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Limited Market Participation and Volatility of Asset Prices

By FRANKLIN ALLEN AND DOUGLAS GALE*

Traditional asset-pricing theories assume complete market participation, despite considerable empirical evidence that most investors participate in a limited number of markets. We show that once the participation decision is endogenized, market properties change dramatically. First, limited market participation can amplify the effect of liquidity trading relative to full participation; under certain circumstances, an arbitrarily small aggregate liquidity shock can cause significant price volatility. Second, there exist multiple equilibria with very different participation regimes and levels of asset-price volatility. Third, under plausible conditions the equilibria can be Pareto-ranked; the Pareto-preferred equilibrium is characterized by greater participation and lower volatility. (JEL G12, D60)

The prices of financial assets such as stocks are volatile in comparison to many other prices. The traditional explanation for this volatility is the arrival of new information about payoff streams and discount rates. There is a large body of evidence which suggests that information is an important determinant of asset-price volatility (see Eugene F. Fama, 1970; Robert C. Merton, 1987a), but whether it is the only determinant is hotly debated. Stephen F. LeRoy and Richard D. Porter (1981) and Robert J. Shiller (1981), among others, have argued that asset prices are characterized by *excess volatility*: they are more volatile than changes in payoff streams and discount rates would predict. A number of authors have suggested that the degree of excess volatility found by these studies can be attributed to the use of inappropriate econometric tech-

niques (see Merton [1987a] and Kenneth D. West [1988a] for surveys of this literature). However, recent work which avoids these problems still finds that there is excess volatility (John Y. Campbell and Shiller, 1988a,b; West, 1988a,b; LeRoy and William R. Parke, 1992).

One possible explanation of asset-price volatility is liquidity trading.¹ For a variety of reasons, investors have sudden needs for cash, and securities are often sold to meet such needs. If the volume of liquidity trading is high enough, it may be able to account for a significant part of the excess volatility found in empirical studies. However, this account leaves some questions unanswered. If the number of traders in the market is large, one would expect some of their liquidity trades to cancel each other, thus reducing the aggregate impact of liquidity trading. Similarly, in a large market one might expect that the other traders

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¹A number of other theories of excess volatility have been suggested. These include those based on asymmetric information (see e.g., Gerard Genotte and Hayne Leland, 1990; Allen and Gary Gorton, 1993) and those based on noise traders (see e.g., Shiller, 1984; J. Bradford De Long et al., 1990). Recent empirical contributions attempting to categorize the causes of stock-price volatility include Campbell and Shiller (1988a,b) and Campbell and Albert S. Kyle (1993).

would absorb a substantial amount of liquidity without a large change in prices. In other words, there is no explanation of why the volume of liquidity trading is high relative to what the market can absorb.

To explain how liquidity trading leads to substantial asset-price volatility, we develop a theory with two important features. The first is *incomplete market participation*. Most asset-pricing theories assume that all investors trade every asset; in other words, there is complete market participation. However, there is extensive empirical evidence that the assumption of complete market participation is not justified. Most investors do not diversify across different classes of assets. For example, Mervyn A. King and Jonathan I. Leape (1984) analyze data from a 1978 survey of 6,010 U.S. households with average wealth of almost \$250,000. They categorize assets into 36 classes and find that the median number owned is 8. In a more recent study, N. Gregory Mankiw and Stephen P. Zeldes (1991) find that only a small proportion of consumers hold stocks; more surprisingly, perhaps, even among those with large liquid wealth, only a fairly small proportion own stocks; of those with other liquid assets in excess of \$100,000, only 47.7 percent hold stocks.²

²Other studies have found that investors' diversification within equity portfolios is also limited. Marshall E. Blume et al. (1974) develop a measure of portfolio diversification which takes into account the proportion of stocks held in individuals' portfolios. Using this measure, they find that the average amount of diversification is equivalent to having an equally weighted portfolio with two stocks. Blume and Friend (1978) provide more detailed evidence of this lack of diversification. They find that a large proportion of investors have only one or two stocks in their portfolios, and very few have more than ten. This observation cannot be explained by the argument that investors are mainly holding mutual funds. In King and Leape's (1984) study, only 1 percent of investors' wealth was in mutual funds compared to 22.3 percent held directly in equities. The Blume et al. (1974) and Blume and Friend (1978) studies are from an earlier period when it is likely that the ownership of mutual funds was an even smaller proportion of wealth, given the growth in mutual-fund holdings that has occurred.

How can limited market participation be explained? One possible explanation is a fixed setup cost of participating in a market. In order to be active in a market, an investor must initially devote resources to learning about the basic features of the market, such as the distribution of asset returns and so forth and how to monitor changes through time. Michael J. Brennan (1975) has shown that with fixed setup costs of this kind, it is only worth investing in a limited number of assets. King and Leape (1984) find evidence that is consistent with this type of model.³

Limited participation by itself does not explain excess volatility. By definition, a market is *liquid* if it can absorb liquidity trades without large changes in price. What we are looking for is an explanation of market illiquidity. In our model, the liquidity of the market does not depend on the number of investors who participate, that is, on the thickness or thinness of the market. On the contrary, we assume that the market is always "thick." Instead it depends on the amount of cash held by the market participants: this is the amount of cash that is available at short notice to buy stocks from liquidity traders, investors who have experienced a sudden need for liquidity. If there is a lot of "cash in the market," liquidity trades are easily absorbed and have little effect on prices. If there is very little cash in the market, on the other hand, relatively small shocks can have a large effect on prices.

The impact of market liquidity on asset pricing is seen most strikingly in the equilibrium "pricing kernel." In equilibrium, the price of the risky asset is equal to the lesser of two amounts. The first is the standard

³Institutions are responsible for investing a significant fraction of total wealth. Portfolio managers also face fixed costs of entering markets. In addition, agency problems limit the wide participation of institutions. Investors are concerned that the institutions will take undesirable risks with their money and impose limits on the amount they can invest in particular classes of assets. For simplicity, we do not distinguish between individual and institutional investors in what follows.

discounted value of future dividends; it applies when there is no shortage of liquidity in the market. The second is the amount of cash available from buyers divided by the number of shares being sold; it applies when there is a shortage of liquidity. In this case, assets are underpriced, and returns are excessive relative to the standard, discounted-dividends formula. This aspect of our model has similarities to the equity-premium puzzle identified by Rajnish Mehra and Edward Prescott (1985).

The amount of cash in the market will depend on the second important feature of our theory, which is the participants' *liquidity preference*. The higher the average liquidity preference of investors in the market, the greater is the average level of cash held in portfolios and the greater the market's ability to absorb liquidity trading without large price changes. Building on this relationship between liquidity preference and asset-price volatility, we can see how market participation helps determine the degree of volatility in the market. The amount of cash in the market and the amount of liquidity trading both depend on who decides to participate. Thus, the participation decision is one avenue by which large endogenous changes in volatility can be effected, as we shall see.

To explore these relationships, we study a model with three dates and with two securities (money and a long-term asset). There is a nonnegative, fixed cost of entering the market for the long asset, and investors' participation decisions are endogenous. Only investors who have paid the fixed cost can trade the long asset; new investors cannot enter the market in the short run. Liquidity trades are modeled by assuming that investors have random preferences of the type introduced by Douglas W. Diamond and Philip H. Dybvig (1983). At the first date investors are unaware of the timing of their liquidity needs. At the second date a random proportion discover they need to consume immediately; the remainder wait until the third date to consume. It is this aggregate uncertainty about the proportion of early consumers that generates price volatility. There are two types of investors.

Type-A investors are "aggressive" because they are more likely to participate in the market. They have a low probability of being early consumers, hence a low preference for liquidity, and they also have low risk aversion compared to the second type. Type-B investors are "backward" because they are less likely to participate in the market. They have a higher probability of being early consumers, hence a higher preference for liquidity and a higher degree of risk aversion.

When the cost of entering the market is sufficiently small, we can show that there is always *full participation*. All investors enter the market, the average amount of liquidity is high, and as a result asset prices are not excessively volatile. As the cost of participation increases, new types of equilibria emerge. For high entry costs there is no participation. For intermediate entry costs there exists a *limited-participation equilibrium*, in which only the aggressive investors are willing to enter the market. Because the market is dominated by investors with low liquidity preference, holding small reserves of cash, even small variations in the proportion of liquidity traders can cause a significant variation in prices. There are highly liquid investors in the economy, holding large reserves of cash which could potentially dampen this volatility, but they have chosen not to participate in this market, at least in the short run.⁴

The two types of equilibria react quite differently to small liquidity shocks. More precisely, we conduct the following experi-

⁴The costs that prevent instantaneous movement from one asset market to another have been stressed in a somewhat related context by Sanford J. Grossman (1988). His concern is that the use of synthetic securities to replace trading in options and futures markets reduces the amount of information flowing to the market. In particular, it may mean that arbitrageurs are not forewarned of impending demands for liquidity in the market. As a result, their capital may be committed elsewhere, so that they cannot react in time to prevent sharp falls in price. See also Grossman and Merton H. Miller's (1988) explanation of the crash of October 1987 as being the result of a lack of funds to absorb a liquidity shock.

ment. Suppose that the liquidity preference of the aggressive type becomes vanishingly small. This means that in the limited-participation equilibrium the aggregate shocks due to liquidity trading become very small too. However, because the liquidity of the market is shrinking at the same time, asset-price volatility is bounded away from zero and can be large for some parameter values. Limited market participation amplifies price volatility in the sense that the ratio of the variance of prices to the variance of the aggregate shock becomes unboundedly large. In the full-participation equilibrium, this cannot happen. The liquidity provided by investors with high liquidity preference absorbs small liquidity trades, so a small amount of aggregate uncertainty implies a small amount of price volatility. Comparing the two equilibria, limited market participation has the effect of amplifying price volatility relative to the full-participation equilibrium.

