# Contract Model Applications: Bankruptcy

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Contract Model Application to Bankruptcy

1. Consumer Bankruptcy: Livshits, MacGee and Tertilt, *AER*, March 2007
   (a) Dynamic model of consumer bankruptcy: insurance (health, job, divorce) vs. credit market distortion (inability to commit to future repayment)
   (b) Bankruptcy institutions:
      i. US: Fresh start
      ii. Germany: No fresh start (repay all debt in lifetime)
2. Firm Bankruptcy: Herranz, Krasa and Villamil
   (a) Dynamic model of firm bankruptcy: insurance (risky firm return distribution) vs. credit market distortion (inability to commit to future repayment)
   (b) This explains puzzling facts about entrepreneurs (small firms)

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# 1 Introduction

Institutions affect the ability of agents to raise finance. For example, Beim and Calomiris [1] note that financial markets depend on the legal system in four general ways. First, legal systems define property rights. Second, they specify the types of contracts that are permissible and procedures to enforce contracts. Third, they permit a firm to establish itself as a legal entity, with different firm types having different rights and responsibilities (e.g., corporations have limited liability, but the debt of an unincorporated firm is the owner's personal liability). Fourth, legal systems define and restrict the financial system by specifying laws to buy and sell securities, pledge collateral, and resolve firm insolvencies.

Despite recent progress in empirically documenting the links between institutions and finance, an integrated view of how institutions and enforcement affect finance has not yet been developed. Contract models are well suited for understanding this issue. There are at least two branches in the contract literature, but neither branch models the contract enforcement process:[1]

- *Optimal contract models* focus on deriving the form of the contract, but have not yet been used for quantitative policy analysis due to data problems (cf., Freixas and Rochet [3] and the references therein).

- *Dynamic contracting models* have been used for quantitative analysis, but the models usually focus on optimal social insurance when there are incentive or commitment problems (cf., Ljungqvist and Sargent [7], chapter 15, and the references therein).[2]

We will study computable contracting models of consumer and firm bankruptcy that can be used to analyze policy, e.g., the effect of particular legal reforms.

# 2 Consumer Bankruptcy

Consider the model of consumer bankruptcy by Livshits, MacGee and Tertilt [6].[3] They provide an interesting quantitative assessment of alternative consumer bank-

[1] Contract models typically assume either perfect ex post enforcement or self-enforcing contracts. For exceptions, see Krasa and Villamil [5] and Krasa, Sharma and Villamil [4].

[2] These models usually study a social planner who wishes to design an efficient contract to supply insurance, subject to incentive constraints.

[3] See also Chatterjee, Corbae, Nakajima and Rios-Rull [2] for a related model of unsecured consumer bankruptcy with an infinitely lived agent and an exogenous income process.

ruptcy rules in a life cycle model of insurance with incomplete markets calibrated to U.S. data. Households face idiosyncratic uncertainty about net asset holdings and labor income. The paper quantitatively analyzes two different bankruptcy institutions for unsecured consumer credit:

- Fresh Start: Debtors discharge debt via bankruptcy and all claims on future income are extinguished. When bankruptcy occurs under Chapter 7 of the U.S. Bankruptcy Code, the debtor gives up all non-exempt property owned at the time the bankruptcy petition is filed. If the court grants a discharge, the debtor is not liable for any other pre-bankruptcy debts and no claims can be made against future earnings. Chapter 7 simultaneously liquidates all non-exempt assets for the benefit of creditors and protects the insolvent debtor.[4]

- No Fresh Start: Consumer bankruptcy restructures consumer debt payments and limits the amount of wage income that can be seized. This is a counterfactual experiment to evaluate the effect solely of removing the option to discharge debt.[5]

Livshits, MacGee and Tertilt [6] argue that bankruptcy has two effects:

1. Credit Market Distortion: Bankruptcy weakens agents' ability to commit to repay debt in the future. This drives up interest rates, which limits their ability to borrow to smooth consumption across *time.*

2. Insurance: When markets are incomplete, bankruptcy increases the ability to smooth across *states* because it introduces contingencies into non-contingent debt contracts. The easier it is to discharge debt, the greater the insurance against bad shocks – divorce, job loss, medical problems.

Taking contracts and markets as given, Livshits, MacGee and Tertilt argue that any evaluation of bankruptcy must consider the quantitative tradeoff between the

[4] In the U.S. there are five types of bankruptcy, Chapters 7, 9, 11, 12 and 13. Chapter 7 is the most common type of bankruptcy for individuals; Chapter 13 is the second. Chapter 13 requires debtors to repay creditors under a court approved plan, and is used when a debtor is better off repaying but needs more time than creditors will allow. For example, if a debtor misses mortgage payments and faces foreclosure due to a temporary job loss, Chapter 13 allows the debtor up to 3 years to repay. Chapter 11 is designed for corporations seeking to reorganize debts while continuing to operate, Chapter 12 is the analog for family farms and Chapter 9 is for government bodies. Businesses can file Chapter 7 or 11, but not 13.

[5] Prior to 1999 consumers in Germany were liable for any debt until the end of life. Current law allows for discharge of debt after a 7-year court approved payment period.

negative effect of the credit market distortion over time and the positive effect from insuring against bad shocks across states. They consider a heterogeneous agents life cycle model with *unsecured* consumer credit where each period agents[6]

- make consumption-savings decisions
- decide whether to file for bankruptcy, given the bankruptcy rule (i.e., whether debt is discharged or rescheduled).

Agents have heterogeneous incomes due to idiosyncratic shocks to endowments (persistent productivity and transitory components) and expenses (uninsured medical bills, divorce costs, and unplanned children). Default is costly because income is seized when bankruptcy is filed and bankrupts are excluded from credit markets (i.e., cannot borrow or save in a default period). They find that a U.S. style "fresh start" bankruptcy system facilitates insurance across states while a "non-fresh start" European system makes life cycle smoothing easier.

## 2.1 The Model

The economy consists of overlapping generations of ex ante identical households that live for J periods. The households face idiosyncratic uncertainty, but there is no aggregate uncertainty. Markets are exogenously incomplete: The only asset is a one-period non-contingent bond.

**Household:** Utility in period $j$ is given by:[7]

$$\sum_{j=1}^{J} u(\frac{c_j}{n_j})$$

$c_j$ is consumption
$n_j$ is the size of a household of age $j$
$\beta$ is the discount factor

The income of household $i$ at age $j$ depends on productivity and an endowment:
$y^i_j = a^i_j \bar{e}_j$, with $a^i_j = z^i_j \eta^i_j$
$\bar{e}_j$ is the household's deterministic endowment of efficiency units of labor
$a_j$ is the household's *stochastic* productivity parameter at age $j$, which is the product of persistent shock $z^i_j$ and transitory shock $\eta^i_j$.

[6]Key model features are that consumers cannot commit to future repayment or fully insure.
[7]$u(\cdot)$ is strictly increasing and concave.

