

Are Entrepreneurs Different?

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April 15, 2007

Abstract

How important for firm performance are differences in owner's personal characteristics (risk tolerance or optimism) versus the institutional environment in which the firm operates (bankruptcy institutions, access to credit or return distributions)? To answer this question we examine SSBF data on small incorporated firms and construct a dynamic, computable model with heterogeneous agents where entrepreneurs weigh the firm's current return against expected future returns. The model accounts for three puzzles in the data: the risk/return trade-off from operating a small firm seems unattractive; owners' personal investments are poorly diversified; and many owners "bail out" their firm rather than declare bankruptcy. We show that entrepreneurs need not have personal characteristics that are significantly different from standard values, but the environment in which they operate matters greatly. The option to declare bankruptcy insures an entrepreneur against extreme current loss and the ability to bail out the firm with personal funds preserves the potential for high future gains. Welfare gains from bankruptcy reform or improved access to credit are equivalent to increasing net-worth by 35% for some agents.

JEL Classification Numbers:

Keywords: Entrepreneur; Bankruptcy; Risk Aversion; Optimism; Legal Environment; Credit Constraints; Small Firms

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We thank Dan Bernhardt, Tim Kehoe, Michele Tertilt and Vincenzo Quadrini. We gratefully acknowledge financial support from National Science Foundation grant SES-031839, NCSA computation grant SES050001, the Center for Private Equity Research at the University of Illinois and Kauffman Foundation grant 20061258. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the National Science Foundation or any other organization.

1 Introduction

How important for firm performance are differences in owner personal characteristics versus the institutional environment in which a firm operates? This question is important because bankruptcy institutions, access to credit and a firm's capital structure can be affected by policy. In contrast, innate characteristics such as risk tolerance or optimism cannot.¹ In this paper we derive facts and theory to assess the impact of these personal characteristics and institutions on firms. We first analyze data from the Survey of Small Business Finance (SSBF) and find three puzzles: entrepreneurs face a seemingly unattractive risk/return trade-off, have poorly diversified personal investments, and many have negative equity. Next we construct a dynamic, computable model with three features: (i) Forward looking entrepreneurs weigh current gains/losses against expected future returns. (ii) The distribution of firm returns from SSBF data is peaked in the middle, with "fat" asymmetric tails. (iii) Bankruptcy protection insures entrepreneurs against poor returns, but permits upside gain. The SSBF data make the analysis challenging because risk aversion is central to the debate on entrepreneurship, but individual coefficients and the distribution of risk aversion are not directly observable. The data also document substantial heterogeneity in entrepreneur behavior, hence we cannot use a representative agent model.

We describe the owners of small firms as entrepreneurs and highlight their importance. The U.S. Small Business Administration defines a firm as small if it has less than 500 employees; these firms account for 99.7 percent of all employer firms (89.3 percent in the U.S. for firms with less than 20 employees). Small firms account for more than 50 percent of non-farm private U.S. GDP, employ half of all private sector employees and pay 45 percent of total private payroll. They are a source of "good jobs", generating 60 to 80 percent of net new jobs annually over the last decade, employing 41 percent of high tech workers (scientists, engineers, and computer workers) and producing 13 to 14 times more patents per employee than large patenting firms.² Thus, small firms are a significant and vital part of the macroeconomy.

We study these firms in a model economy with many long lived agents and a competitive investor (bank). Individuals differ in their willingness to bear risk and entrepreneurs are agents that choose to operate a risky production technology. The risk neutral bank (with an elastic supply of funds)

¹There is a large literature on entrepreneurship. For example, Kihlstrom and Laffont (1979) show that more risk averse individuals become workers and less risk averse become entrepreneurs. Puri and Robinson (2007) use data from the Survey of Consumer Finance to show that the self employed are more optimistic than wage earners, but Hoelzl and Rustichini (2005) show that people are not overconfident when a task is unfamiliar and money is at stake, as is the case for entrepreneurs. There are many models of credit constraints, e.g. Evans and Jovanovic (1989). We will assess each factor – risk aversion, optimism and credit constraints.

²See <http://www.sba.gov/advo/stats/sbfaq.pdf>.

makes risky short term loans to entrepreneurs. Each period, the heterogeneous entrepreneurs decide the scale of their firm and the composition of firm finance: the mix of personal funds and bank loans. At the end of the period entrepreneurs observe their firm’s return and decide whether to repay the loan or default. If default occurs, the bank recovers only a fraction of the loan and the firm cannot operate for several periods. We solve an individual agent’s problem for consumption, the amount of personal net-worth to invest, firm scale, and firm debt-equity structure.³ We next derive cumulative probability distribution functions (cdfs) to account for heterogeneous risk aversion and uncertain firm returns. We then use the model to predict the cdfs of firms’ capital structure, assets, and the percent of personal net-worth entrepreneurs invest in their firms. The discipline imposed by the check for consistency between the distributions predicted by the model and those observed in the data is the analog of matching moments predicted by models with summary statistics from data in quantitative macroeconomic models (cf., Prescott (2006)).

There is a large literature on how institutions affect firms’ ability to raise finance. For example, Beim and Calomiris (2001) note that legal systems define property rights, specify procedures to enforce contracts, establish firms as legal entities (e.g., corporations have limited liability but debt of an unincorporated firm is a personal liability), and specify laws to resolve firm insolvencies, all of which affect entrepreneur decisions. There is also a recent literature on the quantitative effects of consumer bankruptcy rules in dynamic models with limited commitment, e.g. Livshits, MacGee, and Tertilt (2007) and Chatterjee, Corbae, Nakajima, and Rios-Rull (2003). To our knowledge, our paper is the first to examine the problem for firms. Livshits, MacGee and Tertilt analyze quantitatively two different bankruptcy institutions for unsecured consumer credit when markets are incomplete. In U.S. Chapter 7 “fresh start” bankruptcy, debt and all claims on future income are extinguished. The alternative policy restructures debt payments and permits garnishment of future income.⁴ They show that bankruptcy provides consumers with partial insurance against bad luck but also drives up interest rates, which impedes consumption smoothing. We find that more lenient bankruptcy provides a firm with more insurance against bad return realizations but drives up interest rates, raising the cost of future firm finance. This affects firm size, capital structure, access to credit and owner net worth.⁵

³Models with representative agents are aggregated by multiplying the optimal decision rules from the individual’s problem by the number of (identical) agents. This is not possible in our setting because differences in willingness to bear risk (i.e., heterogeneity with respect to the risk aversion parameter) are central to the debate on entrepreneurship. As in Krusell and Smith (1998), heterogeneity requires us to construct a distribution.

⁴Prior to 1999 consumers in Germany were liable for any debt until the end of life. The current law allows for discharge of debt after a 7-year court approved payment period.

⁵We focus on heterogeneity in willingness to bear risk, rather than ability e.g., Cagetti and DeNardi (2006) and Quadrini (2000). Note, however, that although initial net-worth and the return distribution are identical across firms ex-ante in our model, net-worth and consumption evolve differently over time due to differences in risk aversion and project realizations. See section 8.

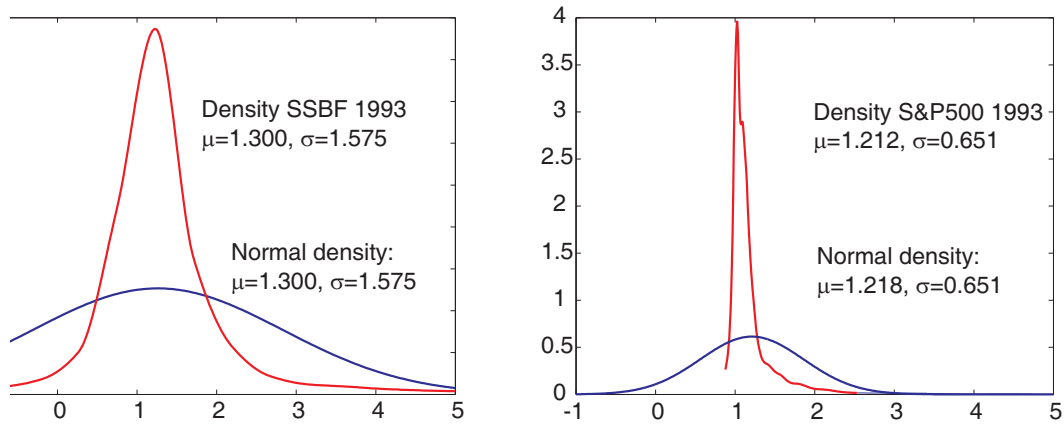


Figure 1: pdfs of return on assets for firms in SSBF 1993 and S&P500 (Compustat) versus normal pdfs

The paper proceeds as follows. Section 2 derives facts about small firms from the SSBF and Section 3 contains the model. Section 4 has theoretical results to obtain a computable problem. Section 5 constructs the distributions predicted by the model. The exogenous model parameters are specified in Section 6, including the derivations of firm return and risk aversion distributions. Section 7 reports the main results – model predictions for the endogenous parameters, median risk aversion and the borrowing constraint, and their implications for firm capital structure, size and owner net-worth. Section 8 reports computational experiments regarding bankruptcy institutions, credit constraints, optimism and the firm’s return distribution; it also analyzes entrepreneur ability in the SSBF data. Section 9 concludes.

2 Facts About Small Firms

The SSBF is a survey administered by The Board of Governors of the Federal Reserve System and the U.S. Small Business Administration. Conducted in 1987, 1993 and 1998, each survey is a cross sectional sample of about 4000 non-farm, non-financial, non-real estate small businesses that represent about 5 million firms.⁶ The surveys contain information on the characteristics of small firms and their primary owner (e.g., owner age, industry, type of business organization), firm income statements and balance sheets, details on the use and source of financial services, and recent firm borrowing experience (including trade credit and capital injections such as equity).

We document several facts from this survey:

Fact 1: *Small firm returns are very risky.*

⁶The 2003 survey was recently released. All surveys are available at <http://www.federalreserve.gov>.

Table 1 provides summary statistics about return on assets for small firms in the 1993 SSBF.⁷ As a benchmark we compare this return to that of a typical firm in the S&P500. The median return, skewness and kurtosis are roughly similar. SSBF firms are noticeably more risky, as the standard deviation indicates, with the higher risk somewhat compensated by a higher mean. About 12% of SSBF firms lost more than 20% of assets invested in the firm (debt plus equity), 7.4% lost more than 40%, and 3.8% lost more than 100%. Returns can also be substantial: 20.7% exceeded 50%, 10.4% exceeded 100%, and 3.8% exceeded 200%. Losses and gains are less extreme for S&P500 firms, as Figure 1 shows.

Table 1: Real Firm Return Summary Statistics, 1993 SSBF and S&P500

moment	median	mean	standard dev.	skewness	kurtosis
1993 SSBF	1.094	1.30	1.57	13.2	290
95% conf.	[1.08, 1.11]	[1.22, 1.38]	[0.95, 2.13]	[2.3, 17.3]	[29, 488]
1993 S&P500	1.093	1.21	0.65	13.1	221
95% conf.	[1.07, 1.10]	[1.16, 1.28]	[0.28, 1.02]	[3.1, 14.6]	[20, 277]

Figure 1 also shows that neither distribution is normal. It compares the empirical ROA distribution to a normal distribution with the same mean and variance; the left panel is for SSBF data and the right is for S&P500 data. Both empirical distributions are tighter around the median than a normal distribution because variance is generated by some firms that do exceptionally well. This, in turn, generates the high kurtosis.⁸

Fact 2: *Owners invest substantial personal net-worth in their firms.*

Table 2 reports the percentage of personal net-worth invested by entrepreneurs in their firm in the 1998 SSBF.⁹ The median amount of net-worth invested is 21%, but the data indicate a surprising lack of diversification for some entrepreneurs: 3% invest more than 80% of personal net-worth in their firm, 11% invest more than 60% and 25% invest more than 40%. This concentration of personal funds in a business is puzzling in view of the risky returns documented by table 1.