We can also generate multiple equilibria: for a nonnegligible set of entry costs, equilibria with full participation and with limited participation coexist. If asset prices are expected to be highly volatile, backward investors will not participate. As a result, the market will be dominated by aggressive investors who hold illiquid portfolios, which ensures that the market is illiquid and generates the expected volatility. In this way, beliefs become self-confirming. On the other hand, if asset prices are expected to be stable, they will all participate, the market will be liquid because the average investor is holding a more liquid portfolio, and the expectation of stability becomes self-confirming.

The existence of multiple equilibria gives rise to the possibility of coordination failure. Comparing equilibria, we see that one has lower volatility than the other. For some parameter values, these differences ensure that we can Pareto-rank the two types of equilibria. Backward investors are clearly better off when there is full participation, because they could have stayed out but chose to enter. Except when there are perverse income effects, the reduction in volatility also benefits the aggressive in-

vestors. In addition to "statistical" excess volatility, limited participation leads to excess volatility in a welfare sense. The fact that the prices of financial assets are more volatile than other prices may or may not be socially undesirable. However, if there exists a Pareto-preferred equilibrium with lower volatility, we can say that high volatility represents a market failure.

Our analysis is related to a number of other papers in the literature. The assumption of a fixed cost of participating in a market has been used in various settings. The closest are Merton (1987b), David Hirshleifer (1988), and Charles J. Cuny (1993). In Merton (1987b), there is a fixed cost of acquiring information about the returns to an asset, which causes traders to invest in a limited number of assets. Among other things, Merton shows that this can lead to an empirically significant effect on asset returns. Hirshleifer (1988) assumes that there is a fixed cost of participating in a futures market. These costs result in thin markets, which in turn cause the residual risk to be priced. In this case, equilibrium risk premia are quite different from those in perfect, frictionless markets. Cuny (1993) considers the optimal design of futures contracts assuming that investors can only participate in one market. This assumption causes liquidity to vary across markets, as in our model, and makes the design problem a nontrivial one.⁵

Marco Pagano (1989) develops a related model which captures the relationship between market thinness and volatility. In his model, a finite number of investors receive idiosyncratic demand shocks. Thus, individual demands are more volatile than aggregate demands. If investors believe that the market will have few traders and will be volatile, only a few traders enter. If investors believe that a market will have many traders and stable asset prices, many traders will enter. In each case, beliefs will be self-

⁵See also Satyajit Chatterjee and Dean Corbae (1992), who develop a model where there is a fixed cost of participating in the bond market and use it to address a number of issues in monetary theory.

fulfilling because the volatility of demand and hence prices is inversely related to the number of traders. The theory is thus able to explain the well-documented empirical finding that thin markets are more volatile than thick ones (see e.g., Kalman J. Cohen et al., 1976; Lester G. Telser and Harlow N. Higinbotham, 1977; George E. Tauchen and Mark Pitts, 1983; Pagano, 1986). The empirical studies of excess volatility are based on data from the New York Stock Exchange and other major exchanges. Pagano's explanation of high volatility may not be relevant to these markets, which are usually considered to be thick. In contrast, what matters in our model are the kinds of trader who enter a market. Markets are always thick in our model because there are large numbers of traders of each type. Prices are volatile when the only investors in the market have a low preference for liquidity.

Stephen D. Williamson (1991) also considers a related model. The main difference is that there is a fixed cost of trading an illiquid asset at the second date in his model, rather than a cost of entering a market. He shows that equilibrium is unique but is not necessarily (constrained) efficient. If the probability of consuming early is high, so liquidity matters a lot, there can be *too much* participation in the market for the illiquid asset. Excessive participation is caused by a pecuniary externality. An increase in participation in the market for the illiquid asset lowers the demand for the liquid asset at the second date, which in turn reduces its price. This makes the liquid asset too unattractive, and as a result, equilibrium is inefficient. Everybody would be better off if they just traded the liquid asset. This type of inefficiency is very different from that considered here where there is too little participation, rather than too much.

I. The Model

The introduction of fixed costs threatens to make even simple asset-market models intractable. For this reason, we focus on the simplest model that allows us to illustrate the essential ideas. The model has only two

assets, two types of investors, and three dates.

A. Investment Technology

One of the assets, the *short* asset, yields a return after each period. The other asset, the *long* asset, yields a return after two periods. In what follows we shall call the short asset "cash" and assume its net return is zero. Where there is no risk of ambiguity, we refer to the long asset simply as the asset. Nothing of importance hangs on these conventions, and the short asset could be any interest-bearing, liquid security. Consumption and asset returns are measured in terms of cash, which serves as the numeraire.

Each investor has an initial endowment of W units of cash at date 0. A constant-returns-to-scale technology allows investors to transform one unit of cash into one unit of the long asset.

In order to hold and trade the long asset, investors must have access to the *asset market*. There is a fixed entry cost $c > 0$. Investors who do not pay the entry cost can only hold cash. Let $W_0 \equiv W - c$ denote an investor's wealth net of the fixed cost of entering the asset market.

B. Asset Returns

The returns to the two assets are described in Table 1. A unit of cash held at the end of date $t = 0, 1$ pays out one unit of cash at date $t + 1$. A unit of the long asset held at the end of date 0 yields nothing at date 1 and a possibly random amount of cash R at date 2.⁶ Neither cash nor the long asset can be sold short.

⁶Cash can be interpreted as a claim on storable consumption which is available in fixed supply. The long asset can be interpreted as a claim on productive assets which are produced from storable consumption at date 0; its price at date 0 is fixed by the technology of production.

TABLE 1—ASSET RETURNS

Dates:	0		1		2
Cash:	1	→	1	→	1
Long asset:	1		→		R

C. Preferences

There are two types of investors and $N_i > 0$ investors of each type. We label the types $i = A, B$, where it is useful to think of type A as aggressive and type B as backward. For reasons that will become clear presently, type A is more eager to enter the asset market than type B. Investors are assumed to have Diamond-Dybvig preferences. At date 0, investors of a given type are identical. At the second date, they receive a preference shock which turns some of them into early consumers and the rest into late consumers. By definition, an early consumer only values consumption at date $t = 1$, and a late consumer only values consumption at $t = 2$. Let λ_i be the fraction of investors of type i who become early investors at date 1, and let $1 - \lambda_i$ be the fraction who become late investors; λ_i is also the probability that an investor of type i is an early consumer.

If a type- i investor consumes C_{it} at date t , his utility is given by $U_i(C_{it})$. Investors maximize the expected utility of consumption.

D. Information

At date 1, all investors observe the proportions of early and late consumers and the asset returns. At the same time, individual investors learn whether they are early or late consumers. Thus, all uncertainty is resolved at the beginning of date 1, and from that point on, all equilibrium variables are deterministic.

E. Assumptions

Without essential loss of generality, we can assume that the preference parameters λ_A and λ_B and the asset return R are functions of some macroeconomic variable θ . The variable θ has a bounded, nondegen-

erate support $[0, \bar{\theta}]$ and a positive density. The particular representation of the parameter distributions is innocuous. A single random variable is used to represent all uncertainty for simplicity. The important restriction is that all uncertainty is resolved when the value of θ is observed at the beginning of date 1. The following assumptions are maintained throughout.

ASSUMPTION 1: λ_A , λ_B , and R are continuous functions of θ satisfying $0 < \lambda_A(\theta) < \lambda_B(\theta) < 1$ and $R(\theta) > 1$ for all θ .

The parameter $\lambda_i(\theta)$ has two functions. First, it represents an aggregate shock. By determining the proportion of early consumers (i.e., the proportion of investors who liquidate their portfolios at date 1), it affects the aggregate demand for liquidity. Second, it determines the probability that a type- i investor will be an early consumer, which in turn affects the investor's liquidity preference (i.e., his demand for cash at date 0). Assumption 1 says that type-A investors have a strictly lower probability of being early consumers than type-B investors. In other words, type-B investors have strictly higher liquidity preference. The restriction that $R(\theta) > 1$ ensures the long asset is always a more productive investment than cash over the long run.

ASSUMPTION 2: $U_A = U$ is an increasing and concave function; U_B is strictly more concave than U ; that is, $U_B(C) = V(U(C))$ for any C , where V is an increasing, strictly concave function. Both U_A and U_B are differentiable.