There are two sources of uncertainty:

1. *Income shock* $z_j$: The household productivity parameter follows a Markov chain with an age independent transition matrix $\Pi(z'|z)$. Initial productivity at age 0, $z_0$, is drawn from a stationary distribution. The transitory component $\eta$ has finite support and is iid over time.

2. *Expense shock* $\kappa$: Idiosyncratic expense shock $\kappa \geq 0$, $\kappa \in K$ where $K$ is a finite set of all possible expense shocks. The probability of each shock $\kappa_i$ is $\pi(\kappa)$. These shocks are iid and independent of the income shock.

**Financial Markets:** The borrowing and lending market is perfectly competitive. Financial intermediaries accept deposits from savers and make one-period, non-contingent loans with face value $d$. Debt obligation $d$ is the amount to be repaid next period, where $d > 0$ is borrowing and $d < 0$ is saving. Loans are non-contingent, but bankruptcy introduces a partial contingency.

Intermediaries observe the borrower's total level of borrowing $d$, current persistent productivity shock $z$, and age.
$q^b(d, z, j)$ is the price of the loan issued by the bank to the household
$\tau$ is the transaction cost of making a loan that is proportional to loan size

Each period, the intermediary maximizes expected profit. Because the market is competitive, intermediaries earn zero expected profit on loans. Thus, the expected value of repayments must equal the cost of the loan to the intermediary. Further, cross subsidization of interest rates across different borrowers will not occur and the interest paid to savers satisfies $q^s = \frac{1}{1+r}$.

**Bankruptcy:** A bankruptcy rule has three elements:

i. A law of motion for a bankrupt's debt.

ii. A repayment (garnishment) rule that specifies the amount of assets and earnings that can be seized by creditors.

iii. Limited access to financial markets.[8]

[8] This is meant to capture Chapter 7 bankruptcy. During bankruptcy, bankrupts are temporarily excluded from the credit market (cannot borrow or save) and a percentage $\gamma$ of income is seized (the total amount garnished is $\Gamma = \gamma y$, where $\gamma$ is the marginal rate of garnishment and $y$ is earnings.

Livshits, MacGee and Tertilt consider two bankruptcy systems:

1. **Fresh Start (FS):** Full discharge of all debts.

2. **No Fresh Start (NFS):** Remaining debt, after seizure of income, is rolled over at a specified interest rate $\bar{r}$ with bond price $\bar{q} = \frac{1}{1+\bar{r}}$. Debt is not discharged, but is rescheduled (payments are postponed and the interest rate may be reduced). All household assets can be seized and the wage garnishment rule is linear: $\Gamma = \gamma y$

   $\Gamma$ is the total amount of earnings transferred to creditors and $\gamma \in [0, 1]$ is the marginal rate of garnishment. The garnishment technology is costless.

Under both systems, the household faces the further punishment that bankrupts cannot save or borrow during the default period.

**Timing within a Period:** At the beginning of the period, each household observes shocks $z$ (productivity) and $\kappa$ (expense). If the household receives an expense shock, debt is increased by $\kappa$ (or savings decreased).

The household decides to:

- File for bankruptcy or not, $I = 0, 1$.
- How much to consume, $c \geq 0$.
- Next period's debt (savings), $d'$, taking bond prices as given.

Income ($y_j^i = \bar{e}_j z_j^i \eta_j^i$) is directly deposited into a "bank account." If the household is bankrupt, the amount garnished $\Gamma$ is deducted and the household keeps the rest. $I$ is the household's default decision. The problem will be stated for each state (solvency and default).

- Bankrupts cannot save in the period they declare bankruptcy and consume all assets net of garnishment $\Gamma$. New debt levels depend on the bankruptcy rule.
- Non-bankrupts choose current consumption and next period net assets.

**Household Problem. Fresh Start**

**The value of repaying debt** $d$ given shock realizations $(z, \eta, \kappa)$ is $(I = 0)$:

$$V_j(d, z, \eta, \kappa) = \max_{c,d'} u(\frac{c}{n_j}) + \beta E \max\{V_{j+1}(d', z', \eta', \kappa'), \bar{V}_{j+1}(z', \eta')\}$$

subject to: $c + d + \kappa = \bar{e}_j z\eta + q^b(d', z, j)d'$

**The value of bankruptcy** $\bar{V}$ is $(I = 1)$:

$$\bar{V}_j(z, \eta) = u(\frac{c}{n_j}) + \beta E \max\{V_{j+1}(0, z', \eta', \kappa'), \bar{W}_{j+1}(z', \eta', \kappa')\}$$

where $c = \bar{e}_j z\eta - \Gamma$, and $\Gamma = \gamma\bar{e}_j z\eta$, and $\bar{W}$ is the value of not repaying debt due to the expense shock in the period after bankruptcy:
$\bar{W}_j(z, \eta, \kappa) = u(\frac{c}{n_j}) + \beta E \max\{V_{j+1}(d', z', \eta', \kappa'), \bar{V}_{j+1}(z', \eta')\}$
where $c = \bar{e}_j z\eta(1 - \gamma)$ and $d' = (\kappa - \gamma\bar{e}_j z\eta)(1 + \bar{r})$

Point: Borrowers default only if the value of bankruptcy is strictly greater than the value of repayment.

**Household Problem. No Fresh Start**

$$V_j^{NFS}(d, z, \eta, \kappa) = \max_{c,d',I} u(\frac{c}{n_j}) + \beta E V_{j+1}^{NFS}(d', z', \eta', \kappa')$$

subject to:

$$c + d + \kappa = \bar{e}_j z\eta + q^b(d', z, j)d', \text{ if } I = 0$$

$$c = (1 - \gamma)\bar{e}_j z\eta - \Gamma, \text{ if } I = 1$$

$$d' = \max\{[d + \kappa - \gamma\bar{e}_j z\eta], 0\}(1 + \bar{r}), \text{ if } I = 1$$

**Intermediaries.** There is no aggregate risk, so the price of a bond is determined by the default probability and the risk free bond price. The household is of age $j$, with current productivity shock $z$, and total debt $d'$.

$\theta(d', z, j)$ is the probability that a household declares bankruptcy tomorrow.

$\bar{q}^b = \frac{1}{1+r^s+\tau}$ is the price of a bond with zero default probability.

The *FS zero profit condition* without garnishment and full discharge of debt is

$q^b(d', z, j) = (1 - \theta(d', z, j))\bar{q}^b$

The bond price under FS with wage garnishment is:

$$q^{FS}(d', z, j) = (1 - \theta(d', z, j))\bar{q}^b + \theta(d', z, j)E[\frac{\Gamma}{d' + \kappa'}|I = 1]\bar{q}^b$$

The *bond price under NFS* with wage garnishment is:

$$q^{NFS}(d', z, j) = (1-\theta(d', z, j))\bar{q}^b+\theta(d', z, j)E[\frac{\Gamma + q(d'', z', j + 1)d''}{d' + \kappa'}|I = 1]\bar{q}^b$$

where $d'' = \frac{\max\{d'+\kappa'-\Gamma,0\}}{\bar{q}}$ and $q(\cdot)d''$ is the value of rolled over debt.

