecasting inflation and the output gap. Specifically, nonlinear forecasts are generated through equations of the forms

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \alpha_4\pi_{t-4} + \alpha_5\widetilde{y}_{t-1} + \alpha_6\widetilde{y}^2_{t-1} + \alpha_7\widetilde{y}^3_{t-1} + u_{1t} \tag{17}$$

$$\widetilde{y}_t = \beta_0 + \beta_1\widetilde{y}_{t-1} + \beta_2\widetilde{y}_{t-2} + \beta_3\widetilde{y}_{t-3} + \beta_4(\bar{i}_{t-1} - \overline{\pi}_{t-1}) + \beta_5(\bar{i}_{t-1} - \overline{\pi}_{t-1})^2 + \beta_6(\bar{i}_{t-1} - \overline{\pi}_{t-1})^3 + u_{2t} \tag{18}$$

estimated over the entire sample series. These equations can be viewed as Taylor series approximations to general nonlinear specifications of the AS/AD equations (9) and (10) respectively. Both the CBO and HP output gap measures are used in equation (18).

The results in Table A.I using actual inflation forecasts are generally in line with our results from Table I, i.e. we do not reject linearity over either the whole sample or for the Volcker–Greenspan subsample. However, one potentially interesting finding is that the Greenbook forecasts yield a p-value that is significant at 5% for this subsample. Although we do not pursue it here, this points to the potential value of further investigation into whether the use of these inflation forecasts constructed within the Fed may shed light on possible nonlinearity in the monetary policy rule over the Volcker–Greenspan period. As also evidenced by Table A.I, our nonlinearity conclusions are robust to the use of the output gap as measured using the HP-filtered GDP series[15] and to the use of nonlinear forecast values from equations (17) and (18).

Since our second subsample includes the period when the Fed targeted nonborrowed reserves rather than Federal Funds rates, and interest rates were high and volatile, it is worth investigating whether this abnormal period has influenced the overall results for the Volcker–Greenspan era. To do this, we exclude 1979:III–1982:IV and repeat the linearity test of Table I for this shorter subsample. These results, also in Table A.I, confirm the lack of significance of the nonlinearity at the conventional level for the generated linear inflation forecasts or those of the *Survey of Professional Forecasters* and for both output gap measures (as Greenbook data are available only to 1996:IV, the test is not conducted using this inflation forecast series over this shorter period).

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(NBER), with the Federal Reserve Bank of Philadelphia taking it over in 1990. The Greenbook is produced before each meeting of the Federal Open Market Committee containing projections by the research staff at the Board of Governors about how the economy will fare in the future. These projections are made available to the public after a lag of 5 years, and hence our Greenbook data ends in 1996:IV.

[14] When HP-detrended output is used in the linear equation analogous to equation (14), the lag length selected by the AIC is unchanged at three. The discussion paper version of this paper (Kim *et al*., 2002) presents estimated models using the HP-filtered output gap for the pre-Volcker subsample.

[15] In addition, when the HP-filtered output gap series is used in the backward-looking rule, the p-values for the nonlinearity test statistic are 0.988, 0.013 and 0.176 for the whole sample, the pre-Volcker subsample and the Volcker–Greenspan subsample respectively. The backward-looking specification estimated using the HP-filtered series from 1983 yields a p-value of 0.292 for this statistic.

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