The fact that U_B is a strictly concave function of U_A means that type-B investors are strictly more risk-averse than type-A investors. This assumption guarantees the existence of multiple equilibria in Proposition 8 below; without it multiple equilibria may or may not exist. Holding the long asset is risky for two reasons. First, there is rate-of-return risk because $R(\theta)$ is unknown at date 0, but in addition there is market risk because of the uncertain price of the asset at date 1. To the extent that asset-price

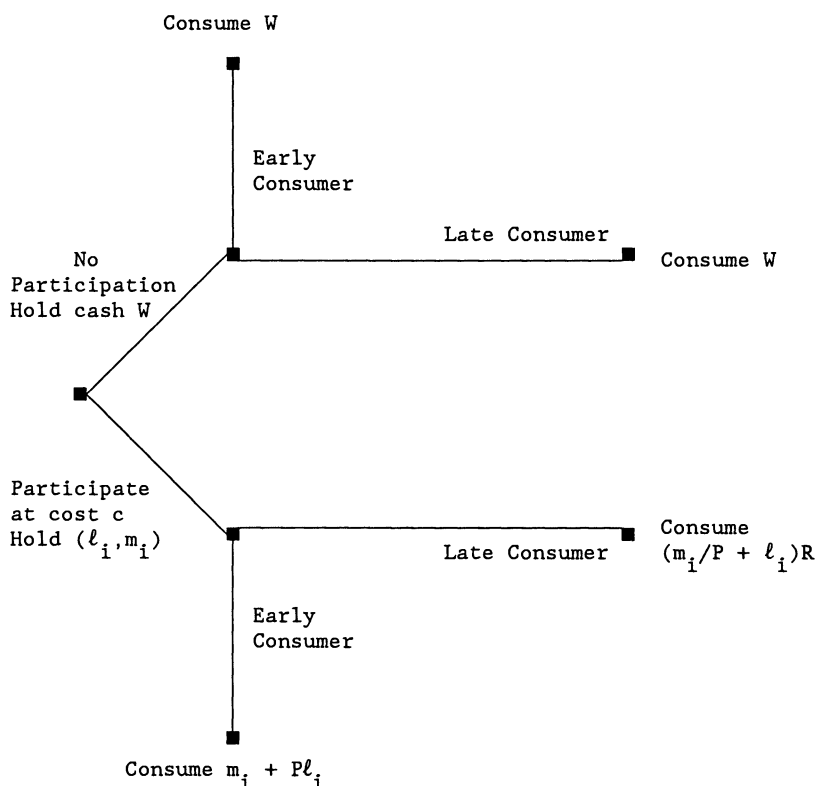


FIGURE 1. INVESTORS' PARTICIPATION DECISION

volatility is a factor in investors' decisions to participate in the asset market, type-B investors with their higher risk aversion and higher liquidity preference seem less likely to participate. We shall later see the precise sense in which this turns out to be the case.

II. Equilibrium

At date 0, an investor has to decide whether or not to participate in the asset market. If he decides not to enter the market, he can only hold cash. Since the return on cash is 1 per period, his consumption is equal to his wealth W regardless of whether he is an early or a late consumer.

If the investor decides to enter the asset market, he pays the fixed cost c and allocates his remaining wealth W_0 between cash and the asset. At date 1 he learns whether

he is an early consumer or a late consumer. If he is an early consumer he supplies his holding of the asset inelastically in exchange for cash and then consumes the liquidated value of his portfolio. If he is a late consumer, he can adjust his portfolio at date 1. At date 2, after the return on the asset has been realized, he consumes the terminal value of his portfolio.

The asset is traded in an auction market at the second date. The prevailing price of the asset clears the market. The price is a function of the information available at date 1 (i.e., a function of θ). Let $P(\theta)$ denote the market-clearing price in state θ . At date 0, the investors who have entered the asset market choose a portfolio to maximize the expected utility of future consumption, taking as given the future asset prices and returns. Although investors do not know the asset prices that will prevail at date 1 when

they make their portfolio choice at date 0, they are assumed to know the price function $P(\theta)$. Given the price function $P(\theta)$ and the distribution of returns $R(\theta)$, investors can calculate the maximum expected utility they can achieve if they enter the asset market, and they use this value to make an optimal participation decision. The determination of equilibrium and the investors' choices are illustrated in Figure 1.

From the preceding discussion it is apparent that equilibrium is characterized by the following three conditions:

- (E1) $P(\theta)$ clears the asset market at date 1, for every θ .
- (E2) In the asset market, investors' portfolios maximize their expected utility at date 0.
- (E3) Investors' participation decisions (i.e., whether to enter the asset market) maximize their expected utility at date 0.

A. Properties of Equilibrium

Before analyzing the portfolio choice problem in detail, we note some useful properties of equilibrium in the long market. Let n_i denote the number of investors of type i who choose to enter the long market at date 0 and suppose that a positive number of investors enter the market at date 0 (i.e., $n_A + n_B > 0$).

Property 1: In any equilibrium the aggregate amount of the long asset held at date 0 must be positive.

To see this, suppose that only cash is held at date 0. At date 1, the demand for the long asset must be zero. This is possible only if cash is undominated [i.e., $P(\theta) \geq R(\theta)$ for every θ]; but since $R(\theta) > 1$ for every θ , this means that $P(\theta) > 1$ for every θ , so cash is dominated by the long asset between date 0 and date 1. This contradicts our supposition that only cash is held at date 0.

Property 2: In any equilibrium, the aggregate amount of cash held at date 0 by investors in the long asset market must be positive.

If not, there would be no demand for the quantity of the asset supplied by the early consumers at date 1 [we know this amount must be positive since $\lambda_i(\theta) > 0$]. Then price would fall to zero for every θ , implying that the asset is dominated by cash between date 0 and date 1, a contradiction.

Property 3: In any equilibrium, $P(\theta) \leq R(\theta)$ for all θ .

Suppose that $P(\theta) > R(\theta)$ for some θ . For this value of θ , cash dominates the asset between date 1 and date 2 so the demand for the asset at date 1 must be zero. Then market-clearing requires that the supply of the asset be zero, contradicting Property 1 above.

Property 4: If $P(\theta) \geq 1$ for all θ then $P(\theta) = 1$ for all θ .

If not, cash is dominated by the long asset between date 0 and date 1, and no cash would be held, contradicting Property 2 above.

B. Market Clearing

Suppose that n_i investors of type i decide to enter the long market at date 0. Let (ℓ_i, m_i) denote the portfolio an investor chooses, where ℓ_i is the amount of the asset held and m_i is the amount of cash or money held. Short selling and borrowing are not allowed so both ℓ_i and m_i are nonnegative.

At date 1, the total amount of the long asset held by early consumers is

$$(1) \quad L(\theta) \equiv \sum_i \lambda_i(\theta) n_i \ell_i.$$

The total amount of cash held by the late consumers is

$$(2) \quad M(\theta) \equiv \sum_i [1 - \lambda_i(\theta)] n_i m_i.$$

Therefore, market clearing requires that $P(\theta)L(\theta) \leq M(\theta)$; that is, the value of the asset supplied by the early consumers can-

not exceed the amount of cash held by the late consumers.

There are two cases to be considered. If $M(\theta)/L(\theta) < R(\theta)$, then the inequality just derived implies that $P(\theta) < R(\theta)$. Then the one-period return on cash is less than that on the asset, and late consumers supply their entire holding of cash inelastically. In this case, $P(\theta) = M(\theta)/L(\theta)$.

On the other hand, if $R(\theta) \leq M(\theta)/L(\theta)$ then $P(\theta) = R(\theta)$. To see this, note that $P(\theta) < R(\theta)$ would imply that cash is dominated between date 1 and date 2. All cash would be inelastically supplied to the market so $P(\theta) = M(\theta)/L(\theta) \geq R(\theta)$, a contradiction. This shows that, for any θ ,

$$(3) \quad P(\theta) = \min\{R(\theta), M(\theta)/L(\theta)\}.$$

When this condition is satisfied, the market must clear. If $P(\theta) = M(\theta)/L(\theta) < R(\theta)$, late consumers want to get rid of their entire holding of cash, so market clearing occurs if and only if $P(\theta)L(\theta) = M(\theta)$. If $P(\theta) = R(\theta) \leq M(\theta)/L(\theta)$, then late consumers are indifferent between holding cash and the asset, so they are willing to supply just what is demanded; that is, they will supply exactly $P(\theta)L(\theta) \leq M(\theta)$. Thus, the equilibrium asset-pricing equation or pricing kernel (3) completely characterizes market clearing in the asset market at date 1.

The pricing kernel illustrates quite clearly the difference between cash-in-the-market pricing and the more usual discounting theories. When $P(\theta) = R(\theta)$, the asset price at date 1 is determined by future returns, as implied by standard discounting theories. When $P(\theta) = M(\theta)/L(\theta)$, the asset price is determined by the amount of cash in the market. There is an asymmetry in the effect of these two forces, however, because the asset price is never greater than the value predicted by future returns. Thus, the impact of cash in the market always leads to "underpricing."