**FS Equilibrium.** Given a bankruptcy rule $(\bar{r}, \gamma)$ and risk free bond prices $(q^s, \bar{q}^b)$, a recursive *competitive equilibrium* with FS is value functions $V$, $\bar{V}$, $\bar{W}$, policy functions $c$, $d'$, $I(d, z, \eta)$, a default probability $\theta(d', z, j)$, and a pricing function $q^b(\cdot)$ such that

1. The value functions satisfy the FS functional equations and $c$, $d'$, $I(\cdot)$ are the associated optimal policy functions.
2. Bond prices $q^b(\cdot)$ are determined by the FS zero profit condition.
3. The default probabilities are given by $q^{FS}(d', z, j) = (1 - \theta(d', z, j))\bar{q}^b + \theta(d', z, j)E[\frac{\Gamma}{d'+\kappa'}|I = 1]\bar{q}^b$

**NFS Equilibrium.** Given a bankruptcy rule $(\bar{r}, \gamma)$ and risk free bond prices $(q^s, \bar{q}^b)$, a recursive *competitive equilibrium* with NFS is a value function $V_j^{NFS}$, policy functions $c$, $d'$, $I(\cdot)$, a default probability $\theta(d', z, j)$, and a pricing function $q^b(\cdot)$ such that

1. The value function satisfies the NFS functional equation and $c$, $d'$, $I(\cdot)$ are the associated optimal policy functions.
2. Bond prices $q^b(\cdot)$ are determined by the NFS zero profit condition.
3. The default probabilities are given by $q^{NFS}(d', z, j) = (1 - \theta(\cdot))\bar{q}^b + \theta(\cdot)E[\frac{\Gamma+q(d'',z',j+1)d''}{d'+\kappa'}|I = 1]\bar{q}^b$

Under a debt contract the value of declaring bankruptcy is independent of the debt level, but the value of repaying is decreasing in debt. Thus, the FS bankruptcy decision follows a simple threshold rule. For every age and income realization there is a unique level of debt $\bar{d}(z, \eta)$ which solves $V_j(\bar{d}, z, \eta, 0) = \bar{V}_j(z, \eta)$. The threshold is: households repay debt $d$ iff $d \leq \bar{d}$.

## 2.2 Calibration

Livshits, MacGee and Tertilt [6] calibrate the model for the US.

**Preferences:** Households live for 18 periods and the length of a period is 3 years. Life begins at age 20. Agents work and receive income shocks in the first 15 periods. They are retired in the last 3 periods. The utility function is

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

The parameters are standard (e.g., $\sigma = 2$) where $1/\sigma$ is the intertemporal elasticity of substitution. The discount factor is 0.94. The savings interest rate is 4%, and the transaction cost is 4% (based on data from credit card companies).

**Income Process:** Livshits, MacGee and Tertilt use Gourinchas and Parker (*Econometrica* 2000) for the life cycle profile of labor income.

$$\ln y_j^i = \ln z_j^i + \ln \eta_j^i + \ln g(X_j^i)$$

Income has a deterministic component, $g(X)$, a transitory earnings shock ,$\eta$, and a persistent productivity shock $z$. In this literature, the log of persistent idiosyncratic shocks follows an AR(1) process:

$$\ln z_j^i = \rho \ln z_{t-1}^i + \epsilon_j^i$$

Productivity is modled as a 5 state Markov process $\{z_1, z_2, z_3, z_4, z_5\}$ with transition matrix $\Pi(z'|z)$. The transition probability is the probability of moving between earning quintiles over a 5 year period. There are no income or expense shocks during retirement.

**Expense Uncertainty:** Expense shocks take on three values, $\kappa \in \{0, \kappa_1, \kappa_2\}$ and there are two probabilities. The shocks are medical expense, divorce, and the cost of an unwanted child. See the paper for calibration.

**Bankruptcy Rules:** The parameters are $\gamma$, $\bar{r}$. Garnishment parameter $\gamma$ is calibrated so that $\frac{d}{y}$ equals the average ratio of unsecured debt to personal disposable income over 1995-1999, 8.4%. The annual rollover interest rate $\bar{r}$ is 20%.

## 2.3 Results

The Livshits, MacGee and Tertilt [6] model does a good job of matching consumer default rates (less than 1%), life cycle consumption, hump shaped borrowing, and different interest rates for each loan amount (u-shaped).

1. They compare the FS system to the counter factual case where the option to discharge debt in bankruptcy is taken away (debt is rescheduled). Welfare is measured as the % increase in lifetime consumption required to equalize expected lifetime utility in FS and NFS. This is the equivalent consumption variation. In the benchmark model, FS is marginally better than NFS. This means that the insurance effect (smoothing across states) dominates the distortion in intertemporal credit markets (smoothing across time).[9] Defaults, and hence borrowing constraints, are worse under FS than NFS.

2. The welfare implications of different bankruptcy rules are sensitive to the type and size of a consumer's uncertainty and to life-cycle considerations.

**Expense Uncertainty.** Livshits, MacGee and Tertilt find:
When there is no expense uncertainty, NFS is better than FS.
When expense shocks are sufficiently high, FS is better than NFS.

Point: When there is expense uncertainty, the debt forgiveness inherent in FS may be welfare improving because it provides valuable insurance (a safety net against bad luck).

**Earnings Uncertainty.** There are two types of shocks:

Transitory shocks to earnings have little effect on life-time wealth and can be smoothed over time. As the variance of the temporary shock increases, NFS becomes better than FS.

Persistent shocks are more complex and depends on the degree of persistence in the income process. Increases in the variance of the persistent shock:

- Make FS less attractive to low income households (want to borrow)
- Make FS more attractive to high income household (precautionary savers so the tighter borrowing constraint under FS is less important)

The magnitude of the effect and cut-off group varies with the level of persistence.

**Life Cycle.**

If the life cycle earnings profile were flat, there would be no need to smooth over time and FS dominates NFS.

If family size did not vary over the life-cycle (hump shaped), NFS dominates FS because the young wish to borrow more against future income.

[9] The problem is that there is no credible way for an agent to commit to not default.

**Conclusions.**

1. Welfare comparisons of different bankruptcy laws vary with differences in the extent and nature of the uncertainty that households face over the life cycle (the stochastic processes for income and expenses). Expense shocks tend to make FS more valuable because it provides insurance.

2. They recommend future work to determine:

- What is the effect of bankruptcy on households' incentives to insure against or take actions to mitigate shocks?
- What is the effect of durable assets (e.g., houses) on the market for unsecured credit? Secured assets affect a household's ability to smooth consumption in response to transitory shocks to income and wealth.

The Livshits, MacGee and Tertilt model with unsecured lending and default is solely for personal bankruptcy. Clearly, it would be useful to have an analog for firms in order to understand the implications of firm bankruptcy for firm size, capital structure, and owner asset diversification. Herranz, Krasa and Villamil [8] propose such a model of small firm finance with secured lending.

# 3 Firm Bankruptcy

**Question:** How important are differences in owner personal characteristics versus the institutional environment in which a firm operates for firm performance?