Fact 3: *Most owners work at their firms.*

⁷We use 1993 data because interest expenses are required to compute the return on assets; they are listed only in the 1993 data. Section 6.1 explains how to compute the ROA distributions for the SSBF and S&P500. We assume that all firms have access to the same constant returns to scale “blue print” technology. As a consequence, the return per unit of asset for a particular firm is a sample point from the distribution of the blue print technology.

⁸In table 1 the 95% confidence bands are computed for each moment using bootstrap sampling, except for the median where the interquartile range is reported.

⁹Owner net-worth is listed only in the 1998 SSBF, consisting of personal net-worth plus home equity.

Table 2: Net-Worth Invested, 1998 SSBF

% net-worth invested	$\geq 20\%$	$\geq 40\%$	$\geq 60\%$	$\geq 80\%$	mean	median
% of entrepreneurs	52%	25%	11%	3%	27%	21%

In the SSBF data for incorporated firms, the percentage of primary owners who work at their firms was 79% in 1993 and 89% in 1998. This compounds the risk return puzzle because if the firm fails, owners lose the funds invested and their jobs.¹⁰

Fact 4: *Negative equity for incorporated firms was 15.7% in 1993 and 21.0% in 1998.*

Negative equity means that the firm uses non-business assets to cover business losses (e.g., personal funds or unpaid bills absorbed by creditors). Entrepreneurs' willingness to use personal funds to "bail out" their firms seems puzzling since we consider only incorporated firms, which are protected by limited liability in bankruptcy. Is firm forbearance in the face of such poor performance rational? Put differently, why do these entrepreneurs not simply default on their loans?

Fact 5: *The average annual default rate on SBA loans is 3.5%.*

The low default rate documented by Glennon and Nigro (2005) on small business loans guaranteed by the Small Business Administration (SBA) deepens the puzzle.

Fact 6: *The distribution of firm capital structure is uniform.*

Figure 2 shows that the cdfs of Equity/Assets in the 1993 and 1998 SSBF are approximately uniform. Total assets consist of debt plus equity, thus Equity/Assets is a measure of firm capital structure. The approximately uniform cdfs, observed in both data sets, indicate that all capital structures are equally likely. This Modigliani-Miller empirical fact for the distribution of all firms, of course, does not preclude a particular firm from having a determinate structure.¹¹

¹⁰Moskowitz and Vissing-Jorgensen (2002) show that a similar risk-return puzzle exists for private equity investors in the Survey of Consumer Finance for large firms. However, the SCF data only allow them to determine the mean of the return distribution. We argue that more complete knowledge of the distribution, combined with bankruptcy protection and a dynamic decision problem are important for understanding the small firm puzzles.

¹¹If individual firm capital structure is optimal but the distribution is uniform, this suggests agent heterogeneity.

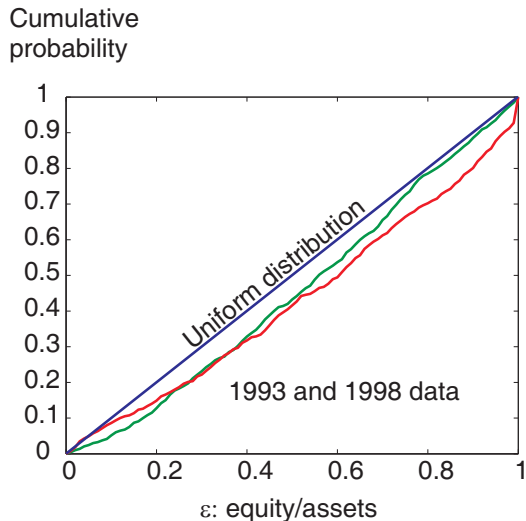


Figure 2: Equity/Assets for firms with positive equity: 1993 and 1998 SSBF Data

3 The Model

Consider an economy with $t = 0, 1, \dots$ time periods. A risk-neutral competitive bank has an elastic supply of funds and makes one-period loans.¹² There are many infinitely lived risk averse agents who discount the future at common rate β , each with CRRA utility function over consumption

$$u(c) = \frac{c^{1-\rho}}{1-\rho}.$$

Preferences are heterogeneous with respect to risk aversion parameter ρ , with $\rho \sim N(\mu, \sigma^2)$.

Agents have a given initial endowment w_0 and access to an ex-ante identical technology. If operated, the constant returns to scale technology produces a random output. For asset investment A , the firm's return on assets is given by random variable X , with cumulative distribution function $F(x)$ and probability density function $f(x)$ which is strictly positive on support $[\underline{x}, \bar{x}]$, with $\underline{x} \leq 0$ and $\bar{x} > 0$. A negative realization means that a firm's losses in a year exceed its current level of assets, and the owner must either use personal funds to stay solvent or default. Net-worth w_t is derived from the return on investment in all periods $t \geq 1$, known at the beginning of the period, and illiquid.¹³ We call an agent who chooses to operate a firm an entrepreneur, which formally means $A > 0$. An agent who chooses not to operate a firm sets $A = 0$. All agents have access to an outside investment opportunity that earns return r .

¹²Small firms may not have access to long-term loans because they lack payment or profit histories, audited financial statements, or verifiable contracts with workers, input suppliers or customers.

¹³For example, equity in a house is illiquid but can be used to invest in a business. We assume that a loan against this equity is riskless because it is fully collateralized. Although w_0 and the risky business technology are ex-ante identical, net-worth and consumption will evolve stochastically over time.

Entrepreneurs can raise assets to invest in the firm at time t in two ways:

Equity: Use personal net-worth w_t to self-finance at riskless real opportunity cost r .

Debt: Obtain a loan, secured by business assets, which gives the bank reservation return $1 + r_B$. The interest rate on the loan is determined endogenously for each entrepreneur by the model and will be higher than r_B when the bankruptcy probability is positive.

We permit $r \neq r_B$. Equality would prevail if all net-worth were liquid (e.g., cash). Because agents are long-lived and hence can invest long-term, the opportunity cost r of using personal net-worth to fund the firm will generally be higher than bank funds, $r > r_B$, where r_B is the bank's opportunity cost of short-term funds.¹⁴ Given a particular level of business assets A in a period, an entrepreneur determines the optimal mix of firm finance by choosing the percentage of self-finance ϵ . Thus, total equity is ϵA and debt is $(1-\epsilon)A$ at the beginning of the period. We introduce a borrowing constraint, i.e., a business loan cannot exceed a percentage b of the entrepreneur's net-worth, $(1 - \epsilon)A \leq bw$. The value of b is determined endogenously.

At the end of each period assets are Ax and the entrepreneur owes $A\bar{v}$. The entrepreneur can choose whether or not to default (repay the loan \bar{v}).¹⁵ If the entrepreneur does not repay, the bank can request judicial enforcement of the contract by a court described by δ, T . The court determines the total value of firm assets and transfers $(1 - \delta)$ percent to the bank, where δ is a deadweight bankruptcy loss (e.g., firm assets are sold at a loss). The entrepreneur is protected by limited liability (only firm assets can be seized), but has the option to pay firm debt with personal funds if this is optimal. If bankruptcy occurs, the firm is unable to operate for T periods. This assumption reflects the fact that bankruptcy remains on a credit record for a period of time; hence creditors and customers may be unwilling to do business with the entrepreneur.

The timing of events is as follows:

1. At the beginning of period t (*ex-ante*), entrepreneur net-worth is w . There are two cases:
 - (a) *The entrepreneur has not declared bankruptcy in any of the previous T periods.* The entrepreneur chooses consumption c ; firm assets A ; the percentage of self-finance ϵ (debt is $1 - \epsilon$); amount \bar{v} to repay such that the bank's ex-ante net return is $(1 - \epsilon)(1 + r_B)$.

¹⁴Consider home equity used to finance a business loan via a second mortgage. The rate on the second mortgage is typically higher than on a primary mortgage, which in turn exceeds the riskless rate. Other personal assets, such as retirement savings, have even higher penalties for early withdrawal.

¹⁵The firm may default for two reasons. It may be unable to repay loan \bar{v} if firm plus personal assets are less than $A\bar{v}$, and unwilling to repay otherwise. The entrepreneur can "bail out the firm" by using personal assets to forestall bankruptcy (but cannot be forced to do so). In contrast to Kehoe and Levine (1993) and Kocherlakota (1996), default occurs in equilibrium in our model.

(b) *The entrepreneur declared bankruptcy k periods ago.* The firm cannot operate for the next $T - k$ periods. Hence, only current consumption is chosen.

2. At the end of period t (*ex-post*) the firm's return on assets is realized, xA , and total end-of-period firm assets are Ax . The entrepreneur must decide whether or not to default.

(a) *Default:* Only firm assets are seized; the entrepreneur is left with personal net-worth $(1 + r)(w - \epsilon A - c)$, where personal assets are invested at outside interest rate r .

(b) *No Default:* Entrepreneur net-worth is $A(x - \bar{v}) + (1 + r)(w - \epsilon A - c)$, which includes both net-equity in the firm and the return on personal assets.

4 An Individual Agent's Problem

In this section we specify the optimization problem for an individual agent, with a given coefficient of risk aversion ρ . The objective is to determine the structure of the value function. We state the problem recursively. Let w be the entrepreneur's net-worth at the beginning of the period. If bankruptcy occurred in the previous T periods, then the state is given by (B, k, w) where k where is the number of periods since default. Otherwise, the state is given by (S, w) . Denote the value functions by $V_{B,k}(w)$ and $V_S(w)$, respectively. After T periods the firm can restart, thus $V_{B,T}(w) = V_S(w)$. Let \mathfrak{B} denote the set of realizations x for which bankruptcy occurs, with complement \mathfrak{B}^c .

If the firm did not default in the previous T periods, the agent solves:

Problem 1
$$V_S(w) = \max_{c,A,\epsilon,\bar{v}} u(c) + \beta \left[\int_{\mathfrak{B}} V_{B,1}((1+r)(w - \epsilon A - c)) dF(x) + \int_{\mathfrak{B}^c} V_S(A(x - \bar{v}) + (1+r)(w - \epsilon A - c)) dF(x) \right]$$

Subject to:

$$\int_{\mathfrak{B} \cap \mathbb{R}_-} x dF(x) + \int_{\mathfrak{B} \cap \mathbb{R}_+} (1 - \delta)x dF(x) + \int_{\mathfrak{B}^c} \bar{v} dF(x) \geq (1 - \epsilon)(1 + r_B) \quad (1)$$

$$x \in \mathfrak{B} \text{ if and only if } V_{B,1} \left((1+r)(w - \epsilon A - c) \right) > V_S \left(A(x - \bar{v}) + (1+r)(w - \epsilon A - c) \right) \quad (2)$$

$$(1 - \epsilon)A \leq bw \quad (3)$$

$$c \geq 0, A \geq 0, 0 \leq \epsilon \leq 1. \quad (4)$$

The objective is an agent's expected utility of current consumption and the expected discounted value of net-worth; the latter is the return on personal assets and net equity in the firm when it is bankrupt and solvent. Constraint (1) ensures the bank is willing to supply funds. The right-hand-side indicates that the $1 - \epsilon$ percent of funds the lender invests in the firm earn at least reservation return $1 + r_B$. The left-hand side is the lender's expected return: the first term accounts for the fact that the lender may absorb some losses when the firm's return is negative,¹⁶ the second term is the net amount transferred in bankruptcy states with positive net returns (deadweight default loss δ arises only if realization x is positive and the firm has not lost more than the value of its assets during the period), and the third term is the fixed debt repayment in solvency states. Constraint (2) specifies ex-post optimality of the default decision: An entrepreneur will default if and only if the expected discounted value of future consumption after default exceeds that from solvency.¹⁷ Constraint (3) is a standard borrowing constraint, see Evans and Jovanovic (1989), which captures the fact that in practice there are limits on the amount of debt a firm can raise: the bank does not grant an entrepreneur a loan that exceeds b percent of net worth. Finally, (4) ensures non-negativity and that ϵ is a percentage.