The preceding discussion assumes that $n_A + n_B > 0$, and the equilibria we focus on in the sequel have this property. If no investors enter the long market, so $n_A + n_B = 0$, demand and supply are both identically zero, and the market-clearing price is not

uniquely determined; any price such that there is no entry is an equilibrium price. In such cases, it is convenient to assume that the equilibrium price is determined *as if* there were only type-A investors in the market; that is, we choose a price function P that will clear the market when $n_A > 0$ and $n_B = 0$. This refinement of equilibrium is suggested by Proposition 2 (below), which shows that if type-B investors are willing to enter the asset market, type-A investors strictly prefer to enter. Conversely, if type-A investors are willing to stay out, type-B investors strictly prefer to stay out. In this sense, type-A investors are always more willing to enter the asset market, so it seems plausible to treat the empty market as if it contained type-A investors. This refinement is not important; investment decisions and utilities, which are the main variables of interest, are uniquely defined independently of equilibrium prices when none of the investors chooses to enter.

C. Portfolio Choice

Consider now the decision of an investor of type i who chooses a portfolio (ℓ_i, m_i) at date 0. The quantities of each asset held are nonnegative and satisfy the budget constraint $\ell_i + m_i = W_0$.

At date 1, the investor learns whether he is an early consumer or a late consumer. If he is an early consumer, he liquidates his holding of the long asset at the prevailing price $P(\theta)$ and consumes the liquidated value of his portfolio

$$(4) \quad C_{i1}(\theta) = P(\theta)\ell_i + m_i.$$

On the other hand, if he learns that he is a late consumer, we know that it is weakly optimal for him to hold only the asset since $P(\theta) \leq R(\theta)$. Therefore, in calculating his expected utility, there is no loss of generality in assuming that he holds only the asset. In that case, he holds $\ell_i + m_i/P(\theta)$ units of the asset at the end of date 1, and his consumption at date 2 will be

$$(5) \quad C_{i2}(\theta) = R(\theta)[\ell_i + m_i/P(\theta)].$$

Then the investor's expected utility from the portfolio (ℓ_i, m_i) is given by

$$(6) \quad V_i(\ell_i, m_i; P) \\ \equiv E\{\lambda_i(\theta)U_i(C_{i1}(\theta)) \\ + [1 - \lambda_i(\theta)]U_i(C_{i2}(\theta))\}$$

and the optimal portfolio solves the problem

$$(7) \quad \max V_i(\ell_i, m_i; P)$$

subject to $\ell_i \geq 0$, $m_i \geq 0$, and $\ell_i + m_i = W_0$, and taking the price function P as given. Let $V_i^*(P)$ denote the maximized expected utility from the optimization problem when the price function is P .

D. Market Participation

If an investor decides not to enter the asset market at date 0, he is restricted to holding his wealth in the form of cash in each period. Whether he is an early consumer or a late consumer, his consumption is $W = W_0 + c$, and his utility is $U_i(W_0 + c)$. Thus, the expected utility of an investor who does not enter the asset market is

$$(8) \quad \bar{u}_i = U_i(W_0 + c).$$

This provides a lower bound to the expected utility of an investor in equilibrium. An investor will enter the asset market only if he can ensure himself at least \bar{u}_i , and he will always enter if he can ensure himself more. Thus, the equilibrium participation decisions must satisfy $0 \leq n_i \leq N_i$ and

$$(9) \quad n_i = \begin{cases} 0 & \text{if } V_i^*(P) < \bar{u}_i \\ N_i & \text{if } V_i^*(P) > \bar{u}_i. \end{cases}$$

Standard arguments suffice to show that the model has an equilibrium for all parameter values.

PROPOSITION 1: *Under the maintained assumptions, there exists an equilibrium of the model for all parameter values.*

The proof is given in the Appendix.

III. Participation

For large entry costs it is clear that in equilibrium none of the investors will enter the market for the long-term asset, while for zero costs they will all enter. In this section, we investigate the investors' decision to participate in the long market focusing on the way in which participation changes as the entry cost changes. Other things being equal, type-A investors get a larger benefit from participation in the long market because they are less risk-averse and have lower liquidity preference. Our first result shows that type-A investors are more willing to participate in the long market than type-B investors. More precisely, we show that for any set of parameter values, if type-B investors weakly prefer to enter the long market, type-A investors strictly prefer to enter. Conversely, if type-A investors weakly prefer not to enter, type-B investors strictly prefer not to enter.

PROPOSITION 2: *Investors with low liquidity preference and low risk aversion (type A) are more willing to enter the asset market than investors with high liquidity preference and high risk aversion (type B). More precisely, in any equilibrium, $n_A < N_A$ implies $n_B = 0$, and conversely, $n_B > 0$ implies $n_A = N_A$.*

The proof is in the Appendix. Note that the proposition rationalizes the refinement of equilibrium adopted earlier.

Depending on the costs of entry and other parameters of the model, we expect to observe different levels of participation in the long market. Obviously, for extremely high values of c , there will be no participation in equilibrium. Nothing more needs to be said about this case. It follows from Proposition 2 that for smaller values there are only two possibilities: we may have either *full participation* ($n_i = N_i$, for $i = A, B$) or *limited par-*

participation ($n_B < N_B$). We begin by analyzing the conditions under which we have full participation.

A. Full Participation

For technical reasons, we find it convenient to treat W_0 , rather than W , as the model parameter in what follows. Then W is defined to be the sum of W_0 and the entry cost c , so a ceteris paribus change in c implies a change in investors' initial wealth $W \equiv W_0 + c$.

Consider the case in which there is no cost of entry ($c = 0$). An investor cannot be worse off entering the asset market since he can always choose to hold only cash. However, the investor can do better than this. In fact, he is strictly better off if he participates in the asset market. To see this, suppose it were optimal for the investor to hold only cash at both dates. If $P(\theta) < R(\theta)$ for some θ , then cash is dominated between date 1 and date 2; but if $P(\theta) = R(\theta) > 1$ for all θ , then cash is dominated between date 0 and date 1. Either way, it cannot be optimal for an investor to hold only cash, and this means that he must be strictly better off entering the asset market. Obviously the same must be true when the entry cost is positive but sufficiently small. Then we have proved the following result.

PROPOSITION 3: *For some $c_0 > 0$ and any entry cost $0 \leq c < c_0$, all equilibria are characterized by full participation in the asset market (i.e., $n_i = N_i$ for $i = A, B$).*

B. Limited Participation

For intermediate values of the entry cost, there are also equilibria with limited participation. Without essential loss of generality, we focus on limited-participation equilibrium in which $n_A = N_A$. If $n_A < N_A$ then Proposition 2 tells us that $n_B = 0$. In that case, the market-clearing condition is independent of n_A . Since type-A investors are indifferent between entering the asset market and staying out when $0 < n_A < N_A$, we can set $n_A = N_A$ without disturbing the equilibrium conditions. In what follows, we

always assume $n_A = N_A$ when the number of investors in the market is positive.

Suppose we are given an arbitrary allocation of investors $n = (n_A, n_B)$. We can define a partial equilibrium for the long market, taking as given the participation decisions n . A *partial equilibrium relative to n* consists of a price function $P(\theta)$ and portfolio choices (ℓ_i, m_i) for each type of investor such that the equilibrium conditions (E1) and (E2) are satisfied. In other words, if the numbers of investors entering the asset market are given by n , then (ℓ_i, m_i) is an optimal portfolio for type- i investors, given the price function $P(\theta)$, and $P(\theta)$ is market-clearing when type- i investors choose the portfolio (ℓ_i, m_i) . The arguments used to prove Proposition 1 can be used to show that there exists a partial equilibrium for every possible n .

PROPOSITION 4: *For any values of $n_A > 0$ and $n_B \geq 0$, we can find portfolios (ℓ_A, m_A) and (ℓ_B, m_B) and a price function $P(\theta)$ such that (i) (ℓ_i, m_i) is optimal for investors of type $i = A, B$ and (ii) $P(\theta)$ is market-clearing when investors hold the portfolios (ℓ_A, m_A) and (ℓ_B, m_B) .*

The proof is in the Appendix.

To show that limited participation does occur for some values of c , we examine the special case in which all the type-B investors stay out. If we arbitrarily put $n_A = N_A$ and $n_B = 0$, Proposition 4 ensures us that there exists a partial equilibrium

$$(P, (\ell_A, m_A), (\ell_B, m_B))$$

relative to n . To show that this is an equilibrium of the full model, it only remains to check that the assumed participation decisions are optimal, that is, satisfy (E3). It is clear that the investors' decisions will be optimal if

$$(10) \quad V_B^*(P) \leq U_B(W) = V(U(W_0 + c))$$

and

$$(11) \quad V_A^*(P) \geq U_A(W) = U(W_0 + c).$$

Suppose we choose c so that the second inequality is just satisfied (we can clearly do this since $U(\cdot)$ is continuous and increasing). From Proposition 2, $U(W_0 + c) = V_A^*(P)$ implies $V_B^*(P) < U_B(W_0 + c)$. By continuity, a small reduction in c will make the type-A investors more eager to enter without attracting the type-B investors. This shows that the required inequalities are satisfied for values of c lying in a nondegenerate interval.