This question is important because:

- Bankruptcy institutions, access to credit and a firm's capital structure can be affected by policy.
- Innate personal characteristics such as risk tolerance or optimism cannot.

Herranz, Krasa and Villamil [8] derive facts and theory to assess the impact of owner personal characteristics and institutions on small firms.

1. They analyze data from the Survey of Small Business Finance (SSBF) and find three puzzles. Entrepreneurs
   (i) face a seemingly unattractive risk/return trade-off,
   (ii) have poorly diversified personal investments, and
   (iii) have negative equity (16-21% of firms).

Risk aversion is central to the debate on entrepreneurship, but individual coefficients and the distribution of risk aversion are not directly observable in the SSBF. The data also document substantial heterogeneity in entrepreneur behavior, hence we cannot use a representative agent model.

2. They construct a dynamic, computable model with three features:
   (i) Forward looking entrepreneurs weigh current gains/losses against expected future returns.
   (ii) The distribution of firm returns from SSBF data is peaked in the middle, with "fat" asymmetric tails.
   (iii) Bankruptcy protection insures entrepreneurs against poor returns, but permits upside gain.

**Entrepreneurship.** The puzzles in the data are relevant for evaluating theories about why people become entrepreneurs. Standard arguments are that entrepreneurs are very:

1. Willing to bear risk: Kihlstrom and Laffont JPE 1979
2. Optimistic: Puri and Robinson JFE 2007, Hoelzl and Rustichini EJ 2005
3. Credit constrained: Evans and Jovanovic JPE 1989, Hopenhayn and Vereshchagina 2005
4. Irrational? Seemingly modest returns for risk

**Goals:**
*Examine data on small firms
*Construct a dynamic, computable model to organize the data.

**Preview of SSBF Facts:** We document 4 facts from the SSBF

**Fact 1.** Small firm returns are very risky.

**Fact 2.** Owners invest substantial personal net worth in their firms.

**Fact 3.** Most owners work at their firms.

**Fact 4.** Owners use personal funds to cover firm losses.

**SSBF Data Set:** FRB & SBA 1987, 1993, 1998, 2003
* CS 4000 small firms: non-farm, finance, real est
* Represents about 5 million firms
* Information: firm & owner (owner age, industry, business type), income statement & balance sheet, source/use of financial funds, borrowing experience (trade credit, equity injection, etc.)

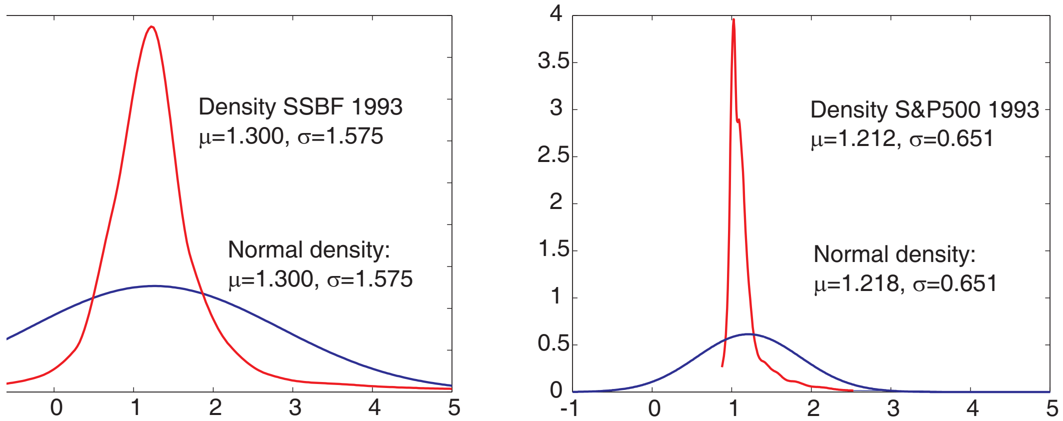


Figure 1: pdfs of ROA for firms in SSBF 1993 and S&P500 (Compustat) vs normal pdfs

**Fact 1.** Small firm returns are very risky (real, after tax).

Table 1: Real Firm Return Summary Statistics, 1993 SSBF and S&P500

| **moment** | median | mean | standard dev. | skewness | kurtosis |
|---|---|---|---|---|---|
| **1993 SSBF** | 1.094 | 1.30 | 1.57 | 13.2 | 290 |
| 95% conf. | [1.08, 1.11] | [1.22, 1.38] | [0.95, 2.13] | [2.3, 17.3] | [29, 488] |
| **1993 S&P500** | 1.093 | 1.21 | 0.65 | 13.1 | 221 |
| 95% conf. | [1.07, 1.10] | [1.16, 1.28] | [0.28, 1.02] | [3.1, 14.6] | [20, 277] |

* Median return: 9.4%
* Very risky: big standard deviation
* Skewed right with big upper tail: high kurtosis

**Real Firm Losses and Gains**: 1993 SSBF

| **loss %** | $\geq 20$ | $\geq 40$ | $\geq 60$ | $\geq 80$ | $\geq 100$ |
|---|---|---|---|---|---|
| % firm | 12 | 7.4 | 5.1 | 4.2 | 3.8 |
| **gain** | $\geq 50$ | $\geq 100$ | $\geq 200$ | $\geq 300$ | $\geq 1000$ |
| % firm | 20.7 | 10.4 | 3.8 | 2 | 0.3 |

For funds invested (D+E), this means possibly
* Big losses: 12% of firms lost more than 20%
* Bigger gains: 21% of firms return more than 50%

**Point:** The return distribution is not normal; this may be good.

**Fact 2.** Owners invest substantial personal net worth in their business.[10]

**Net Worth Invested:** 1998 SSBF

| % invest | $\geq 20$ | $\geq 40$ | $\geq 60$ | $\geq 80$ | mean | median |
|---|---|---|---|---|---|---|
| % Ent. | 52 | 25 | 11 | 3 | 27 | 21 |

Poorly diversified: 25% invest $\geq$ 40% net-worth

This is puzzling because:
* Returns are very risky
* If firm fails: lose funds invested & job

**Fact 3.** Most owners work at their firms.

SSBF, inc. firms: % of owners who work at firm
* 79% in 1993
* 89% in 1998

[10] Moskovitz and Vissing-Jorgensen find a risk/return puzzle for non-traded equity investors in the SCF for large firms. SCF data only allow them to get the mean of the return distribution.