Now consider the problem of a firm that defaulted $k \leq T$ periods ago. Recall that $V_{B,T}(\cdot) = V_S(\cdot)$, i.e., after T periods the firm can operate again. Let w' denote net-worth in the next period.

Problem 2 $V_{B,k}(w) = \max_{c,w'} u(c) + \beta V_{B,k+1}(w')$

Subject to:

$$c(1+r) + w' \leq w(1+r); \tag{5}$$

$$c, w' \geq 0. \tag{6}$$

The objective of problem 2 is the agent's expected ex-ante utility. If default occurred, a firm cannot operate for T periods and the entrepreneur can choose only consumption and saving, consistent with budget constraint (5). (6) is the non-negativity constraint.

We use the fact that entrepreneurs have CRRA utility to determine the structure of the value function.

Proposition 1 *Suppose that the entrepreneur has constant relative risk aversion. Let $v_S = V_S(1)$ and $v_{B,k} = V_{B,k}(1)$. Then $V_S(w) = w^{1-\rho}v_S$ and $V_{B,k}(w) = w^{1-\rho}v_{B,k}$.*

¹⁶This can occur if the loan has an overdraft provision or the firm has some trade credit.

¹⁷Bailing out the firm with personal funds means that the entrepreneur continues to operate the firm even if $x < \bar{v}$. In a one period model (instead of the dynamic model) both $V_{B,1}$ and V_S would be the identity mapping, and (2) would reduce to $x \in \mathfrak{B}$ if and only if $(1+r)(w - \epsilon A - c) > A(x - \bar{v}) + (1+r)(w - \epsilon A - c)$, which implies $x \in \mathfrak{B}$ if and only if $x < \bar{v}$ (bankruptcy occurs only if the firm's return is less than its debt plus interest).

Applying Proposition 1 to Problem 2 it is straightforward to compute $v_{B,k}$ (as a function of v_S). Further, Lemma 1 and Lemma 2 in Appendix B prove that the investor's constraint binds and bankruptcy set \mathfrak{B} is a lower interval, with cutoff x^* . Thus, the entrepreneur's optimization problem can be rewritten as follows.

$$\begin{aligned} \mathbf{Problem\ 3} \quad v_S = & \max_{c,A,\epsilon,\bar{v}} u(c) + \beta v_B \int_{\underline{x}}^{x^*} \left[(1+r)(1-\epsilon A - c) \right]^{1-\rho} dF(x) \\ & + \beta v_S \int_{x^*}^{\bar{x}} \left[A(x - \bar{v}) + (1+r)(1-\epsilon A - c) \right]^{1-\rho} dF(x) \end{aligned}$$

Subject to:

$$\int_{\underline{x}}^0 x dF(x) + \int_0^{x^*} (1-\delta)x dF(x) + \int_{x^*}^{\bar{x}} \bar{v} dF(x) = (1-\epsilon)(1+r_B) \quad (7)$$

$$x^* = \max \left\{ \bar{v} - \left[1 - \left(\frac{v_B}{v_S} \right)^{\frac{1}{1-\rho}} \right] \frac{(1+r)(1-\epsilon A - c)}{A}, \underline{x} \right\} \quad (8)$$

$$c + \epsilon A \leq 1 \quad (9)$$

$$(1-\epsilon)A \leq b \quad (10)$$

$$c \geq 0, A \geq 0, 0 \leq \epsilon \leq 1. \quad (11)$$

The objective is to maximize the utility of current consumption and the expected discounted value of future net-worth in firm bankruptcy and solvency states. Constraint (7) corresponds to bank individual rationality constraint (1), and binds by Lemma 1 in Appendix B. Constraint (8) is the optimal default cutoff and follows from (2) by Lemma 2 in Appendix B. When w is normalized to 1, constraint (9) ensures feasibility; (10) is the borrowing constraint and (11) is obvious. Problem 3 is non-convex because the timing of decisions leads to a commitment problem: c , A , ϵ , \bar{v} are chosen ex-ante, but the bankruptcy decision is made ex-post and the entrepreneur cannot commit to not declare bankruptcy. This implies that default set cutoff x^* is determined by constraint (8). Lotteries cannot be used to convexify the problem because independent randomization over A , ϵ , c , \bar{v} and x^* is not possible. See Krasa and Villamil (2000), Krasa and Villamil (2003) for an analysis of randomization and commitment.

Proposition 2 *There exist $\underline{\rho} < 1$ and $\bar{r} > \frac{1}{\beta} - 1$ such that Problem 3 has a solution for all $\rho \geq \underline{\rho}$ and for all $r \leq \bar{r}$.*

Let $\Gamma(v_S)$ be the expected utility given continuation value v_S . In general $\Gamma'(v_S) > 1$ for all v_S close to 0. Thus, Γ is not a contraction mapping because net-worth is unbounded.¹⁸ In the proof

¹⁸Thus, a standard existence argument like maximizing a continuous function over a compact set cannot be used.

of Proposition 2 in Appendix B, we show that $\Gamma(0) \leq 0$ and there exists v_S such that $\Gamma(v_S) \geq 0$. As a consequence, continuity of Γ implies that Γ has a fixed point.

5 Heterogeneous Entrepreneurs & Model cdf Predictions

In the SSBF data we observe the distribution of end-of-period asset values, personal net-worth invested in the firm, and the ratio of equity over assets (firm capital structure). In order to derive predictions for these three cdfs in our model, we use the distribution of project returns, $f(x)$, to account for uncertainty and the distribution of entrepreneur risk aversion, $g_{\mu,\sigma}(\rho)$, to account for heterogeneity in risk aversion, with $\underline{\rho}$ the lowest parameter for which a model solution exists.

Distribution of Net-Worth: After project realization x occurs, firm assets are given by $A(\rho)x$ and firm debt is $A(\rho)\bar{v}$. Equity in the firm is $A(\rho)(x - \bar{v}(\rho))$, which is positive if $x \geq \bar{v}(\rho)$. The owner's personal net-worth outside the firm is $(1+r)(1-c_S(\rho) - \epsilon(\rho)A(\rho))$. Thus, the percentage of total net-worth invested is

$$\mathbf{w} = \frac{A(\rho)(x - \bar{v}(\rho))}{A(\rho)(x - \bar{v}(\rho)) + (1+r)(1-c_S(\rho) - \epsilon(\rho)A(\rho))}. \quad (12)$$

It follows immediately that \mathbf{w} is strictly increasing in x . We can solve equation (12) for $x = x(\mathbf{w}, \rho)$. The percentage of net-worth invested is less than or equal to \mathbf{w} for all $x \leq x(\mathbf{w}, \rho)$. The model cdf of the percent of net-worth invested for firms with positive equity is given by integration

$$W_{\mu,\sigma}^m(\mathbf{w}) = \frac{\int_{-\infty}^{\underline{\rho}} \int_{\bar{v}(\rho)}^{x(\mathbf{w},\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho + \int_{\underline{\rho}}^{\infty} \int_{\bar{v}(\rho)}^{x(\mathbf{w},\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho}{\int_{\bar{v}(\rho)}^{\infty} f(x) dx}, \quad (13)$$

where the denominator is the probability that the entrepreneur has positive equity.

Distribution of Equity/Assets: The percentage of equity is given by

$$\mathbf{e} = \frac{A(\rho)(x - \bar{v}(\rho))}{A(\rho)x}. \quad (14)$$

Solve this equation for $x = x(\mathbf{e}, \rho)$. For firms with positive equity, integrate the model cdf to get

$$E_{\mu,\sigma}^m(\mathbf{e}) = \frac{\int_{-\infty}^{\underline{\rho}} \int_{\bar{v}(\rho)}^{x(\mathbf{e},\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho + \int_{\underline{\rho}}^{\infty} \int_{\bar{v}(\rho)}^{x(\mathbf{e},\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho}{\int_{\bar{v}(\rho)}^{\infty} f(x) dx}. \quad (15)$$

End of Period Assets as a percentage of net-worth outside the firm: Let \mathbf{a} be the current realization of end of period assets, where

$$\mathbf{a} = \frac{A(\rho)x}{(1+r)(1-c_S(\rho) - \epsilon(\rho)A(\rho))}$$

Solving this equation for $x = x(\mathbf{a}, \rho)$ and integrating gives the model cdf for end of period assets

$$A_{\mu,\sigma}^m(\mathbf{a}) = \int_{-\infty}^{\underline{\rho}} \int_{\underline{x}}^{x(\mathbf{a},\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho + \int_{\underline{\rho}}^{\infty} \int_{\underline{x}}^{x(\mathbf{a},\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho. \quad (16)$$

6 Empirical Analysis

We now parameterize the model and construct distributions. We use the 1993 SSBF to compute the return on firm assets (ROA) because it includes interest payments. We use the 1998 SSBF to determine the percentage of net-worth invested in the firm because unlike the other surveys, it contains information about the principal firm owner’s net-worth. We use both surveys to compute the firm’s ratio of equity over assets (capital structure) because both contain equity and debt.

6.1 Construction of the Distribution of Firm Returns

We construct the distribution of firm returns for incorporated firms. We exclude unincorporated firms because the SSBF data do not account for the entrepreneur’s wage from running the firm. We compute the firm’s nominal after-tax ROA as follows:¹⁹

$$x = \frac{\text{Profit after taxes} + \text{Interest Paid}}{\text{Assets}} + 1. \quad (17)$$

Interest paid is added to after tax profit because the ROA must include payments to both debt and equity holders.²⁰ The nominal rate is adjusted by 3% for inflation in 1993 (BLS CPI). We compute return on assets instead of return on equity because about 16% of firms in the 1993 SSBF and 21% in 1998 had negative equity. Many of these firms stay in business because owners use personal funds to “bail out the firm.” Computing a ROA and modeling owners’ allocations of equity and debt accounts for this.²¹

¹⁹Section 2 reported the S&P500 ROA in 1993 (for comparison with the SSBF), computed as follows from Compustat’s Research Insight 7.6 database (440 companies with complete data):

$$x = \frac{IBCOM + XINT + DVP}{AT} + 1$$

IBCOM is income before extraordinary items, XINT is interest expense, DVP is preferred stock and dividends and AT is total assets. The median, mean, standard deviation, skewness and kurtosis are computed and confidence intervals are calculated using 5000 bootstrap iterations.

²⁰We consider after tax returns since this is relevant to an entrepreneur’s decision of how much net-equity to invest.