PROPOSITION 5: *For some numbers $c_2 > c_1 > 0$ and every entry cost c in the interval $[c_1, c_2]$, there exists an equilibrium with limited participation, in fact, an equilibrium satisfying $n_A = N_A$ and $n_B = 0$.*

A number of interesting questions arise now that we have established the existence of equilibria belonging to different participation regimes. How does asset-price volatility vary across these equilibria? Do there exist multiple equilibria, belonging to different participation regimes? Can the equilibria be Pareto-ranked? If so, what is the intuition for the market failure? These questions are addressed in the following sections.

IV. Asset-Price Volatility

To develop the intuition behind our story, we impose some additional structure on the model. We assume that the preference shocks $\lambda_i(\theta)$ are random for type A, but not for type B:

$$\lambda_A(\theta) = k\theta$$

$$\lambda_B(\theta) = \bar{\lambda}_B$$

for all θ and some constants k and $\bar{\lambda}_B$.

By assuming that the proportion of early consumers among the type-B investors is nonstochastic, we eliminate one source of asset-price volatility. It is only fluctuations in the proportion of early consumers of type A that cause the asset price $P(\theta)$ to deviate from $R(\theta)$. This suggests that the degree of

asset-price volatility will be significantly affected by the participation decisions of the different types of investors. When the type-B investors enter the asset market, their lack of aggregate uncertainty should have a stabilizing effect on asset prices. Conversely, when they remain outside the asset market, volatility should be higher.

To emphasize the role of liquidity shocks, we could specialize the model further by assuming that the asset returns $R(\theta)$ are nonstochastic, in which case fluctuations in the proportion of early consumers of type A are the only source of asset-price volatility. An example of this sort is studied in Section V.

A. Volatility with Full Participation

Consider first what happens when both types enter the asset market in equilibrium, so $n_A = N_A$ and $n_B = N_B$. Property 4 tells us that if the equilibrium price $P(\theta)$ is never less than 1, then $P(\theta) = 1$ for all θ . This rules out the possibility that $P(\theta) = R(\theta)$ for every θ , so it must be the case that $P(\theta) < R(\theta)$ for some θ . When there is full participation the pricing formula requires $P(\theta) = \min\{R(\theta), Q(\theta)\}$ where

$$(12) \quad Q(\theta) = \frac{\sum_i [1 - \lambda_i(\theta)] N_i m_i}{\sum_i \lambda_i(\theta) N_i \ell_i}$$

Then as $k \rightarrow 0$, $\lambda_A(\theta) \rightarrow 0$, and $m_A \rightarrow 0$, so

$$(13) \quad Q(\theta) \rightarrow (1 - \bar{\lambda}_B) m_B / \bar{\lambda}_B \ell_B = \bar{Q}$$

which is independent of θ . In the limit, $P(\theta) = \min\{R(\theta), \bar{Q}\}$. Recall that $R(\theta) > 1$ for all θ . Then $\bar{Q} > 1$ implies $P(\theta) > 1$ for all θ , contradicting Property 4. On the other hand, if $\bar{Q} < 1$ then $P(\theta) < 1$ for all θ and the long asset is dominated between date 0 and date 1, contradicting Property 1. Therefore $P(\theta) = \bar{Q} = 1$ for all θ .

PROPOSITION 6: *As $k \rightarrow 0$, asset-price volatility vanishes in a full-participation equilibrium. In fact, $P(\theta) \rightarrow 1$ in probability.*

A detailed proof is given in the Appendix.

In addition, it can be seen that as $N_B \rightarrow \infty$, holding N_A constant, $P(\theta)$ tends to the same limit, and a similar result holds. As the number of type B's goes up, the amount of cash in the market becomes large relative to the fluctuations in liquidity trading, and this dampens volatility.

B. Volatility with Limited Participation

Consider next the limited-participation equilibrium in which $n_A = N_A$ and $n_B = 0$. The market-clearing price is determined by the demands of the type-A investors only. When $P(\theta) < R(\theta)$, the pricing equation reduces to

$$(14) \quad P(\theta) = \frac{1 - \lambda_A(\theta)}{\lambda_A(\theta)} \left(\frac{W_0 - \ell_A}{\ell_A} \right).$$

From Properties 1 and 2, we know that both assets are held at date 0 (i.e., $0 < \ell_A < W_0$), so the pricing function has the form $P(\theta) = a/\theta - b$, where $a, b > 0$ are constants. In other words, the asset price is stochastic.

We can sharpen this point by considering what happens as $k \rightarrow 0$. Clearly,

$$(15) \quad \lim_{k \rightarrow 0} (\text{Var } \lambda_A) = \lim_{k \rightarrow 0} (k^2 \text{Var } \theta) = 0$$

and in this sense aggregate uncertainty vanishes. But price uncertainty does not vanish. From the pricing equation above we can see that the expected value of $P(\theta)$ converges to 0 only if ℓ_A converges to W_0 ; but if $P(\theta)$ converges to 0 in probability, the one-period holding return on the asset is eventually dominated by cash, so $\ell_A = 0$, a contradiction. This proves that the expected value of $P(\theta)$ is bounded away from 0. From that fact and the pricing equation it follows that the variance of prices is also bounded away from 0.

PROPOSITION 7: *As $k \rightarrow 0$ the amount of aggregate uncertainty in the model converges to zero. In particular, $\text{Var } \lambda_A \rightarrow 0$. However,*

the volatility of asset prices in a limited-participation equilibrium remains nonnegligible: $\text{Var } P$ is bounded away from zero.

A detailed proof is in the Appendix.

As k becomes vanishingly small, the type-A investors become unwilling to hold cash. In fact, their demand for cash becomes small relative to the variance of λ_A . Because there is so little cash in the market, the asset price P becomes highly volatile. When λ_A is relatively high, there is no demand for the quantity of the asset supplied by early consumers, so the price plunges. However, the expected value of the asset cannot go to zero at date 1 or no one would be willing to hold it at date 0. Thus, when λ_A is relatively low the asset price must be high to compensate for its low value in other states.

It can be seen from Proposition 7 that as $k \rightarrow 0$, the ratio of the variance of prices to the variance of aggregate uncertainty, $\text{Var } P_L / \text{Var } \lambda_A$, becomes unboundedly large. Limited market participation amplifies volatility in the sense that the fluctuation in prices is large relative to the aggregate uncertainty. A comparison of Propositions 6 and 7 illustrates a second sense in which limited market participation amplifies price volatility. In the full-participation equilibrium, price volatility becomes negligible as aggregate uncertainty tends to zero. In contrast, in the corresponding limited-participation equilibrium price volatility is bounded away from zero in the same circumstances. Thus the ratio of the variance of the prices in the two equilibria, $\text{Var } P_L / \text{Var } P_F$, becomes unboundedly large.

We have compared the price volatility of different equilibria under the assumption that k is very small. This case is interesting because it generates sharp results quite easily. Similar results could have been obtained for other configurations of parameter values.

V. Multiple Equilibria

In the preceding section, we saw that the participation decision has an important impact on the degree of asset-price volatility.

When type-B investors enter the asset market, volatility is low; when they stay out, volatility is high. Since type-B investors are more sensitive to volatility, the stage is set for existence of multiple equilibria: either set of participation decisions may be self-justifying. What is interesting here is not the multiplicity of equilibria as such, but rather the possibility of developing a plausible set of conditions under which there exist equilibria with very different price volatilities for a given set of parameter values.

To simplify the analysis we assume that the returns to the long asset are nonstochastic:

$$R(\theta) = \bar{R} > 1$$

where \bar{R} is a constant, for all θ . By making the returns to the asset nonstochastic, we remove one source of asset-price volatility. The asset price is now driven by liquidity shocks alone.

PROPOSITION 8: *When k is sufficiently small and type-B investors are sufficiently risk-averse relative to type-A investors, there exist two equilibria for a nondegenerate interval of entry costs c . In one equilibrium there is full participation. In the other there is limited participation: type-A investors enter the asset market, and type-B investors stay out.*

Only a sketch of the proof is given here; the details are in the Appendix.

Proposition 4 tells us that, for any arbitrary participation decisions $n = (n_A, n_B)$, there exists a partial equilibrium relative to n . So we begin by specifying the two participation regimes, one with full participation $n^F = (N_A, N_B)$ and one with limited participation $n^L = (N_A, 0)$, and then we consider the corresponding partial equilibria $(P^F, \{(\ell_i^F, m_i^F)\})$ and $(P^L, \{(\ell_i^L, m_i^L)\})$. To show that these correspond to full equilibria, we need to show that the participation decisions are in fact optimal given the putative market-clearing prices.

To do this, we use the results from Section IV that characterize the equilibria belonging to different regimes for particular

ranges of parameter values. In the full-participation equilibrium, we saw that as $k \rightarrow 0$, $P^F(\theta)$ converges to 1 in probability (Proposition 6). Then it is easy to show that

$$(16) \quad V_i^*(P^F) \rightarrow \bar{\lambda}_i U_i(W_0) + (1 - \bar{\lambda}_i) U_i(W_0 \bar{R})$$

where $\bar{\lambda}_i$ is the expected value of $\lambda_i(\theta)$, for $i = A, B$. For the type-A investors, this reduces to $V_A^*(P^F) \rightarrow U_A(W_0 \bar{R})$.