**Fact 4.** Owners use personal funds to cover business losses.

SSBF, inc. firms: % of firms with negative equity
* 16 % in 1993
* 21 % in 1998

**Negative equity:** Firms use non-business assets to cover business losses
* use personal funds to "bail out" firm
* unpaid bills absorbed by creditors

This is puzzling because:
* We consider only incorporated firms
* These firms enjoy limited liability

Model must be consistent with the data and other facts:

**Fact 5.** The average default rate on SBA loans is 3.5% (Glennon and Nigro JMCB 2005).

**Fact 6.** The % change in consumption in response to a % change in wealth is about 4%: Abel and Bernanke (2001)

**Fact 7.** Median risk aversion is $\rho = 1.7$ for men (Mazzocco 2006) and generally $0 < \rho < 3$.

**Question:** Is it rational to be an entrepreneur?
* Median firm returns are modest
* The return distribution is very risky
* Ventures lose firm assets, owner funds & job

**Answer:** Yes, this is optimal for an owner if

1. Dynamic model: Tradeoff current return v. expected future returns

2. SSBF ROA distribution

3. Bankruptcy protection: Insures against loss, but why are default rates low & even firm bailouts occur?
* Shutdown precludes (high) future returns
* This helps explain negative equity

**Model.** $t = 0, 1, \ldots$ periods; common discount at rate $\beta$

* many entrepreneurs: heterogeneous CRRA $u^i(\cdot)$
* competitive RN investor: elastic S of funds

**Preferences:** Heterogeneous CRRA

$$u^i(c) = \frac{c^{1-\rho^i}}{1-\rho^i}$$

Risk aversion coefficient $\rho \sim N(\mu, \sigma^2)$, pdf $g_{\mu,\sigma}(\rho)$

**Endowment:** Illiquid wealth $w_t$ (ex ante identical)

**Technology:** CRS, random output, pdf $f(x)$, $[-A, X]$, $x \in [\underline{x}, \bar{x}]$, $\underline{x} \leq 0$, $\bar{x} > 0$

**Information:** At start of $t$, observe current $w_t$, $x_t$.
History: Know prior defaults, but not past returns.

**Funds:** Firms raise $A = D + E$; $\epsilon\%$ personal funds

1. Self-finance: use riskless personal funds $\epsilon A$
2. Debt: Secured by risky firm assets, $D = (1 - \epsilon)A$

**Borrowing Constraint:** debt limited to $(1 - \epsilon)A \leq bw$

**Bankruptcy:** Limited liability & if default
* exclude for $T$ periods
* deadweight default loss $\delta$ (if $Ax > 0$)

We solve problem of solvent RA: $c(\rho)$, $A(\rho)$, $\epsilon(\rho)$, $\bar{v}(\rho)$

**Problem 1** *If the firm is solvent the agent solves*

$$V_S(w) = \max_{c,A,\epsilon,\bar{v}} u(c) + \beta\Big[\int_{\mathcal{B}} V_{B,1}((1+r)(w-\epsilon A - c))\, dF(x)$$
$$+ \int_{\mathcal{B}^c} V_S(A(x-\bar{v}) + (1+r)(w-\epsilon A - c))\, dF(x)\Big]$$

*Subject to:*

$$\int_{\mathcal{B}\cap\mathbb{R}_-} x\, dF(x) + \int_{\mathcal{B}\cap\mathbb{R}_+} (1-\delta)x\, dF(x) + \int_{\mathcal{B}^c} \bar{v}\, dF(x) \geq (1-\epsilon)(1+r_B) \quad (1)$$

$$x \in \mathcal{B} \text{ iff } V_{B,1}\Big((1+r)\big(w-\epsilon A - c\big)\Big) > V_S\Big(A(x-\bar{v}) + (1+r)\big(w-\epsilon A - c\big)\Big) \quad (2)$$

$$(1-\epsilon)A \leq bw \quad (3)$$

$$c, A \geq 0, \;\; 0 \leq \epsilon \leq 1. \quad (4)$$

*Objective:* Entrepreneur expected payoff = $u(c) + \beta E[V(c')]$

*Constraints:*
1. Bank IR (default loss $\delta$ relevant iff $Ax > 0$):
expected return on loan $\geq$ opp. cost of bank funds[11]

2. Default decision is optimal: Default iff expected value of default > solvency[12]

3. Firm borrowing constraint, $b$ is %

4. Non-negativity constraints and $\epsilon$ is % the expected discounted value of future consumption in bankruptcy and solvency.

[11](1) ensures the bank is willing to supply funds: the expected return on the $1-\epsilon\%$ of funds invested in the firm is at least $1+r_B$. The DWL arises only if $x > 0$ and the firm has not lost more during the period than the value of its assets.

[12]Bailing out the firm with personal funds means the owner runs the firm even if $x < \bar{v}$. In a one period model $V_{B,1}$ and $V_S$ would be the identity mapping, and (2) reduces to $x \in \mathcal{B}$ iff $(1+r)(w-\epsilon A-c) > A(x-\bar{v}) + (1+r)(w-\epsilon A-c)$, which implies $x \in \mathcal{B}$ if and only if $x < \bar{v}$.

We solve the problem for $c(\rho)$, $w'(\rho)$ if the firm defaulted $k \leq T$ periods ago.

$V_{B,T}(\cdot) = V_S(\cdot)$, i.e., after $T$ periods the firm can operate again.

Let $w'$ denote wealth in the next period. Then

**Problem 2** *If the firm is bankrupt the agent solves*

$$V_{B,k}(w) = \max_{c,w'} u_E(c) + \beta V_{B,k+1}(w')$$

*Subject to:*

$$c(1+r) + w' \leq w(1+r); \tag{5}$$

$$c, w' \geq 0. \tag{6}$$

*Objective:* Entrepreneur expected payoff = $u(c) + \beta E[V(c')]$

*Constraints:* When default occurred, the firm cannot operate for $T$ periods.

(5) is the standard budget constraint.

(6) ensures that consumption and net-worth are non-negative.

**Technical Results.** To get a single, recursive problem we use:

Prop. 1 to simplify the objective (the value function is scalable: $u(wx) = w^{1-\rho}u(x)$.)

*Lemma 1 & 2.* simplify the constraints

**Prop. 1** Assume CRRA, $v_S = V_S(1)$, $v_{B,k} = V_{B,k}(1)$. Then $V_S(w) = w^{1-\rho}v_S$ and $V_{B,k}(w) = w^{1-\rho}v_{B,k}$.

**Lemma 1.** Constraint 1 of Problem 1 binds.

**Lemma 2.** Let bankruptcy set $\mathcal{B}$ be non-empty and

$$x^* = \bar{v} - \left[1 - \left(\frac{v_B}{v_S}\right)^{\frac{1}{1-\rho}}\right] \frac{(1+r)(1-\epsilon A - c)}{A} \tag{7}$$

Then $\mathcal{B} = \{x | \underline{x} \leq x < x^*\}$. Conversely, if $x^* > \underline{x}$, $\mathcal{B} = \{x | \underline{x} \leq x < x^*\}$ is non-empty.

At $x^*$ the firm is indifferent to default or not; thus constraint 2 holds with equality.