²¹Computing a ROE also can be misleading if a firm is near distress. If the firm records a low but positive level of equity, even a small profit will result in a high percentage return. Furthermore, many of the loans may be collateralized and the book value of equity understates the actual contribution of the firm’s owner, i.e., the “correct” value of equity. We restrict attention to firms with at least \$50,000 in assets.

6.2 Model Parameters & Construction of the Distribution of Risk Aversion

The model has eight parameters which we report in two tables. Table 3 reports the five exogenous parameters, with values taken from U.S. data or other studies: discount rate, β ; institutional parameters, T and δ ; bank real return on alternative investments, r_B ; and entrepreneur opportunity cost of using personal net-worth to fund the firm, r . We identify r_B , the bank's opportunity cost of short-term funds, with the average real return on 6 month Treasury bills between 1992 and 2006 because this period includes the SSBF data.²² The interest rate charged by the bank will be strictly higher than r_B because of bankruptcy costs. We identify the entrepreneur's opportunity cost of funds r with the real rate on 30 year mortgages over the period; the cost of using home equity to finance a business loan will also be strictly higher. The value for β is 1 divided by the rate of return. Weighing r and r_B equally, gives $\beta = 1/(1 + 0.5r_B + 0.5r) = 0.97$. We set $T = 11$ because in the U.S., after 10 years past default is removed from a credit record. We use Boyd and Smith (1994) to set deadweight bankruptcy loss δ at 10%.²³

Table 3: Exogenous Parameters

Parameter	Interpretation	Value	Comment/ Observations
β	discount factor	0.97	determined from r and r_B
T	default exclusion period	11	U.S. credit record
δ	default deadweight loss	0.10	Boyd-Smith (1994)
r_B	bank opportunity cost	1.2%	real rate, 6 mo T-Bill, 1992-2006
r	entrepreneur opportunity cost	4.5%	real rate, 30 year mortgage, 1992-2006

Endogenous parameters b , μ and σ are estimated from the data as follows. We choose the parameters to minimize a measure of the difference between the predicted and empirical cdfs of the percentage of net worth invested and the median asset level. Since we exclude firms with negative equity when determining W^e , net-worth invested ranges between 0% and 100%. In contrast, assets are unbounded.²⁴ This lack of a well defined upper bound for assets is a problem because tail behavior would greatly impact model prediction; we solve it by requiring the median asset level to lie within its empirical 95% confidence interval range of 43.1 to 51.9%. See equation (18) below.

We proceed as follows:

- (i) The empirical cdf of the percentage of net-worth invested, $W^e(\mathbf{w})$, is computed directly from

²²We use monthly data for T-Bill rates and deduct for each month the CPI reported by the BLS.

²³Section 10 in Appendix A shows that our results are largely insensitive to changes in δ , due to the low equilibrium default rate. Ex-ante deadweight loss δ is multiplied by the (small) bankruptcy probability times expected bankruptcy payments. These values are small, thus the product is very small.

²⁴For example, 5% of firms had assets as a function of ownership share that exceeded owner net-worth by 500%.

the 1998 SSBF by constructing:

$$\frac{\text{owners' share * equity}}{\text{Net-worth outside the firm} + \text{owners' share * equity}}.$$

$W^e(\mathbf{w})$ is the number of observations, accounting for sample weights, at which the percent of net-worth invested is less than or equal to \mathbf{w} .

(ii) The cdf of net-worth invested implied by the model, $W_{\mu,\sigma}^m(\mathbf{w})$, is given by (13) in section 5.

(iii) The empirical distribution of end-of-period assets per unit of net-worth is $A^e(\mathbf{a})$, where

$$\frac{\text{owners' share * asset}}{\text{Net-worth outside the firm}}.$$

The 95% confidence interval for the median asset level is [0.431, 0.519]. The model-predicted median assets are $\mathbf{a}_{\mu,\sigma}$ such that $A_{\mu,\sigma}^m(\mathbf{a}_{\mu,\sigma}) = 0.5$.

(iv) Choose parameters b , μ and σ to minimize the supnorm distance between the distribution implied by the model and the empirical distribution:

$$\min_{\mu,\sigma,b \geq 0} \|W_{\mu,\sigma}^m(\mathbf{w}) - W^e(\mathbf{w})\|_{\infty} + (0.431 - \mathbf{a}_{\mu,\sigma})^+ + (\mathbf{a}_{\mu,\sigma} - 0.519)^+, \quad (18)$$

where $\|\cdot\|_{\infty}$ is the supremum norm, taken over all non-negative percentages of net-worth.²⁵ The second and third terms impose penalties only for asset values outside the 95% confidence interval.

(iv) Use μ , σ and the assumption that the distribution of risk aversion is normal to construct density function $g_{\mu,\sigma}(\rho)$. Of course, when $\sigma = 0$ the population collapses to a single agent model.²⁶

6.3 Default Probability, Average Consumption & the Distribution of $\frac{\text{Equity}}{\text{Assets}}$

The percentage of equity finance is $\frac{\text{Equity}}{\text{Assets}}$. Empirical cdf, $E^e(\epsilon)$, is constructed by computing the percentage of firms (adjusted by sample weights) whose percentage of equity finance is less than or equal to ϵ .

The point estimate for the percentage of net-worth spent on consumption is given by

$$\text{Expected \% of net-worth consumed} = \int_{-\infty}^{\underline{\rho}} c_S(\underline{\rho}) g_{\mu,\sigma}(\rho) d\rho + \int_{\underline{\rho}}^{\infty} c_S(\rho) g_{\mu,\sigma}(\rho) d\rho \quad (19)$$

²⁵To compute the supremum norm we evaluate $|W_{\mu,\sigma}^m(\mathbf{w}) - W^e(\mathbf{w})|$ at 1,000 equidistant points between 0 and 1, and take the maximum. In Appendix C we show that the estimates are not affected by using the square distance

$$\sqrt{\int (W_{\mu,\sigma}^m(\mathbf{w}) - W^e(\mathbf{w}))^2 d\mathbf{w} + ((0.431 - \mathbf{a}_{\mu,\sigma})^+)^2 + ((\mathbf{a}_{\mu,\sigma} - 0.519)^+)^2}$$

²⁶To examine whether willingness to bear risk is a distinguishing feature of entrepreneurs, we permit entrepreneurs' coefficients of risk aversion to differ, ρ . Risk preference is not directly observable in the SSBF, and we have no information about the shape of this distribution. We assume that ρ is normally distributed in the population of firm owners, with mean μ and standard deviation σ , because this distribution has maximum entropy.

The point estimate for the probability of default is given by

$$\text{Expected default probability} = \int_{-\infty}^{\underline{\rho}} \int_{\underline{x}}^{x^*(\epsilon, \rho)} f(x) g_{\mu, \sigma}(\rho) dx d\rho + \int_{\underline{\rho}}^{\infty} \int_{\underline{x}}^{x^*(\epsilon, \rho)} f(x) g_{\mu, \sigma}(\rho) dx d\rho \quad (20)$$

6.4 Numerical Procedure

Given a particular choice of model parameters, solutions to problem 3 are computed as follows. For fixed v_S , use the first order conditions to solve for the optimum. Constraint (9) is always slack, since $c + \epsilon A = 1$ would imply zero future consumption. Thus, we need only verify whether constraint (10) and (or) (11) binds, by computing the first order conditions and checking for positive Lagrange multipliers. Inserting the solution of the first order conditions into the objective yields $\Gamma(v_S)$. To find a fixed point, compute slope $\Gamma'(v_S)$ either by using the Envelope Theorem or by computing the difference of Γ between v_S and some other point v'_S . The solution is the ex-ante percent of equity finance ϵ , assets A , consumption c_S , and debt \bar{v} . To go from these point estimates to cdfs that can be compared to the data, compute the end-of-period values of these parameters and use $f(x)$ and $g_{\mu, \sigma}(\rho)$ as explained in section 5. Compute $\underline{\rho}$ from the first order condition using the fact that $v_S \rightarrow \infty$ as $\rho \downarrow \underline{\rho}$.²⁷

7 Results

Table 4: Endogenous Parameters

Parameter	Interpretation	Est. Value
$b\%$	borrowing constraint: loan $\leq bw$	21.5
μ	median of distribution of risk aversion	1.55
σ	standard deviation of distribution of risk aversion	0.83

Parameters b , μ , σ are chosen to minimize (18), the distance between the model-predicted and empirical net-worth cdfs when assets are in the 95% confidence interval. The resulting values are reported in Table 4. The model predicts a maximal loan size of 21.5% of entrepreneur net-worth. These loans are secured by risky business assets because the firm is incorporated; the bank cannot seize personal assets in default. The median risk aversion is 1.55, with a standard deviation of 0.83. Thus, about 75% of all entrepreneurs have a coefficient of risk aversion that lies between 1 and 3, the range in real business cycle models. Using the Consumer Expenditure Survey, Mazzocco (2006)

²⁷In the computation we choose a very large value for v_S , and then solve for the remaining parameters including ρ , which approximates $\underline{\rho}$. In other words, rather than solving the fixed point problem for v_S , we solve it for ρ .

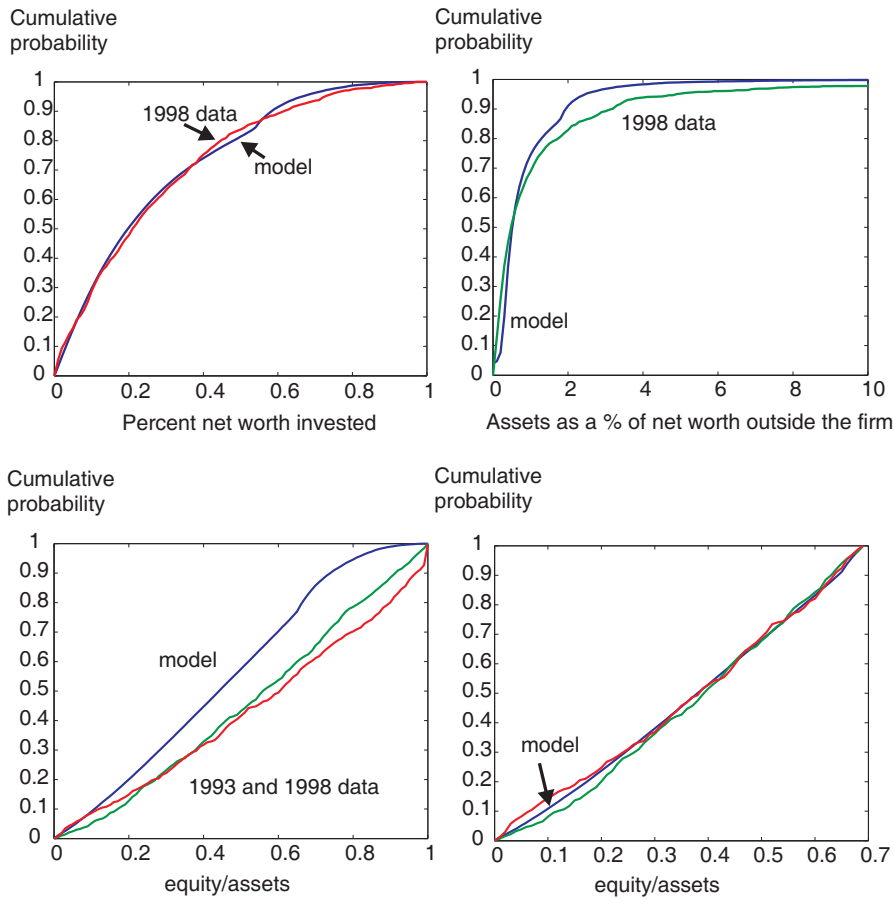


Figure 3: Model Predictions and SSBF Data: cdfs

estimates a median coefficient of risk aversion of 1.7 for men and 5 for women. We would expect entrepreneurs to be somewhat less risk averse than the general population and only about $x\%$ in the sample are women; our estimate for ρ is line with this.²⁸

Figure 3 compares the cdfs predicted by the model from section 5 with the relevant SSBF data. The first panel shows the model-predicted and empirical cdfs of the percent of net-worth an owner invests in the firm. Since we fit to this empirical cdf we would expect to see a match, but the match is surprisingly good given that we use only three parameters.