Similarly, in the limited-participation equilibrium, we can show that in the limit as $k \rightarrow 0$ the type-A investors are certain of being late consumers, their liquidity preference falls to 0, they hold only the long asset, and

$$(17) \quad V_A^*(P^L) \rightarrow U_A(W_0 \bar{R})$$

which is the same as in the full-participation equilibrium. In particular, they are indifferent to asset-price volatility. On the other hand, the type-B investors are not indifferent. It can be shown that their demand for the long asset is bounded away from zero, so they care about the market risk, which is nonnegligible according to Proposition 7. Then if the type-B investors are sufficiently risk-averse relative to the type-A investors, they will be worse off in the limited-participation equilibrium than in the full-participation equilibrium:

$$(18) \quad V_B^*(P^L) < V_B^*(P^F).$$

Then we can choose a value of c such that

$$(19) \quad V_B^*(P^L) < U_B(W_0 + c) < V_B^*(P^F).$$

From Proposition 2 we know that $n_B > 0$ implies $n_A = N_A$, so we have

$$(20) \quad U(W_0 + c) < V_A^*(P^L) \approx V_A^*(P^F).$$

This completes the proof.

Again, we have demonstrated the existence of multiple equilibria only for the case where k is small. This case is interesting because the analysis is simple and it gives sharp results. Similar results can be ob-

tained for a variety of parameter configurations.

VI. Pareto-Ranked Equilibria

Given the simultaneous existence of full-participation and limited-participation equilibria, a natural question is how the two compare in welfare terms. Although no general result is possible, it can be shown that in certain circumstances they can be Pareto-ranked.

PROPOSITION 9: *The type-B investors strictly prefer the full-participation equilibrium to the limited-participation equilibrium. In the limit as $k \rightarrow 0$, the type-A investors are indifferent between the two so the full-participation equilibrium is Pareto-preferred.*

To see this result, first note that as shown in the previous section the type-B investors always prefer the full-participation equilibrium. They have the option of just holding the short asset, as they do in the limited-participation equilibrium, but choose not to do so. The reason the full-participation equilibrium is better for them is that it allows them to obtain the benefits of the long asset's higher return.

As for the type-A investors, it was shown in the previous section that as $k \rightarrow 0$, $V_A^*(P^F) \rightarrow U_A(W_0 \bar{R})$ in the full-participation equilibrium and $V_A^*(P^L) \rightarrow U_A(W_0 \bar{R})$ in the limited-participation equilibrium. In the limit, the type-A investors are indifferent between the two equilibria. The full-participation equilibrium is (weakly) Pareto-preferred.

A general comparison of the utilities of the type-A investors in the two equilibria is more complex. The reason is that there are risk and income effects. The risk effect arises

from the fact that prices are more variable in the limited- than in the full-participation equilibrium. This tends to make the type-A investors prefer the full-participation equilibrium. However, in addition there is an income effect which can more than offset this risk effect.

In the limited-participation equilibrium the type-A investors who are early consumers trade with the type-A investors who are late consumers. Their initial expected utility averages over these two states, and there are no transfers between the type-A investors and type-B investors. In the full-participation equilibrium, however, $\lambda_A(\theta) < \lambda_B(\theta)$ so proportionately, there are fewer type-A investors than type-B investors who are early consumers, and more who are late consumers. On average, the type-A investors buy more of the long-term asset from the type-B investors than they sell to them. The price of the long-term asset determines whether there is a transfer from the type-A investors to the type-B investors or vice versa. It is possible that the transfer of income from the type-A investors to type-B investors in the full-participation equilibrium outweighs the risk effect and the type-A investors prefer the limited-participation equilibrium. Thus, the equilibria cannot be Pareto-ranked in general.

To illustrate this and the working of the model for a range of parameter values a numerical example is considered. For simplicity, a two-point distribution is assumed: $\theta = 1$ with probability π and 0 with probability $1 - \pi$. The parameter values for the example are shown in Table 2. In the limited-participation equilibrium, it can be shown from the type-A investors' first-order condition and the market-clearing condition that the distribution of prices as $k \rightarrow 0$ is given by equation (21), below.

$$(21) \quad \lim_{k \rightarrow 0} P = \begin{cases} \frac{\pi \bar{R}}{\bar{R} - (1 - \pi)} < 1 & \text{with probability } \pi \text{ (i.e., when } \theta = 1) \\ \bar{R} > 1 & \text{with probability } 1 - \pi \text{ (i.e., when } \theta = 0) \end{cases}$$

TABLE 2—A NUMERICAL EXAMPLE

Investors:		
Parameter	Type A's	Type B's
Utility function	$U = -\exp(-C)$	$U = -\exp(-5C)$
Initial wealth	$W_A = 1$	$W_B = 1$
Group size	$N_A = 1$	$N_B = 10$
Probability of being an early consumer	$\lambda_A = k\theta$	$\bar{\lambda}_B = 0.5$
Distribution of θ	$\theta = 1$ with probability π $\theta = 0$ with probability $1 - \pi$	
Assets:		
Return on long asset:		$\bar{R} = 1.1$
Market:		
Cost of entry:		$c = 0.04$

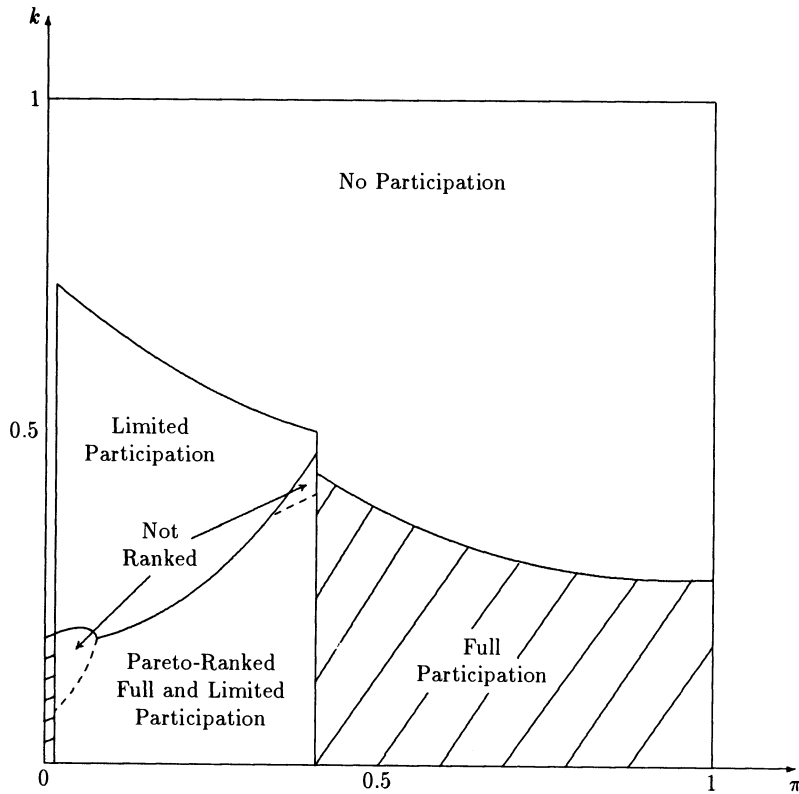


FIGURE 2. A NUMERICAL EXAMPLE

When π is near 1, the price corresponding to $\theta = 1$ is just below 1. As π falls, the price in this state also falls, and the variation in prices can be large even though k is small. In particular, prices can vary from almost zero to \bar{R} .

Figure 2 shows the types of equilibrium obtained for various values of k and π . For large values of k , there is considerable price volatility, and both types stay out of the market for the long asset; there is no participation. For low k and large values of π , there is relatively little price volatility, and there is a unique equilibrium with full participation. For low k and π , there exist both limited- and full-participation equilibria. In most cases the full-participation equilibrium is Pareto-preferred to the limited-participation one. However, this is not always the case as explained above. For larger k , full-participation equilibria are not Pareto-preferred, and in the areas marked "not ranked" the type-A investors' expected consumption and expected utility are higher with limited participation than with full participation. For larger k and moderate π , there is only a limited-participation equilibrium, because even with both types in the market the price volatility is too large for type B's to want to stay. For very small π , there is full participation in the hatched region near the origin; otherwise there is no entry.

The example illustrates that multiple equilibria can exist for a wide range of parameter values. One has limited participation and volatile prices; the other has full participation but little price volatility. Usually, but not always, the full-participation equilibrium is Pareto-preferred to the limited-participation equilibrium.