**RA Problem 3.** Choose $c(\rho)$, $A(\rho)$, $\epsilon(\rho)$, $\bar{v}(\rho)$ to

$$\max u(c) + \beta v_B \int_{\underline{x}}^{x^*} \Big[(1+r)\Big(1-\epsilon A - c\Big)\Big]^{1-\rho} \, dF(x)$$
$$+\beta v_S \int_{x^*}^{\bar{x}} \Big[A(x-\bar{v}) + (1+r)\Big(1-\epsilon A - c\Big)\Big]^{1-\rho} \, dF(x)\Big]$$

Subject to:

$$\int_{\underline{x}}^{0} x\, dF(x) + \int_{0}^{x^*} (1-\delta) x\, dF(x) + \int_{x^*}^{\bar{x}} \bar{v}\, dF(x) = (1-\epsilon)(1+r_B)$$
$$x^* = \max\left\{\bar{v} - \left[1 - \left(\frac{v_B}{v_S}\right)^{\frac{1}{1-\rho}}\right] \frac{(1+r)(1-\epsilon A - c)}{A}, \underline{x}\right\}$$
$$c + \epsilon A \leq 1$$
$$(1-\epsilon)A \leq b$$
$$c \geq 0,\ A \geq 0,\ 0 \leq \epsilon \leq 1$$

*Objective:* The entrepreneur's expected payoff is $U(c) + \beta E[v(c')]$, where $v_B(v_S)$ and $v_S$ is a number.

*Constraints:*

1. Bank IR: expected return on loan = risk-free return on funds (default loss $\delta$ is relevant iff $Ax > 0$)

2. The firm's default decision is optimal.

3. This is the owner's feasibility constraint, where we normalize $w = 1$.

4. This is the firm borrowing constraint, where we normalize $w = 1$.

5. These are non-negativity constraints and $\epsilon$ is a %.

Prop. 1 $\rightarrow$ write $v_B$ in terms of $v_S$, since $u(\cdot)$ is scalable in wealth. Then
 * Replace value *functions*, $V_{B,1}$ and $V_S$,
 * with *number* $v_S$.

**Result:** Problem 3 is a 1-dimensional fixed point problem in $v_S$.

Prop. 2 establishes existence of a solution.

**Prop. 2.** There exist $\underline{\rho} < 1$ and $\bar{r} > \frac{1}{\beta} - 1$ such that Prob. 3 has a solution $\forall\ \rho \geq \underline{\rho}$ and $\forall\ r \leq \bar{r}$.

## 3.1 Heterogeneous Entrepreneurs & Model cdf Predictions

**SSBF data:** The data make predictions for *distributions*, not a representative agent. Thus, the model must predict cdfs, not just moments:

1. *Net-worth invested*: $W^m_{\mu,\sigma}(w) = \frac{\int_{-\infty}^{\underline{\rho}} \int_{\bar{v}(\rho)}^{x(w,\underline{\rho})} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho + \int_{\underline{\rho}}^{\infty} \int_{\bar{v}(\rho)}^{x(w,\rho)} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho}{\int_{\bar{v}(\rho)}^{\infty} f(x)\, dx}$

2. *Equity/Assets*: $E^m_{\mu,\sigma}(e) = \frac{\int_{-\infty}^{\underline{\rho}} \int_{\bar{v}(\rho)}^{x(e,\underline{\rho})} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho + \int_{\underline{\rho}}^{\infty} \int_{\bar{v}(\rho)}^{x(e,\rho)} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho}{\int_{\bar{v}(\rho)}^{\infty} f(x)\, dx}$

3. *Asset value*: $A^m_{\mu,\sigma}(a) = \int_{-\infty}^{\underline{\rho}} \int_{\underline{x}}^{x(a,\underline{\rho})} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho + \int_{\underline{\rho}}^{\infty} \int_{\underline{x}}^{x(a,\rho)} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho$

**Strategy:** We need 2 pdfs to construct these cdfs
* $g_{\mu,\sigma}(\rho)$: accounts for heterogeneity in $\rho$ (RA)
* $f(x)$: accounts for uncertainty about firm returns

Note:
$\int_{\bar{v}(\rho)}^{\infty} f(x)\, dx$: Prob. firm equity is positive
$\underline{\rho}$: lowest $\rho$ at which a model solution exists

**Net Worth Invested.** After $x$ is realized, firm assets are $A(\rho)x$ & debt is $A(\rho)\bar{v}$. Then:
* Owner equity: $A(\rho)(x - \bar{v}(\rho)) > 0$, if $x \geq \bar{v}(\rho)$
* Owner net worth: $(1 + r)(1 - c_S(\rho) - \epsilon(\rho)A(\rho))$

The % total net worth an owner invests is

$$w = \frac{A(\rho)(x-\bar{v}(\rho))}{A(\rho)(x-\bar{v}(\rho))+(1+r)(1-c_S(\rho)-\epsilon(\rho)A(\rho))}$$

*Model cdf:* Solve $w$ for $x(w, \rho)$, integrate over $f(x)$ and $g_{\mu,\sigma}(\rho)$, to get $W^m_{\mu,\sigma}(w)$.

**Equity/Assets.** The % equity in a firm is

$$e = \frac{A(\rho)(x-\bar{v}(\rho))}{A(\rho)x}$$

*Model cdf:* Solve $e$ for $x(e, \rho)$, integrate to get $E^m_{\mu,\sigma}(e)$.

**End of Period Assets** The current realization is

$$a = \frac{A(\rho)x}{(1+r)(1-c_S(\rho)-\epsilon(\rho)A(\rho))}$$

*Model cdf:* Solve $a$ for $x(a, \rho)$, integrate to get $A^m_{\mu,\sigma}(a)$

## 3.2 Empirical Analysis

**Construct $f(x)$:** Distribution of real firm returns
(1993 SSBF inc. firms, after tax, adjust $\pi = 3\%$). Firm return on assets:

$$x = \frac{\text{Profit after taxes+Interest Paid}}{\text{Assets}} + 1$$

**Construct $g_{\mu,\sigma}(\rho)$:** Distribution of Risk Aversion
Assume: $\rho \sim N(\mu, \sigma^2)$
Estimate $\mu$,$\sigma$,$b$ to min the supnorm distance * net-worth invested in firm cdf implied by model
* net-worth invested in firm cdf from SSBF data
Use these $\mu$,$\sigma$ to construct pdf $g_{\mu,\sigma}(\rho)$

The equation is

$$\min_{\mu,\sigma,b\geq 0} ||W^m_{\mu,\sigma}(w) - W^e(w)||_\infty + (0.431 - a_{\mu,\sigma})^+ + (a_{\mu,\sigma} - 0.519)^+$$

where $||.||_\infty$ is the supremum norm, taken over all non-negative % of net-worth. The second and third terms impose penalties only for asset values outside the 95% confidence interval.

Note: If $\sigma = 0 \rightarrow$ representative agent model.[13]

**Table 2: Exogenous Parameters**

| Parameter | Interpretation | Value | Comment/ Observations |
|---|---|---|---|
| $\beta$ | discount factor | 0.97 | determined from $r$ and $r_B$ |
| $T$ | default exclusion period | 11 | U.S. credit record |
| $\delta$ | default deadweight loss | 0.10 | Boyd-Smith (1994) |
| $r_B$ | bank opportunity cost | 1.2% | real rate, 6 mo T-Bill, 1992-2006 |
| $r$ | entrepreneur opportunity cost | 4.5% | real rate, 30 year mortgage, 1992-2006 |

These parameters are taken from US data.