The next panel compares the predicted cdf of firm assets to its empirical counterpart, where assets are normalized by owner net-worth outside the firm. Recall that we use the median asset level in our fitting procedure (18). Because the asset level of 47.8% predicted by the model is well within the 95% confidence interval of [43.1, 51.9], the penalty term in (18) is not relevant in the

²⁸Since Mazzocco (2006) does not estimate the distribution of risk aversion, his estimate of the standard deviation of 0.96 is close, but not directly comparable to our estimate.

neighborhood of the optimal parameters. The match between the two asset cdfs is quite good, except the model under predicts a few large firms. This occurs because no model solutions exist below $\underline{\rho} = 0.74$, and as explained above, we assign point mass of $\mu(\{\rho \leq \bar{\rho}\})$ to $\bar{\rho}$. At $\bar{\rho}$, the ex-ante level of ϵ and A are 0.720 and 0.766, respectively, while c is close to 0. Thus, end of period net-worth outside the firm, $(1 - \epsilon A - c_S)(1 + r)$ is 0.470. Recall that the median level of x is $\bar{x} = 1.094$. Thus, the median value net-worth invested for risk aversion level $\bar{\rho}$ is $A\bar{x}/(1 - \epsilon A - c_S)(1 + r) = 1.786$. As the graph indicates, this is the range where the model predicted curve moves away from the data.

The bottom panels of figure 3 compare the model prediction for firm capital structure to the empirical cdfs for 1993 and 1998. The left panel shows that the model over predicts equity/assets, but only in the upper tail. This occurs because no model solutions exist below $\underline{\rho}$ and (15) assigns point mass to these values. Under the benchmark parameters in table 3, $\underline{\rho} = 0.74$ with an associated value of $\bar{v} = 0.335$. At median return level $x = 1.09$, this gives $(x - \bar{v})/x = 0.7$, which is about where the kink in the left panel occurs. Below 0.7, the model does an almost perfect job in replicating the approximately uniform distribution of equity/assets among firms as the right panel shows (conditioned on equity/asset less than 0.7).

Table 5: Model Point Estimates

Parameter	Interpretation	Value
median $A\%$	median firm assets (size)	48.1
default %	firm default rate	4.4
cons. %	consumption as a percent of net worth	3.6
neg. Eq. %	negative equity in the firm	10.6

The model predicts that 10.6% of firms have negative equity (table 5). This number is below the empirical values of 15.7% and 21.0% in 1993 and 1998. Given the high percentage of firms with negative equity, it may seem surprising that the model predicts a default rate of only 4.4% (though this is close to the empirical level). In a dynamic model an entrepreneur may not default, and hence continue to operate a firm that performs poorly today, if the firm's expected discounted continuation value is sufficiently high. While the benchmark model's predicted level of negative equity falls short of the values observed in the SSBF, section 8.3 shows that they are compatible with our model if entrepreneurs are slightly optimistic. Finally, the model also predicts an average consumption level of 3.6%, which is in the range of 3-5% documented for the U.S.

8 Computational Experiments

We argue that a dynamic model, the distinctive SSBF return distribution, and bankruptcy are essential for understanding entrepreneurs. We now conduct computational experiments to better understand the effect of bankruptcy rules, liquidity constraints, and entrepreneur optimism in explaining the data, and the distinctive SSBF return distribution. All tables are in Section 10: Appendix A.

8.1 Bankruptcy Institution: Experiment 1 in Appendix A

Bankruptcy Exclusion Period T : Experiment 1 evaluates the effect of the exclusion period on the results. Longer exclusion raises the penalty of bankruptcy. The U.S. benchmark is $T = 11$.

(i) Table 8: We re-estimate the model for values of T from 10 to 20. The results indicate that the estimates of μ and σ are very stable: as T increases, μ remains between 1.5 and 1.6 and σ between 0.74 and 0.9. Liquidity constraint b decreases a bit more significantly because the penalty increases with T ; entrepreneurs become more cautious and run smaller firms (lower A). In order to fit the model, the optimization procedure lowers b to ensure entrepreneurs use enough personal funds to finance the firm. Default decreases with T because it is more costly to the entrepreneur.

(ii) Table 9: These comparative statics with respect to T , fix μ , σ , b to the benchmark case, do not re-optimize, and evaluate T from 6 to 20. The default probability decreases by 50% and firm size, measured by median asset level A , decreases. Because b is fixed, the decrease in total investment results in a decrease in equity and an increase in debt, which raises negative equity.

(iii) Table 10: We wish to consider the effect of changes in T on welfare, but agents are heterogeneous and we cannot simply compare utilities. We use an equivalent variation: fix the base case at $T = 11$, consider changing T to T' to get the change in net-worth at $T = 11$ that would be equivalent to an actual change in the bankruptcy exclusion period to T' . Formally, let $V_S^T(w)$ be the value function in a solvency state for bankruptcy exclusion period T . The equivalent variation is given by λ such that $V_S^T(\lambda w) = V_S^{T'}(w)$. Proposition 1 implies that $V_S^T(\lambda w) = \lambda^{1-\rho} V_S^T(w)$. Further, $V_S^T(w) = w^{1-\rho} V_S^T(1) = v_S^T$ and $V_S^{T'}(w) = w^{1-\rho} V_S^{T'}(1) = v_S^{T'}$. The change in welfare is

$$\lambda = \left(\frac{v_S^T}{v_S^{T'}} \right)^{\frac{1}{1-\rho}}.$$

The estimates of μ and σ and to a lesser degree b are relatively insensitive to T , but table 10 shows the bankruptcy exclusion period has a significant impact on welfare. Lowering T from 11 to 10 is equivalent to a 1.1% increase in net-worth to a person with the median level of risk aversion. The

improvement for an agent with risk aversion $\rho = 0.9$ is a much more substantial increase of 6.3%. Decreasing T to 6 is equivalent to a 7.7% increase in net-worth for the median agent and 36.9% for one with $\rho = 0.9$. Similarly, if T is increased to 20, this corresponds to a 5.7% loss of net-worth to the median person, and 20.4% when $\rho = 0.9$.

(iv) Table 11: One of the main economic arguments in support of the recent U.S. bankruptcy reform act was that more stringent bankruptcy rules lower interest rates, and therefore help borrowers. Table 11 shows that the loan rate decreases as T increases. However, the downside of stricter bankruptcy is less insurance against bad realizations. The welfare results in table 10 indicate that this insurance effects strongly dominates the benefit from lower interest rates.

Our results involve a tradeoff between insurance and interest rates. Livshits, MacGee, and Tertilt (2007) show that the insurance aspect can dominate in some (but not all) cases in consumer bankruptcy, but their effect is much more modest than ours. The key reason for the difference is that in our model, more lenient bankruptcy rule encourages more risk taking and entrepreneurs invest more in their firms, operating at a larger scale. This increase in production is the main source of the large welfare effect we find.²⁹ In this sense, even though we do not find extreme variations in ρ , risk interacts with the dynamic decision problem, the return distribution and the bankruptcy institution to have an important effect on some of the heterogeneous agents, namely those that are most invested in their firms.

Bankruptcy Costs δ : Experiment 1 also considers the effects of bankruptcy cost δ . Table 8 is the benchmark case with $\delta = 0.1$. Table 12 solely increases the cost to $\delta = 0.3$ and re-estimates the model. The model estimates of μ , σ , b are almost unaffected. This shows that the model is robust to such changes in bankruptcy costs, and that a detailed measurement of this cost is not essential within this range. Table 13 reports comparative static results in which δ varies between 0 and 100%, fixing b , μ and σ at the benchmark values (i.e., we do not re-estimate the model). Again, δ has almost no impact on endogenous variables — and the impact is significantly smaller than in the comparative statics with respect to T . Table 14 shows that the welfare gains or losses from δ are also minor: For an entrepreneur with the median level of risk aversion the gains/losses are less than 0.1%. The effect of δ is minor because (a) bankruptcy occurs with only a small probability, and (b) since assets A in bankruptcy states tend to be small, the deadweight loss δA will also be small. Clearly, the expected costs, i.e., the product of (a) and (b) is second order.

The results indicate that when $\delta < 0.3$, changes in the cost of bankruptcy (e.g., court efficiency

²⁹In our model bad luck is a poor return realization x while in Livshits, MacGee, and Tertilt (2007) it is a health, divorce or family shock. To our knowledge, the effect of the factors we consider on production has not been examined before.

in liquidating firm assets) has only minor impacts on welfare. However, if δ is very large and if, in addition, there are substantial fixed cost to creditors to recover payment from defaulters, the parties will attempt to avoid costly bankruptcy (e.g., through debt forgiveness or renegotiation) and debtors will be more likely to default. The static model of Krasa, Sharma, and Villamil (2007) shows that these effects can generate substantial deadweight losses when courts are sufficiently inefficient.

8.2 Risk Aversion and Liquidity Constraints: Experiment 2 in Appendix A

Experiment 2 considers the effect of changes in μ and b . Median risk aversion μ cannot be modified by policy changes, but comparative statics with respect to μ help develop intuition for the model. Table 15 shows that higher median risk aversion implies that entrepreneurs run smaller firms, and the effect is pronounced. Since b is kept constant, these smaller firms have higher levels of debt, which in turn explains why negative equity and the default rate rise with μ . Consumption increases because the intertemporal rate of substitution is affected when ρ is increased.

From a policy perspective, changes in b are more relevant. Table 16 shows that increasing b allows firms to borrow more, and hence operate at a larger scale A . The higher levels of firm debt, however, increase the percentage of firms who default or have negative equity. Table 17 shows there are again substantial welfare effects for the least risk averse agents, but not for more risk averse agents because for sufficiently high b the credit constraint does not bind. Comparing the welfare effects of T and b shows that the median entrepreneur receives more benefit from more lenient bankruptcy rules than from reducing the borrowing constraint.

Lowering T or raising b both increase the riskiness of loans. In the U.S., regulation makes it very costly for banks to deal with creditors with high default rates. Bank CAMELS ratings establish portfolio risk, and a poor rating forces a bank to either hold more liquid assets or it raises the cost of funds.³⁰ Because bank deposits are insured in the U.S., these regulations are designed to prevent banks from taking excessive risk (insurance limits bank and depositor downside loss and greater risk taking may be beneficial for the bank but inefficient for society). Thus, b cannot simply be raised by a policy maker, unless it is accompanied by other institutional changes. Similarly, making bankruptcy more lenient by lowering T could also lead banks to reduce loans, i.e., effectively reducing b . While our results clearly indicate that lowering T is beneficial, any policy that makes bankruptcy more lenient would also have to ensure that banks did not simply increase their credit constraints. In other words, in practice T and b may be linked and investigating this is an important

³⁰The acronym CAMELS refers to the components of a bank's condition that are assessed by regulators: Capital adequacy, Asset quality, Management, Earnings, Liquidity and Sensitivity to market risk.

topic for future research in a model with heterogeneity in risk aversion.³¹

8.3 Entrepreneur Optimism: Experiment 3 in Appendix A

How does optimism by entrepreneurs affect our results? Intuition suggests that less risk averse, less optimistic agents will behave similar to more risk averse, more optimistic agents. This leads to an identification problem: optimistic agents may be observationally equivalent to less risk averse, non-optimistic agents. We now investigate whether the model has observable implications that are uniquely induced by optimism. We assume that an optimistic entrepreneur believes the firm’s return exceeds the true return by some fixed percentage \bar{o} . Formally, this implies the entrepreneur assumes that firm returns are $X + \bar{o}$, which yields cdf $H(x - \bar{o})$ in the objective of problem 3. The bank is assumed to use the correct distribution to determine payoff (7) in problem 3.