VII. Concluding Remarks

There is considerable empirical evidence suggesting that investors only participate in a limited number of markets because of transaction costs. This paper has investigated the effect of the interaction of limited market participation and liquidity preference on asset-price volatility. It was shown that for a range of entry costs a full-

participation equilibrium and a limited-participation equilibrium can both exist. In the illustrative case in which there is only aggregate uncertainty associated with the liquidity needs of the low-liquidity types, price volatility becomes negligible as the aggregate uncertainty tends to zero. The ample reserves of cash held by the high-liquidity types ensure that the aggregate uncertainty has little effect on prices. However, for the same parameter values, price volatility is not negligible when there is limited market participation. Price volatility is bounded away from zero as aggregate uncertainty tends to zero. The people who are in the long market have long time horizons and so hold little cash. When liquidity is scarce, asset prices are not determined by discounted cash flows, but rather by the amount of cash in the market. It is the relative variation in the amount of the long asset and cash supplied to the market that is important for price volatility. The absolute amounts of both fall as aggregate uncertainty becomes negligible, but the relative variation remains the same. As far as a welfare comparison of the two equilibria is concerned, the high investment in the full-participation equilibrium and the lack of price volatility mean that the full-participation equilibrium is usually Pareto-preferred to the limited-participation equilibrium.

APPENDIX

PROOF OF PROPOSITION 1:

The proof uses a standard fixed-point argument, so only the general outline is sketched here. For any $\varepsilon > 0$, let

$$(A1) \quad N_\varepsilon = \{(n_A, n_B) \mid \varepsilon_i \leq n_i \leq N_i\}$$

and let

$$(A2) \quad L_\varepsilon = \{(\ell_A, \ell_B) \mid \varepsilon_i \leq \ell_i \leq W_0\}$$

where $\varepsilon_A = \varepsilon$ and $\varepsilon_B = 0$. For any $(\ell, n) \in L_\varepsilon \times N_\varepsilon$ there exists a unique price function

defined by

$$(A3) \quad \Pi(\theta; \ell, n) = \min \left\{ R(\theta), \frac{\sum (1 - \lambda_i) n_i (W_0 + \ell_i)}{\sum \lambda_i n_i \ell_i} \right\}.$$

Define a correspondence from $L_\varepsilon \times N_\varepsilon$ to itself as follows. Let

$$(A4) \quad F_i(\ell, n) = \operatorname{argmax}_{\varepsilon_i \leq \ell'_i \leq W_0} V_i(\ell'_i; \Pi(\ell, n))$$

for $i = A, B$

where $V_i(\ell'_i; P)$ is the expected utility from a portfolio $(\ell'_i, W_0 - \ell'_i)$ when the price function is P . Let

$$(A5) \quad G_i(\ell, n) = \operatorname{argmax}_{\varepsilon_i \leq n'_i \leq N_i} \{n'_i V_i^*(\Pi(\ell, n)) + (N_i - n'_i) \bar{u}_i\}$$

for $i = A, B$.

It is easily checked that $F_A \times F_B \times G_A \times G_B$ maps $L_\varepsilon \times N_\varepsilon$ to itself and satisfies the properties of Kakutani's fixed-point theorem. Let $(\ell^\varepsilon, n^\varepsilon)$ denote a fixed point for each value of $\varepsilon > 0$. Then let $\varepsilon \rightarrow 0$ and consider a subsequence such that $(\ell^\varepsilon, n^\varepsilon) \rightarrow (\ell^0, n^0)$.

To show that (ℓ^0, n^0) defines an equilibrium, we first have to show that the price function $\Pi(\ell^0, n^0)$ is market-clearing. If $\ell_A^0 + \ell_B^0 = 0$ then $\Pi(\theta; \ell^\varepsilon, n^\varepsilon)$ converges almost uniformly to $R(\theta) > 1$, which implies that $\ell_i^\varepsilon = W_0$ for ε sufficiently small, a contradiction. Similarly, if $\ell_A^0 + \ell_B^0 = 2W_0$ then $\Pi(\theta; \ell^\varepsilon, n^\varepsilon)$ converges to 0, which implies that $\ell_i^\varepsilon = 0$ for ε sufficiently small, a contradiction. Thus, $0 < \ell_A^0 + \ell_B^0 < 2W_0$ and it is easily checked that the price function $\Pi(\ell^0, n^0)$ and the portfolios $\{(\ell_i^0, W_0 - \ell_i^0)\}$ satisfy (E1) and (E2).

Similarly, it follows by continuity that n^0 satisfies (E3) when the price function is $\Pi(\ell^0, n^0)$.

PROOF OF PROPOSITION 2:

Suppose to begin with that both types of investor had the same risk preferences represented by the utility function U .

Let (ℓ, m) be an arbitrary portfolio and let $C_1(\theta)$ and $C_2(\theta)$ denote the corresponding consumption levels of an early consumer and a late consumer. Since $P(\theta) \leq R(\theta)$,

$$(A6) \quad C_1(\theta) = P(\theta) \ell + m \leq R(\theta) [\ell + m / P(\theta)] = C_2(\theta)$$

and the inequality is strict when $P(\theta) < R(\theta)$. This shows that a late consumer is always at least as well off as an early consumer, and with positive probability he is strictly better off since Assumption 1 and Property 4 ensure that $P(\theta) = R(\theta)$ is not possible for all θ .

By Assumption 1, in any state θ the type-A investors have a lower probability of being early consumers than type-B investors, that is, $\lambda_A(\theta) < \lambda_B(\theta)$. This means that for any particular value of θ , type-A investors are better off than type-B investors, because they are less likely to be early consumers. Formally,

$$(A7) \quad \lambda_A(\theta) U(C_1(\theta)) + [1 - \lambda_A(\theta)] U(C_2(\theta)) \geq \lambda_B(\theta) U(C_1(\theta)) + [1 - \lambda_B(\theta)] U(C_2(\theta))$$

for every θ , with strict inequality for some θ .

From the perspective of date 0, a type-A investor will get a higher expected utility than a type-B investor from the given portfolio (ℓ, m) . Taking expectations with respect to θ in the above inequality yields

$$(A8) \quad E\{\lambda_A U(C_1) + (1 - \lambda_A) U(C_2)\} > E\{\lambda_B U(C_1) + (1 - \lambda_B) U(C_2)\}.$$

Now suppose that a type-A investor weakly prefers to stay out of the asset market.

Then

$$(A9) \quad \bar{u}_A \geq E\{\lambda_A U(C_1) + (1 - \lambda_A)U(C_2)\}$$

for any available portfolio. If the type-B investor had the same risk preferences, it would follow that he too would (weakly) prefer to stay out. Increasing his risk aversion will only amplify this preference. Formally, apply the transformation $V(\cdot)$ to both sides of the preceding inequality to get

$$(A10) \quad \bar{u}_B \equiv V(\bar{u}_A) \\ \geq V(E\{\lambda_B U(C_1) \\ + (1 - \lambda_B)U(C_2)\}).$$

Since V is strictly concave,

$$(A11) \quad V(E\{\lambda_B U(C_1) + (1 - \lambda_B)U(C_2)\}) \\ > E\{\lambda_B V(U(C_1)) + (1 - \lambda_B)V(U(C_2))\}$$

so

$$(A12) \quad \bar{u}_B > E\{\lambda_B V(U(C_1)) \\ + (1 - \lambda_B)V(U(C_2))\}.$$

Since this inequality holds for any portfolio (ℓ, m) , it is clear that

$$(A13) \quad V_A^*(P) \leq \bar{u}_A \text{ implies } V_B^*(P) < \bar{u}_B$$

for any equilibrium price function P so

$n_A < N_A$ implies $n_B = 0$. It can similarly be shown that when $n_B > 0$, all type-A investors strictly prefer to enter the market for the long asset so $n_A = N_A$.

PROOF OF PROPOSITION 4:

The proof follows the lines of the proof of Proposition 1 except for the fact that $n = (n_A, n_B)$ is taken as fixed. Construct a correspondence H from L_ε to itself by putting $H(\ell) = G_A(\ell, n) \times G_B(\ell, n)$ for every $\ell \in L_\varepsilon$. This mapping has a fixed point ℓ^ε , say, and letting $\varepsilon \rightarrow 0$ we obtain a convergent (sub)sequence $\ell^\varepsilon \rightarrow \ell^0$. The usual argument is used to show that $0 < \ell_A^0 + \ell_B^0 < 2W_0$, and continuity can be used to show that the portfolios defined by ℓ^0 are optimal at the price function $\Pi(\ell^0, n)$.

PROOF OF PROPOSITION 6:

In the usual way let

$$(P^k, (\ell_A^k, m_A^k), (\ell_B^k, m_B^k))$$

denote the partial equilibria corresponding to the economies with $\lambda_A(\theta) = k\theta$, $\lambda_B(\theta) = \bar{\lambda}_B$ and with $n_i = N_i, i = A, B$.