[13] We compute $\sigma = 0$ to show heterogeneity matters.

# 4 Results

## 4.1 Endogenous Parameters:

The Table indicates

$b$: firms can borrow only 21.5% of net-worth

$\mu$: risk aversion in the RBC range & close to Mazzocco (2006) for men (1.7)

Table 3: Endogenous Parameters

| Parameter | Interpretation | Est. Value |
|---|---|---|
| $b\%$ | borrowing constraint: loan $\leq bw$ | 21.5 |
| $\mu$ | median of distribution of risk aversion | 1.55 |
| $\sigma$ | standard deviation of distribution of risk aversion | 0.83 |

Given $\beta = 0.97$, $T = 11$, $\delta = 10\%$, $r_B = 1.2\%$ $r = 4.5\%$, choose $\mu$, $\sigma$, $b$ to

$$\min_{\mu,\sigma,b\geq 0} ||W^m_{\mu,\sigma}(\varpi) - W^e(\varpi)||_\infty + (0.431 - a_{\mu,\sigma})^+ + (a_{\mu,\sigma} - 0.519)^+$$

**Point:** The estimated $\mu$, $\sigma$ allow us to:

*construct *model cdfs* to

*compare to *empirical cdfs* from SSBF data

## 4.2 Match Model Prediction to SSBF data: cdfs

* Net-worth invested in firm (%): $W^m_{\sigma,\mu}(\varpi)$ vs $W^e(\varpi)$

* $\frac{\text{Assets}}{\text{\% net-worth outside firm}}$: $E^m_{\sigma,\mu}(e)$ vs $E^e(e)$

* $\frac{\text{Equity}}{\text{Assets}}$: $A^m_{\sigma,\mu}(a)$ vs $A^e(a)$

Figure 2 compares

*the cdfs predicted by the model with

*the cdfs constructed from the relevant SSBF data

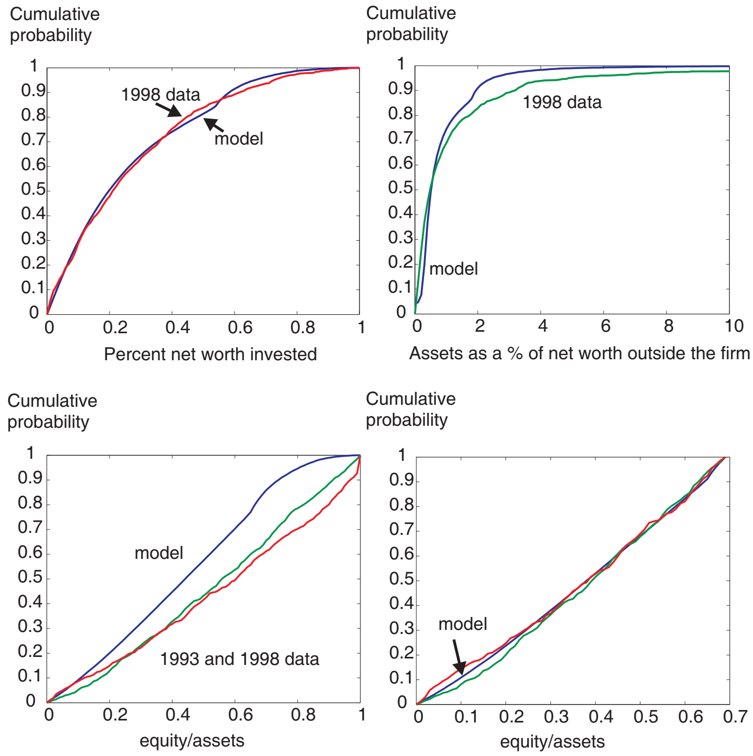


Figure 2: Model Predictions and SSBF Data: cdfs

**Panel 1.** Match model-predicted & SSBF cdfs of owner % of net-worth in firm: This is good, especially since we use only 3 parameters.

**Panel 2.** Match model predicted & SSBF cdfs of firm assets: The match is good, except the model underpredicts a few large firms.[14]

**Panels 3 & 4.** Match model-predicted & SSBF cdfs for firm capital structure: The model over predicts equity/assets, but only in the upper tail.[15]

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[14]This occurs because model solutions do not exist below $\underline{\rho} = 0.74$ so we assign point mass. Median net-worth invested for risk aversion level $\bar{\rho}$ is $A\bar{x}/(1 - \epsilon A - c_S)(1 + r) = 1.786$, which is the range where the model predicted curve moves away from the data.

[15]Again, this occurs because no model solutions exist below $\underline{\rho}$ and the min criterion assigns point mass to these values. $\underline{\rho} = 0.74$ with an associated $\bar{v} = 0.3\overline{35}$. At median return $x = 1.09$, this gives $(x - \bar{v})/x = 0.\overline{7}$, which is where the kink occurs. Below 0.7, the model replicates the approximately uniform distribution of equity/assets among firms. Th right panel shows this, conditioned on equity/asset less than 0.7.

## 4.3 Match Model Prediction to SSBF Data: Point Estimates

Table 4: Model Point Estimates

| **Parameter** | **Interpretation** | **Value** |
|---|---|---|
| median $A\%$ | median firm assets (size) | 48.1 |
| default % | firm default rate | 4.4 |
| cons. % | consumption as a percent of net worth | 3.6 |
| neg. Eq. % | negative equity in the firm | 10.6 |

**Median Assets:** The model predicted $A$ of 47.8% is well within 95% confidence interval [43.1, 51.9]. Thus, the penalty term in the supnorm min criterion is not relevant in a neighborhood of the optimal parameters.

**Default:** The model predicted default of 4.4% is close to 3.5% for SBA loans reported by Glennon and Nigro, JMCB 2005.

**Consumption:** The model predicted consumption as a % of net-worth of 3.6% is in the standard range of 3-5% predicted for the U.S.

**Negative Equity:** The model predicts that 10.6% of firms have negative equity, below the empirical values of 15.7% and 21.0% in 1993 and 1998.[16] We will show that the levels in the SSBF data are compatible with our model if entrepreneurs are slightly optimistic.

*"Luck": correlation between $\rho$ and net-worth invested is 62%. This means that risk-taking explains 62% of the amount of net-worth invested and luck (the realization of $x$) explains the rest.

[16]Given the high negative equity, the low default rate may seem surprising. In a dynamic model an entrepreneur may not default. Rather, it may be optimal to operate a firm that performs poorly today, if the firm's expected discounted continuation value is sufficiently high.

# 5 Computational Experiments

We now conduct a series of experiments to better understand the role of

* bankruptcy institutions
* risk aversion
* liquidity constraints
* entrepreneur optimism
* SSBF return distribution

## 5.1 Bankruptcy Institution: $T$, $\delta$

**Summary of Results for Exclusion Period $T$:** Longer exclusion raises the bankruptcy penalty.

**Table 8.** Benchmark: $\mu$, $\sigma$ are very stable as $T \to 20$.
$b$ decreases because the penalty increases with $T$.
$A$ drops because owners are more cautious.
Default decreases.