In Experiment 3 we vary \bar{o} and fix all other parameters. The exclusion benchmark T is 11, and T is varied from 10 to 20. Table 19 shows that slight optimism (10%) improves the fit in the baseline model with $T = 11$ while keeping μ , σ and the default rate in acceptable ranges. Liquidity constraint parameter b increases slightly, as does A . Negative equity increases to a level more consistent with the SSBF. The intuition for the increase in negative equity is that optimistic entrepreneurs run larger firms because they expect higher future returns relative to the baseline, thereby increasing the total amount of debt \bar{v} . Equity is negative if $x < \bar{v}$. When \bar{v} is higher, $x < \bar{v}$ is more likely and this increases the percentage of projects with negative equity. Mild entrepreneur optimism can explain the level of negative equity observed in the 1993 SSBF (15.7%) and still accommodate the relatively low level of default observed in the data.

8.4 Counterfactual Exercise: Normally Distributed Firm Returns

In order to show the importance of a risky return distribution, we conduct two counterfactual experiments. We replace the empirical ROA distribution computed from SSBF data, keeping all other benchmark settings the same, with two different normal distributions:

Best Fit Normal Distribution. Let $g_{\mu,\sigma}$ be the density of a normal distribution with mean μ and standard deviation σ and f be the density of the empirical distribution. We solve $\min_{\mu,\sigma} \sup_x |g_{\mu,\sigma}(x) - f(x)|$ to find a normal distribution that best approximates the empirical density function. The resulting values are $\mu = 1.193$ and $\sigma = 0.394$ — both distributions are shown in figure 4. In order to fit the “middle” this normal distribution has less mass in the tails and, as a consequence, is

³¹Akyol and Athreya (2007) examine this relationship in an occupational choice model with bankruptcy and two types of human capital but no heterogeneity in willingness to bear risk.

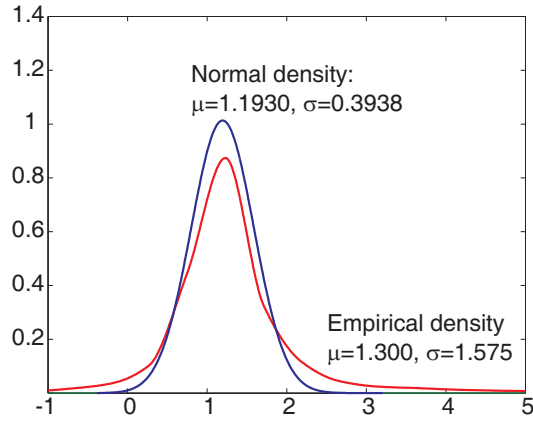


Figure 4: Empirical firm return pdf versus best-fit normal pdf, SSBF 1993

less risky. Median risk aversion increases from 1.55 to 2.33 but at the same time, for given ρ , the lower project risk in the normal distribution encourages entrepreneurs to run firms that are larger. Default is lower, again because the normal distribution has a thinner lower tail. Finally, the thinner upper tail of the normal distribution implies that less firms will be “lucky” and have a very good realization. In order to be able to match the distribution of net-worth invested, firms must be more leveraged: Given two solvent firms with the same realization, a more leveraged firm earns a higher return because the owner receives a higher residual after making the fixed debt payment.³² The somewhat higher level of debt also implies that more low realizations will result in negative equity, i.e., the predicted percentage of firms with negative equity increases from 10.6% to 13.7%.

Normal Distribution with the SSBF μ, σ . Figure 1 compares the empirical distribution with a normal distribution with the same mean and standard deviation. Table 6 shows the results for this distribution are significantly at odds with the data, highlighting the importance of the return distribution. First, the fat tails lead to μ and σ such that all point mass is at $\underline{\rho}$ and $\bar{\rho}$, where $\bar{\rho}$ is the highest value of risk aversion for which we compute a solution. Generally, we can choose $\bar{\rho}$ sufficiently high that the mass above $\bar{\rho}$ is negligible; this is not the case for this normal distribution with fat tails and $\bar{\rho}$ affects the results.³³ Second, the model predictions in the last column of table 6 are clearly implausible.

³²This also explains the higher value of b .

³³Upper bound $\bar{\rho}$ is needed for computational reasons; it is impossible to compute solutions for a fine grid $[\underline{\rho}, \infty]$.

Table 6: Counterfactual Experiment: Normal Distributions

Parameter	Data	Empirical Dist	Best Fit Normal Dist $\mu=1.193, \sigma=0.3938$	μ, σ Normal Dist $\mu=1.300, \sigma=1.193$
μ	1-3	1.55	2.33	$4.4 * 10^8$
σ	NA	.83	1.11	$7.9 * 10^8$
b%	NA	21.5	30.0	23.4
fit	NA	0.042	0.040	.045
median A%	[43.1,51.9]	48.1	54.7	38.6
default %	3.5	4.4	1.5	61.0
cons. %	3-5	3.6	4.9	3.1
neg. Eq %	15.7	10.6	13.7	64.4

8.5 Entrepreneur Ability

This paper has focused on differences among agents in their willingness to bear risk because this is a central theme in discussions of entrepreneurship. There is also a large literature on differences in entrepreneurial ability. We omit this type of heterogeneity from our analysis because we examined the SSBF data for evidence of differences in entrepreneur ability and found none. We assume that all agents have similar ability to be consistent with this data. In the Lucas (1978) “span of control” model, the foundation of entrepreneur ability models, a firm with a more able manager has better return realizations than a firm with a less able manager. Our model implies that a more able entrepreneur (with a better return distribution) would run a firm that is larger in relation to the entrepreneur’s net-worth w than a less able one. Thus, if ability is relevant in our data, then A/w should be positively correlated with realizations x . We use the 1998 SSBF data to test whether a positive correlation exists.³⁴

The level of assets reported in the SSBF corresponds to Aw in our model. We divide assets by entrepreneur net-worth, and compute firm return on assets according to (17) in section 6.1. Consistent with our previous empirical analysis, we consider only incorporated firms with assets over \$50,000. Entrepreneurs with zero or negative net-worth are omitted. The results are not sensitive to entrepreneur equity in the firm. If we include the owner’s equity in the firm, then

$$\text{realization} = 1.4776 + 0.0042A + \text{error}$$

$$(0.0934) \quad (0.0119)$$

³⁴We use 1998 data because the 1993 data set does not contain entrepreneur net-worth. The 1998 data set does not contain interest expenses, which should be added to profits to compute ROA. Since interest expenses in the 1993 set are not large relative to profit, this missing data problem does not have a significant impact on the correlation coefficient.

Otherwise, if we exclude the owner's equity in the firm then

$$\text{realization} = 1.5175 - 0.0068A + \text{error}$$

(0.0926) (0.0101)

The numbers in parenthesis are the estimated standard deviation of the coefficient.

The absence of a positive correlation between assets and realizations indicates that differences in ability is not significant in the SSBF data. While we do not doubt the relevance of manager ability for firm success more broadly, we believe it does not matter in the SSBF data due to a selection bias that is particularly acute for small firms. First, most firms in the data set have operated for a number of years; firms operated by less able entrepreneurs are less likely to survive. Second, most firms with less able managers will not accumulate the required \$50,000 in assets.

9 Concluding Remarks

This paper assesses the impact of differences in innate personal characteristics versus institutions on entrepreneurship – whether to operate a firm, its size, capital structure, terms of finance, owner consumption and net-worth. We examined SSBF data and constructed a dynamic, computable model to organize the facts. The returns generated by small firms are very risky, yet some entrepreneurs invest a substantial portion of personal net-worth in their business and even use personal funds to cover business losses when they would be protected by limited liability if they declared bankruptcy (the default rate is low). We show that intertemporal tradeoffs, the return distribution, and bankruptcy can explain this puzzle. The distribution is important because the combination of asymmetric fat tails and bankruptcy means that the firm is insured against extreme loss, but may enjoy very high future upside gains. Most of the mass is centered around the middle of the distribution, which is attractive to individuals with standard degrees of risk aversion. Entrepreneurs trade off the value of absorbing a current loss against the expected discounted value of future gains from maintaining the firm. Thus, to answer the question posed by the paper — Entrepreneurs need not have significantly different personal characteristics such as willingness to bear risk or optimism to explain the facts in the SSBF data. However, the environment in which they operate is very important (e.g., bankruptcy laws, access to credit and the return distribution), which suggests that policies which effect their environment are important.

10 Appendix A: Experiments

10.1 Experiment 1: Bankruptcy Exclusion Parameter T and Cost δ

Table 8 Benchmark Exogenous Variables: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

T	10	11	12	13	14	15	16	20
μ	1.62	1.55	1.49	1.51	1.52	1.52	1.51	1.50
σ	0.90	0.83	0.75	0.74	0.76	0.76	0.76	0.78
b %	20.6	21.5	22.0	19.8	18.4	17.7	17.3	15.4
fit	0.046	0.042	0.037	0.034	0.034	0.034	0.035	0.036
median A %	46.9	48.1	49.2	47.0	45.3	44.3	43.8	41.3
default %	4.7	4.4	4.2	3.8	3.5	3.3	3.1	2.5
cons. %	3.7	3.6	3.6	3.6	3.6	3.6	3.6	3.5
neg Eq. %	10.2	10.6	10.8	10.5	10.8	11.1	11.6	11.1

Table 9 Comparative statics for T : Fix $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

T	6	7	8	9	10	11	12	13	14	15	16	20
fit	0.095	0.085	0.076	0.066	0.053	0.042	0.054	0.065	0.073	0.079	0.084	0.107
med A %	56.2	54.4	52.7	51.1	49.6	48.1	46.7	45.4	44.3	43.3	42.4	38.9
default %	6.1	5.6	5.3	5.0	4.7	4.4	4.2	4.0	3.8	3.6	3.5	2.9
cons. %	3.7	3.7	3.7	3.7	3.7	3.6	3.6	3.6	3.6	3.6	3.6	3.6
neg Eq. %	8.4	8.8	9.3	9.7	10.0	10.6	11.1	11.9	13.4	15.1	17.0	21.0