Since (ℓ_A^k, ℓ_B^k) is bounded, there is a convergent subsequence $(\ell_A^k, \ell_B^k) \rightarrow (\ell_A^0, \ell_B^0)$, say. From the pricing equation, for every value of θ , P^k converges almost surely to a function P^0 , as shown, for example, in equation (A14), below.

$$(A14) \quad P^k(\theta) = \min \left\{ R(\theta), \frac{[1 - \lambda_A(\theta)](W_0 - \ell_A^k)N_A + (1 - \bar{\lambda}_B)(W_0 - \ell_B^k)N_B}{\lambda_A(\theta)\ell_A^k N_A + \bar{\lambda}_B \ell_B^k N_B} \right\} \\ \rightarrow \min \left\{ R(\theta), \frac{(W_0 - \ell_A^0)N_A + (1 - \bar{\lambda}_B)(W_0 - \ell_B^0)N_B}{\bar{\lambda}_B \ell_B^0 N_B} \right\} \\ = \min\{R(\theta), \bar{Q}\} = P^0(\theta)$$

Using standard arguments, we can show that ℓ_i^0 is an optimal choice for a type- i investor in the limit when the price function is P^0 . Since type-A investors are certain of being late consumers, they will choose $\ell_A^0 = W_0$, so the price function reduces to

$$(A15) \quad P^0(\theta) = \min \left\{ R(\theta), \frac{(1 - \bar{\lambda}_B)(W_0 - \ell_B^0)}{\bar{\lambda}_B \ell_B^0} \right\}.$$

Then it is easy to see that $0 < \ell_B^0 < W_0$, because $\ell_B^0 = 0$ implies $P^0(\theta) = R(\theta)$ for all θ and $\ell_B^0 = W_0$ implies $P^0(\theta) = 0$ for all θ , neither of which is consistent with optimality.

If $\bar{Q} > 1$ then $P^0(\theta) > 1$ for all θ , and money is dominated at the first date; if $\bar{Q} < 1$ then $P^0(\theta) < 1$, and the long asset is dominated at the first date. Since both assets are held in the limit, we must have $\bar{Q} = 1$, that is, $P^0(\theta) = 1$ for all θ as required.

PROOF OF PROPOSITION 7:

For any value of k , let P^k denote the equilibrium price function and (ℓ_i^k, m_i^k) denote the equilibrium portfolio chosen by a type- i investor. The pricing equation implies that

$$(A16) \quad P^k(\theta) = \min\{R(\theta), M(\theta)/L(\theta)\} \\ = \min\{R(\theta), (1 - k\theta)(W_0 - \ell_A^k)/k\theta \ell_A^k\}.$$

Let $Y^k \equiv (W_0 - \ell_A^k)/k \ell_A^k$. Then,

$$(A17) \quad P^k(\theta) = \min \left\{ R(\theta), \frac{Y^k}{\theta} - \frac{W_0 - \ell_A^k}{\ell_A^k} \right\}.$$

We first note that Y^k is bounded. If not, we can choose a subsequence (using the same notation) that diverges to ∞ . Then for all k sufficiently large, $P^k(\theta) = R(\theta)$ for all θ , implying that cash is dominated and $\ell_A^k = W_0$, a contradiction.

Next note that $\{Y^k\}$ is bounded away from 0. If not, we can choose a subsequence

(using the same notation) that converges to 0. Then it is easy to check that P^k converges to 0 in probability, which implies that cash effectively dominates the asset and $\ell_A^k = 0$, contradicting Property 1.

Thus, we can find a convergent subsequence (using the same notation) $Y^k \rightarrow Y^0 \in (0, \infty)$. It follows that P^k converges almost uniformly to

$$(A18) \quad P^0(\theta) = \min\{R(\theta), Y^0/\theta\}.$$

From Property 4, we know that $P^0(\theta) < R(\theta)$ for some θ . On the other hand, P^0 is nonstochastic only if $P^0 = 0$, which contradicts Property 1. Thus, $Y^0 > 0$ and P^0 has a nonzero variance as required.

PROOF OF PROPOSITION 8:

The proposition claims that for some values of c and for k sufficiently small there exist two equilibria, one with full participation and one with limited participation. To prove this we assume the contrary and obtain a contradiction. If the proposition is false, we can find sequences of partial equilibria corresponding to the two participation regimes in which the participation decisions are not optimal. Let (P^{Lk}, ℓ^{Lk}) denote the sequence of price functions and portfolio decisions for the limited-participation regime and (P^{Fk}, ℓ^{Fk}) the sequence of price functions and portfolio decisions for the full-participation regime.

Step 1.—From Proposition 6, we know that P^{Fk} converges almost surely to 1, from which it is immediate that

$$(A19) \quad V_B^*(P^{Fk}) \\ \rightarrow \bar{\lambda}_B U_B(W_0) + (1 - \bar{\lambda}_B) U_B(W_0 \bar{R}).$$

Step 2.—The next step is to show that type-B investors would always choose a risky portfolio if they entered the market (i.e., ℓ_B^{Lk} is bounded away from 0). Since only type-A investors are in the market, equilibrium Properties 1 and 2 require that, for each value of k , $0 < \ell_A^{Lk} < W_0$. Then the

first-order condition

$$(A20) \quad E\{\lambda_A U'(C_{A1}^{Lk})(P^{Lk} - 1) + (1 - \lambda_A)U'(C_{A2}^{Lk})(1 - 1/P^{Lk})\bar{R}\} = 0$$

must be satisfied, where C_{A1}^{Lk} is the equilibrium consumption of type A at date t . Taking limits, we find that

$$(A21) \quad E\{U'(W_0\bar{R})(1 - 1/P^0)\bar{R}\} = 0$$

where P^0 is the limiting price function, which implies that $E\{1/P^0\} = 1$.

If ℓ_B^{Lk} is not bounded away from zero, we can find a convergent subsequence (using the same notation) $\ell_B^{Lk} \rightarrow 0$ and by a standard continuity argument show that $\ell_B = 0$ must be optimal in the limit when the price function is P^0 . But in the limit, the first-order condition for an optimal portfolio choice becomes

$$(A22) \quad E\{\bar{\lambda}_B U'_B(W_0)(P^0 - 1) + (1 - \bar{\lambda}_B)U'_B(W_0\bar{R}/P^0) \times (1 - 1/P^0)\bar{R}\} \leq 0.$$

However, using the fact that $1 = E\{1/P^0\} > 1/E\{P^0\}$, we see that

$$(A23) \quad E\{\bar{\lambda}_B U'_B(W_0)(P^0 - 1)\} > 0.$$

Given that $U'_B(W_0\bar{R}/P^0)$ and $(1 - 1/P^0)$

have positive covariance, then

$$(A24) \quad E\{(1 - \bar{\lambda}_B)U'_B(W_0\bar{R}/P^0)(1 - 1/P^0)\bar{R}\} > E\{(1 - \bar{\lambda}_B)U'_B(W_0\bar{R}/P^0)\} \times E\{(1 - 1/P^0)\bar{R}\} = 0$$

since $1 = E\{1/P^0\}$. This contradicts the first-order condition. Thus, $\{\ell_B^{Lk}\}$ is bounded away from 0.

Step 3.—It remains to show that

$$V_B^*(P^{Fk}) > V_B^*(P^{Lk})$$

for k sufficiently small. Comparing the derived expressions for expected utility in the limit, we need to demonstrate inequality (A25) where $P^{L0}(\theta) = \min\{\bar{R}, Y^0/\theta\}$ and Y^0 is the number defined in the proof of Proposition 7. Since we have shown that $0 < \ell_B^{L0} < W_0$, the type-B investor bears some price risk. By choosing V to be sufficiently concave so that the type-B investors are sufficiently risk-averse relative to the type-A investors, we can ensure that the inequality is satisfied. Then we can clearly choose $c > 0$ in such a way that

$$(A26) \quad V_B^*(P^{Lk}) < U_B(W_0 + c) < V_B^*(P^{Fk})$$

for all k sufficiently small, which proves that it must be the participation decisions of the type-A investors that are not optimal.

Step 4.—In the proof of Proposition 7, the fact that $\{Y^k\}$ is bounded also implies that $\ell_A^{Lk} \rightarrow W_0$, which immediately gives us equation (A27), on the following page.

$$(A25) \quad \bar{\lambda}_B V(U(W_0)) + (1 - \bar{\lambda}_B)V(U(W_0\bar{R})) > E\left\{\bar{\lambda}_B V(U(W_0 - \ell_B^{L0} + P^{L0}\ell_B^{L0})) + (1 - \bar{\lambda}_B)V\left(U\left(\left[\frac{W_0 - \ell_B^{L0}}{P^0} + \ell_B^{L0}\right]\bar{R}\right)\right)\right\}$$

$$(A27) \quad V_A^*(P^{Lk}) = E \left\{ \lambda_A U(W_0 - \ell_A^{Lk} + P^{Lk} \ell_A^{Lk}) + (1 - \lambda_A) U \left(\left[\frac{W_0 - \ell_A^{Lk}}{P^{Lk}} + \ell_A^{Lk} \right] \bar{R} \right) \right\} \\ \rightarrow U(W_0 \bar{R})$$

It follows from the definitions and the fact that P^{Fk} converges almost surely to 1 that

$$(A28) \quad V_A^*(P^{Fk}) \rightarrow U(W_0 \bar{R}).$$

From Proposition 2, $V_B^*(P^{Fk}) > U_B(W_0 + c)$ implies $V_A^*(P^{Fk}) > U(W_0 + c)$ for sufficiently small k . But the fact that $V_A^*(P^{Fk})$ and $V_A^*(P^{Lk})$ converge to the same limit implies that $V_A^*(P^{Lk}) > U(W_0 + c)$ for all sufficiently small k , showing that all the assumed participation decisions are optimal and contradicting our hypothesis. Thus, the proposition must be true.

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