Table 10 Welfare Effect as T Varies: % increase or decrease of net-worth compared to benchmark

risk aversion ρ	0.9	1.2	1.5	1.8	2.1	2.5	3.0	3.5	4.0
$T = 6$	36.9	11.2	7.7	6.1	5.0	3.9	3.1	2.6	2.2
$T = 7$	27.5	8.1	5.6	4.4	3.6	2.8	2.2	1.9	1.6
$T = 8$	19.8	5.5	3.9	3.0	2.4	1.9	1.5	1.3	1.1
$T = 9$	13.5	3.2	2.4	1.8	1.5	1.2	0.9	0.7	0.6
$T = 10$	6.3	1.3	1.1	0.8	0.7	0.5	0.4	0.3	0.3
$T = 11$	—	—	—	—	—	—	—	—	—
$T = 12$	-3.6	-0.7	-0.9	-0.7	-0.5	-0.4	-0.3	-0.3	-0.2
$T = 13$	-4.4	-3.2	-1.5	-1.3	-1.0	-0.8	-0.6	-0.5	-0.4
$T = 14$	-7.6	-4.4	-2.1	-1.8	-1.4	-1.0	-0.8	-0.6	-0.5
$T = 15$	-10.5	-5.5	-3.2	-2.1	-1.7	-1.3	-1.0	-0.8	-0.6
$T = 16$	-12.7	-6.5	-3.8	-2.4	-2.0	-1.5	-1.1	-0.9	-0.7
$T = 20$	-20.4	-9.4	-5.7	-4.0	-2.8	-1.8	-1.5	-1.2	-1.0

Table 11 Interest Rate as T Varies

risk aversion ρ	0.9	1.2	1.5	1.8	2.1	2.5	3.0	3.5	4.0
$T = 6$	18.0	15.3	14.2	14.0	14.3	14.4	14.3	14.1	14.1
$T = 7$	17.7	14.9	13.7	13.5	13.8	13.8	13.7	13.6	13.5
$T = 8$	17.3	14.5	13.3	13.0	13.3	13.3	13.2	13.1	13.0
$T = 9$	17.0	14.1	12.9	12.5	12.8	12.9	12.7	12.6	12.5
$T = 10$	16.6	13.7	12.4	12.1	12.3	12.4	12.2	12.0	11.9
$T = 11$	16.3	13.3	12.0	11.6	11.9	11.9	11.6	11.5	11.4
$T = 12$	16.0	12.9	11.7	11.2	11.4	11.4	11.2	11.0	10.9
$T = 13$	15.6	12.6	11.3	10.8	11.0	10.9	10.7	10.6	10.4
$T = 14$	15.3	12.3	10.9	10.5	10.6	10.5	10.3	10.2	10.1
$T = 15$	15.0	12.0	10.7	10.1	10.2	10.2	10.0	9.8	9.7
$T = 16$	14.7	11.8	10.4	9.8	9.8	9.8	9.6	9.5	9.4
$T = 20$	13.6	10.7	9.3	8.7	8.5	8.8	8.6	8.5	8.4

Table 12 Higher Cost δ : $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.30$

T	10	11	12	13	14	15	16	20
μ	1.79	1.67	1.55	1.50	1.52	1.52	1.51	1.50
σ	1.08	0.95	0.81	0.74	0.76	0.76	0.76	0.78
b %	14.9	16.9	19.8	20.1	18.4	17.6	17.2	15.4
fit	0.052	0.046	0.040	0.035	0.034	0.034	0.035	0.036
median A %	39.8	42.6	46.3	47.3	45.3	44.3	43.6	41.3
default %	4.0	4.0	4.0	3.8	3.5	3.2	3.1	2.5
cons. %	3.8	3.7	3.7	3.6	3.6	3.6	3.6	3.5
neg Eq. %	8.7	9.2	10.2	10.5	10.7	11.0	11.4	11.1

Table 13 Comparative Statics for δ : Fix $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

δ	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.80	1.00
fit	0.042	0.042	0.046	0.050	0.054	0.057	0.060	0.063	0.065
median A %	48.3	48.1	48.0	47.9	47.8	47.8	47.7	47.6	47.5
default %	4.5	4.4	4.4	4.4	4.3	4.3	4.3	4.2	4.2
cons. %	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6
neg Eq. %	10.8	10.6	10.3	10.2	10.1	10.1	10.0	9.9	9.7

Table 14 Welfare Effect as δ Varies: % increase or decrease of net-worth compared to benchmark

risk aversion ρ	0.9	1.2	1.5	1.8	2.1	2.5	3.0	3.5	4.0
$\delta = 0.00$	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.0	0.0
$\delta = 0.10$	—	—	—	—	—	—	—	—	—
$\delta = 0.20$	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0
$\delta = 0.30$	0.0	0.0	0.0	0.0	-0.2	-0.1	-0.1	-0.1	0.0
$\delta = 0.40$	0.0	0.0	0.0	0.0	-0.2	-0.2	-0.1	-0.1	-0.1
$\delta = 0.50$	0.0	0.0	0.0	-0.1	-0.3	-0.2	-0.2	-0.1	-0.1
$\delta = 0.60$	0.0	0.0	0.0	-0.1	-0.4	-0.3	-0.2	-0.1	-0.1
$\delta = 0.80$	0.0	0.0	0.0	-0.2	-0.5	-0.3	-0.2	-0.2	-0.1
$\delta = 1.00$	0.0	0.0	0.0	-0.2	-0.5	-0.4	-0.3	-0.2	-0.1

10.2 Experiment 2: μ and b

Table 15 Comparative Statics for μ : $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

μ	1.15	1.25	1.35	1.45	1.55	1.65	1.75	1.85
fit	0.224	0.146	0.109	0.074	0.042	0.080	0.117	0.153
median A %	74.3	65.4	58.3	52.7	48.1	44.4	41.2	38.6
default %	4.2	4.2	4.3	4.4	4.4	4.5	4.6	4.7
cons. %	2.8	3.0	3.2	3.5	3.6	3.8	4.0	4.1
neg Eq. %	8.4	8.9	9.5	10.0	10.6	11.1	11.7	12.3

Table 16 Comparative Statics for b : $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

b	0.10	0.15	0.20	0.21	0.25	0.30	0.35	0.40	0.50
fit	0.145	0.071	0.048	0.042	0.067	0.094	0.113	0.126	0.145
median A %	46.9	47.1	47.8	48.1	49.0	50.5	51.8	52.4	52.5
default %	3.0	3.6	4.3	4.4	4.8	5.2	5.4	5.6	5.9
cons. %	3.6	3.6	3.6	3.6	3.7	3.7	3.7	3.6	3.6
neg Eq. %	5.7	7.7	10.0	10.6	11.7	12.9	13.6	14.2	15.0

Table 17 Welfare Effect as b Varies: % increase or decrease in net-worth compared to benchmark

Note: More risk averse agents are unaffected because the credit constraint does not bind for them

risk aversion ρ	0.9	1.2	1.5	1.8	2.1	2.5	3.0	3.5	4.0
$b = 0.100$	-13.1	-8.5	-6.2	-4.9	-3.7	-2.2	-0.9	-0.2	-0.1
$b = 0.150$	-6.1	-4.8	-3.2	-2.0	-1.4	-0.4	0.0	0.0	0.0
$b = 0.200$	-1.8	-0.2	-0.6	-0.4	-0.1	0.0	0.0	0.0	0.0
$b = 0.215$	—	—	—	—	—	—	—	—	—
$b = 0.250$	8.2	1.7	1.3	0.6	0.0	0.0	0.0	0.0	0.0
$b = 0.300$	14.8	4.2	2.4	0.7	0.0	0.0	0.0	0.0	0.0
$b = 0.350$	20.9	6.0	2.7	0.7	0.0	0.0	0.0	0.0	0.0
$b = 0.400$	26.6	7.2	2.7	0.7	0.0	0.0	0.0	0.0	0.0
$b = 0.500$	35.0	7.5	2.7	0.7	0.0	0.0	0.0	0.0	0.0

10.3 Experiment 3: Slight optimism is consistent with the data

Table 18 5% Optimism: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=5%

T	10	11	12	13	14	15	16	20
μ	1.69	1.65	1.61	1.58	1.55	1.52	1.50	1.48
σ	0.75	0.71	0.68	0.66	0.64	0.63	0.62	0.62
b %	26.4	26.2	26.3	26.7	27.0	27.3	27.2	24.4
fit	0.032	0.030	0.029	0.028	0.028	0.028	0.028	0.029
median A %	55.1	54.9	54.8	54.7	54.7	54.7	54.5	51.5
default %	4.7	4.4	4.1	3.9	3.7	3.6	3.4	2.7
cons. %	4.4	4.4	4.3	4.3	4.2	4.2	4.1	4.0
neg Eq. %	12.6	13.4	14.5	15.9	17.1	17.7	17.8	16.2

Table 19 10% Optimism: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=10%

T	10	11	12	13	14	15	16	20
μ	1.92	1.89	1.83	1.79	1.76	1.73	1.70	1.61
σ	0.83	0.81	0.77	0.74	0.72	0.70	0.69	0.63
b %	26.6	26.2	27.0	27.2	27.3	27.3	27.3	27.4
fit	0.030	0.030	0.029	0.029	0.029	0.029	0.029	0.028
median A %	54.9	54.1	54.8	54.8	54.8	54.8	54.8	54.7
default %	4.4	4.0	3.8	3.6	3.4	3.3	3.1	2.7
cons. %	5.2	5.1	5.1	5.0	5.0	4.9	4.9	4.7
neg Eq. %	15.8	16.7	17.5	17.8	17.8	17.8	17.7	17.6

Appendix B: Proofs

Proof of Proposition 1. First, substitute $V_S(w) = w^{1-\rho}v_S$ and $V_B(w) = w^{1-\rho}v_B$ into the right-hand side of the objective of problem 1 and in constraint 2. Thus, we get

$$V_S(w) = \max_{c,A,\epsilon,\bar{v}} u(c) + \beta \left[\int_{\mathfrak{B}} ((1+r)(w - \epsilon A - c))^{1-\rho} v_B dF(x) + \int_{\mathfrak{B}^c} (A(x - \bar{v}) + (1+r)(w - \epsilon A - c))^{1-\rho} v_S dF(x) \right];$$

Subject to:

$$\int_{\mathfrak{B}} (1 - \delta)x dF(x) + \int_{\mathfrak{B}^c} \bar{v} dF(x) \geq (1 - \epsilon)1 + r_B \quad (21)$$

$$x \in \mathfrak{B} \iff v_B \left((1+r)(w - \epsilon A - c) \right)^{1-\rho} > v_S \left(A(x - \bar{v}) + (1+r)(w - \epsilon A - c) \right)^{1-\rho} \quad (22)$$

$$(1 - \epsilon)A \leq bw \quad (23)$$

$$c, A \geq 0, \quad 0 \leq \epsilon \leq 1. \quad (24)$$

Let $\lambda > 0$ and let current wealth be w . We must prove that $V_S(\lambda w) = \lambda^{1-\rho}w$.

Suppose that the entrepreneur chooses a level of consumption λc , increases the firm's assets to λA , and keeps ϵ fixed. Then

$$\lambda^{1-\rho}v_B \left((1+r)(w - \epsilon A - c) \right)^{1-\rho} = v_B \left((1+r)(\lambda w - \epsilon \lambda A - \lambda c) \right)^{1-\rho}, \text{ and}$$

$$\lambda^{1-\rho}v_S \left(A(x - \bar{v}) + (1+r)(w - \epsilon A - c) \right)^{1-\rho} = v_S \left(\lambda A(x - \bar{v}) + (1+r)(\lambda w - \epsilon \lambda A - \lambda c) \right)^{1-\rho}$$

This and constraint (22) imply that bankruptcy set \mathfrak{B} remains unchanged. Thus, constraints (21), (23), and (24) are also satisfied. Next, note that the right-hand side of the objective changes by the factor $\lambda^{1-\rho}$. Because $V_S(\lambda w)$ is the maximum utility of the entrepreneur given wealth λw , it follows that

$$V_S(\lambda w) \geq \lambda^{1-\rho}V_S(w), \quad (25)$$

for all $\lambda > 0$. Thus,

$$V_S(w) = V_S \left(\frac{1}{\lambda} \lambda w \right) \geq \frac{1}{\lambda^{1-\rho}} V_S(\lambda w),$$

which implies that (25) holds with equality. Substituting $w = 1$ and $\lambda = w$ in (25) immediately implies that $V_S(w) = w^{1-\rho}v_S$. The proof that $V_B(w) = w^{1-\rho}v_B$ is similar. ■

Lemma 1 *Constraint 1 of Problem 1 binds.*

Lemma 2 *Suppose that \mathfrak{B} is non-empty. Let*

$$x^* = \bar{v} - \left[1 - \left(\frac{v_B}{v_S} \right)^{\frac{1}{1-\rho}} \right] \frac{(1+r)(1-\epsilon A - c)}{A} \quad (26)$$

Then $\mathfrak{B} = \{x | \underline{x} \leq x < x^\}$. Conversely, if $x^* > \underline{x}$, then bankruptcy set $\mathfrak{B} = \{x | \underline{x} \leq x < x^*\}$ is non-empty.³⁵*

Proof of Lemma 1. Immediate: Suppose by way of contradiction that constraint (1) is slack. Then \bar{v} can be lowered thereby increasing $w'_s(x)$, which increases the objective of problem 1.³⁶ ■

Proof of Lemma 2. If the entrepreneur chooses to default, then the entrepreneur's utility is

$$u^B(x) = \left[\eta Ax + (1+r)(1-\epsilon A - c) \right]^{1-\rho} v_B. \quad (27)$$

Otherwise, if the entrepreneur does not default, then the utility is

$$u^S(x) = \left[A(x - \bar{v}) + (1+r)(1-\epsilon A - c) \right]^{1-\rho} v_S. \quad (28)$$

Note that $x \in \mathfrak{B}$ if $u^B(x) > u^S(x)$ and $x \notin \mathfrak{B}$ if $u^S(x) \geq u^B(x)$.

Suppose that $u^S(x) \geq u^B(x)$. We now show that $u^S(x') > u^B(x')$ for all $x' > x$. Note that

$$\frac{d(u^S(x) - u^B(x))}{dx} = \frac{(1-\rho)(1-\eta)Av_S}{\left[\eta Ax + (1+r)(1-\epsilon A - c) \right]^\rho v_B} > 0$$

Thus, $u^S(x) - u^B(x) \geq 0$ implies that $u^S(x') > u^B(x')$ for all $x' > x$. Similarly, $u^B(x) > u^S(x)$ implies $u^B(x') > u^S(x')$ for all $x' < x$. Let x^* solve $u^B(x^*) = u^S(x^*)$. Then the bankruptcy set is given by $\mathfrak{B} = \{x | \underline{x} \leq x < x^*\}$. (27) and (28) imply

$$\begin{aligned} & \left[\eta Ax^* + (1+r)(1-\epsilon A - c) \right] \left(\frac{v_B}{1-\rho} \right)^{1-\rho} \\ & = \left[A(x^* - \bar{v}) + (1+r)(1-\epsilon A - c) \right] \left(\frac{v_S}{1-\rho} \right)^{1-\rho}, \end{aligned}$$

which implies (26).

³⁵At realization x^* , the entrepreneur is indifferent between default and continuing to operate the firm. Thus, (2) must hold with equality. Solving (2) for x^* implies (26).

³⁶The direct effect is to increase the entrepreneur's payoff by decreasing required payments to the bank and the indirect effect is to lower the bankruptcy probability.

Now suppose that x^* is given by (26) and $x^* > \underline{x}$. Then by construction, $u^S(x^*) = u^B(x^*)$. Further, the monotonicity result established above implies $u^B(x) > u^S(x)$ for all $x < x^*$ and $u^S(x) \leq u^B(x)$ for all $x \geq x^*$. Thus, the bankruptcy set is given by $\mathfrak{B} = \{x | \underline{x} \leq x < x^*\}$. ■

Proof of Proposition 2. Let $\Gamma(v_S)$ be the maximum entrepreneur utility in Problem 3. We must prove that there exists v_S^* such that $\Gamma(v_S^*) = v_S^*$.

First let $\rho > 1$. Suppose that $v_S = 0$. Then $v_B < 0$. As a consequence, $\Gamma(0) < 0$. Now let \hat{v}_S be the entrepreneur's expected utility from autarky. i.e.,

$$\hat{v}_S = \max_{c_0, c_1, \dots} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Subject to:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} \leq w$$

$$c_0, c_1, \dots \geq 0,$$

Note that if $v_S = \hat{v}_S$ and we choose $A = 0$ in problem 3 then we get the autarky utility \hat{v}_S . Thus, optimization implies that $\Gamma(\hat{v}_S) \geq \hat{v}_S$. Since Γ is continuous, the intermediate value theorem implies that there exists a fixed point v_S^* .

For $\rho \leq 1$ we re-normalize $u_\rho(x) = (x^{1-\rho} - 1)/(1 - \rho)$. Then $\lim_{\rho \rightarrow 1} u_\rho(x) = \ln(x)$. Suppose that $v_S = 0$ and that $u(x) = \ln(x)$. We show that $\Gamma(v_S) < 0$.

Let $w_0 = 1 - \epsilon A$ be the amount of net-worth not invested in the firm. Because the continuation payoff from non-default is zero we get

$$\Gamma(0) = \max_{c_0, c_1, \dots, c_T} \sum_{t=0}^T \beta^t \ln(c_t) \tag{29}$$

Subject to:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} \leq w_0$$

Furthermore, it is sufficient to prove that the objective of (29) is negative for $w_0 = 1$, because the objective is increasing in w_0 .

The first order conditions immediately reveal that

$$c_t = (1+r)^t \beta^t c_0, \quad c_0 = \frac{1-\beta}{1-\beta^{T+1}}. \tag{30}$$

Substituting (30) into the objective of (29) yields

$$\sum_{t=0}^T \beta^t \ln((1+r)^t \beta^t) + \sum_{t=0}^T \beta^t \ln(c_0). \quad (31)$$

If $\beta(1+r) = 1$ then (31) is strictly less than 0. Thus, there exists $\bar{r}(\beta)$ with $(1+\bar{r}(\beta))\beta > 1$ such that $\Gamma(0) < 0$ for all $r \leq \bar{r}(\beta)$. By continuity there exists $\underline{\rho} < 1$ such that $\Gamma(0) < 0$ for $\rho \geq \underline{\rho}$. Finally, note again that $\Gamma(\hat{v}_S) \geq \hat{v}_S$ for the autarky level of utility \hat{v}_S . Thus, continuity of Γ again implies the existence of a fixed point v_S^* . ■

Appendix C: Match Criterion

We compare criterion (18) to the alternative square distance criterion in the relevant footnote.

Table 20 Supremum Norm: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=0.0%

T	10	11	12	13	14	15	16	20
μ	1.62	1.55	1.49	1.51	1.52	1.52	1.51	1.50
σ	0.90	0.83	0.75	0.74	0.76	0.76	0.76	0.78
b %	20.6	21.5	22.0	19.8	18.4	17.7	17.3	15.4
fit	0.046	0.042	0.037	0.034	0.034	0.034	0.035	0.036
median A %	46.9	48.1	49.2	47.0	45.3	44.3	43.8	41.3
default %	4.7	4.4	4.2	3.8	3.5	3.3	3.1	2.5
cons. %	3.7	3.6	3.6	3.6	3.6	3.6	3.6	3.5
neg Eq. %	10.2	10.6	10.8	10.5	10.8	11.1	11.6	11.1

Table 21 Square Norm: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=0.0%

T	10	11	12	13	14	15	16	20
μ	1.53	1.49	1.47	1.46	1.44	1.41	1.42	1.41
σ	0.74	0.70	0.70	0.69	0.67	0.65	0.67	0.69
b %	21.4	21.8	20.9	20.3	20.3	20.7	19.3	17.3
fit	0.020	0.019	0.019	0.019	0.019	0.019	0.019	0.020
median A %	50.2	50.8	49.6	49.0	48.9	49.3	47.4	44.7
default %	4.7	4.4	4.0	3.8	3.6	3.4	3.2	2.6
cons. %	3.7	3.7	3.6	3.6	3.6	3.5	3.5	3.5
neg Eq. %	9.8	10.2	10.1	10.3	10.9	11.7	11.8	11.5

References

- Akyol, A. and K. Athreya (2007). Unsecured credit and self-employment. unpublished.
- Beim, D. and C. Calomiris (2001). *Emerging Financial Markets*. New York, NY: McGraw-Hill Irwin.
- Boyd, J. and B. Smith (1994). How good are standard debt contracts? Stochastic versus non stochastic monitoring in a costly state verification environment. *Journal of Business* 67, 539–561.
- Cagetti, M. and M. DeNardi (2006). Entrepreneurship, frictions, and wealth. *Journal of Political Economy* 114, 835–870.
- Chatterjee, S., D. Corbae, M. Nakajima, and V. Rios-Rull (2003). A quantitative theory of unsecured consumer credit with risk of default. forthcoming *Econometrica*.
- Evans, D. S. and B. Jovanovic (1989). An estimated model of entrepreneurial choice under liquidity constraints. *Journal of Political Economy* 97, 808–827.
- Glennon, D. and P. Nigro (2005). Measuring the default risk of small business loans: A survival analysis approach. *Journal of Money Credit and Banking* 37, 923–947.
- Hoelzl, E. and A. Rustichini (2005). Overconfident: Do you put your money on it? *The Economic Journal* 115, 305–318.
- Kehoe, T. and D. Levine (1993). Debt-constrained asset markets. *Review of Economic Studies* 60, 865–888.
- Kihlstrom, R. and J. Laffont (1979). A general equilibrium entrepreneurial theory of firm formation based on risk aversion. *Journal of Political Economy* 87, 719–749.
- Kocherlakota, N. (1996). Implications of efficient risk sharing without commitment. *Review of Economic Studies* 63, 595–609.
- Krasa, S., T. Sharma, and A. Villamil (2007). Bankruptcy and firm finance. forthcoming *Economic Theory*.
- Krasa, S. and A. Villamil (2000). Optimal contracts when enforcement is a decision variable. *Econometrica* 68, 119–134.
- Krasa, S. and A. Villamil (2003). Optimal contracts when enforcement is a decision variable: A reply. *Econometrica* 71, 391–393.
- Krusell, P. and A. Smith (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy* 106, 867–896.

- Livshits, I., J. MacGee, and M. Tertilt (2007). Consumer bankruptcy: A fresh start. *American Economic Review* 97, 402–418.
- Lucas, R. E. (1978). On the size distribution of business firms. *Bell Journal of Economics* 9, 508–523.
- Mazzocco, M. (2006). Individual rather than household euler equations: Identification and estimation of individual preferences using household data. <http://www.ccpr.ucla.edu/asp/Mrsch.asp>.
- Moskowitz, T. J. and A. Vissing-Jorgensen (2002). The returns to entrepreneurial investment: A private equity premium puzzle? *American Economic Review* 92, 745–778.
- Prescott, E. C. (2006). The transformation of macro policy and research. *Journal of Political Economy* 114, 203–236.
- Puri, M. and D. Robinson (2007). Optimism and economic choice. forthcoming *Journal of Financial Economics*.
- Quadrini, V. (2000). Entrepreneurship, saving, and social mobility. *Review of Economic Dynamics* 3, 1–40.