**Table 9.** Comparative Statics wrt to $T$: Fix $\mu$, $\sigma$, $b$ and evaluate $T = 6, ..., 20$.
Default and $A$ decrease.

**Table 10.** Welfare (EV): $T$ has a significant impact on welfare. Given $T = 11$,
Lowering $T$ to 10 is equivalent to a 1.1% increase in net-worth to a person with the median $\rho = 1.55$. For $\rho = 0.9$, welfare increases 6.3%.
Decreasing $T$ to 6 is equivalent to a 7.7% increase in net-worth for $\rho = 1.55$ and 36.9% for $\rho = 0.9$.
Increasing $T$ to 20 gives a 5.7% loss of net-worth for $\rho = 1.55$, and 20.4% when $\rho = 0.9$.

**Table 11.** Interest Rate: Decreases as $T$ increases, yet the insurance effect dominates since welfare increases for lower $T$. This is the key tradeoff.

**Summary of Results for DWL:** $\delta$ is not important in the range $\delta \in [.1, .3]$

**Table 12.** Higher Cost: No effect.

**Table 13.** Comparative Statics: Fix $\mu, \sigma, b$ and vary $\delta \in [0, 1]$:
(a) bankruptcy occurs with only a small probability, and
(b) assets $A$ in bankruptcy states are small.
Thus, DWL $\delta A$ is small and expected costs, $(a) \times (b)$, is second order.

**Table 14.** Welfare: Negligible for $\delta \in [.1, .3]$. DWL can be big if $\delta$ is big, KSV.

## 5.2 Risk Aversion and Liquidity Constraints

**Table 15.** $\mu$ Comparative Statics: higher $\mu$ implies lower $A$, with more debt, $-E$ and default.

**Table 16.** $b$ Comparative Statics: Higher $b$ means firms can borrow more, hence $A$ is bigger, as is debt, $-E$ and default.

**Table 17.** $b$ Welfare: This is much higher when $\rho$ is low ($b$ binds).

**Remark:** Raising $T$ and $b$ increases the riskiness of loans. in practice, bank regulations limit this.

## 5.3 Entrepreneur Optimism

**Table 18.** Slight optimism increases $-E$ to the SSBF range without affecting the other parameters (much). Optimistic agents run larger firms because they expect higher future returns. They increase debt $\bar{v}$. When $\bar{v}$ is higher, $x < \bar{v}$ is higher and this increases $-E$.

We assume that firm believes its returns are $X + 10\%$.

The bank uses the correct distribution.

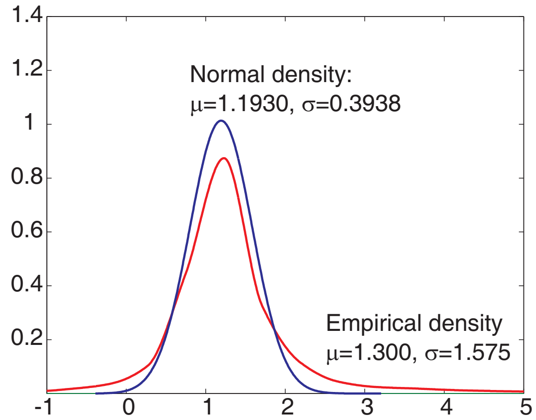


Figure 3: Empirical firm return pdf versus best-fit normal pdf, SSBF 1993

## 5.4 Counterfactual Exercise: Normally Distribution ROA $f(x)$

To show the risky return pdf matters, we conduct two counter factual experiments. We replace the SSBF ROA pdf, keeping all other benchmark settings the same, with two different normal distributions:

**1. Best Fit Normal Distribution.** Let

$g_{\mu,\sigma}$: pdf of a *normal distribution* with mean $\mu$, standard deviation $\sigma$

$f$: pdf of the SSBF ROA.

We solve $\min_{\mu,\sigma} \sup_x |g_{\mu,\sigma}(x) - f(x)|$ to find a normal pdf that best approximates the SSBF pdf. We get $\mu = 1.193$ and $\sigma = 0.394$.

To fit the "middle" this normal pdf has less mass in the tails and is less risky.

$A$: Median risk increases from 1.55 to 2.33 but for given $\rho$, the lower pdf risk encourages entrepreneurs to run firms that are larger.

Default: lower, because this normal pdf has a thinner lower tail.

$-E$: The thinner upper tail implies less firms will be "lucky" and have a very good $x$. To match the cdf of net-worth invested, firms must be more leveraged.[17] The somewhat higher $\bar{v}$ also implies that more low $x$ will result in $-E$, and the prediction increases from 10.6% to 13.7%.

[17] Given two solvent firms with the same $x$, a more leveraged firm earns a higher return because the owner receives a higher residual after paying $\bar{v}$. This also explains the higher $b$.

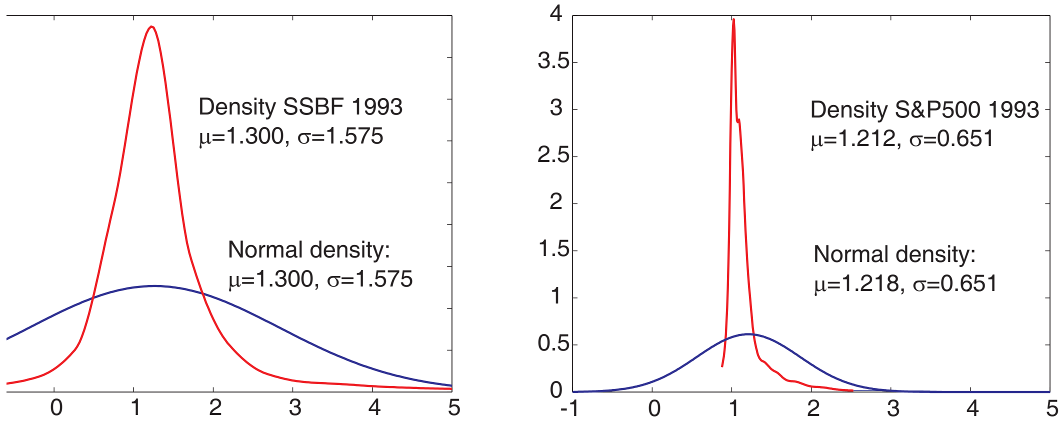


Figure 4: pdfs of ROA for firms in SSBF 1993 & S&P500 (Compustat) vs normal pdfs

**2. Normal Distribution with the SSBF $\mu, \sigma$.** Figure 4 compares the SSBF pdf with a normal pdf with the same $\mu$ and $\sigma$.

The fat tails lead to $\mu$, $\sigma$ with all point mass at $\underline{\rho}$ and $\bar{\rho}$. Generally, we can choose $\bar{\rho}$ sufficiently high that the mass above $\bar{\rho}$ is negligible; this is not the case for this normal pdf with fat tails and $\bar{\rho}$ affects the results.[18]

The results (last column) are implausible.

Table 5: Counterfactual Experiment: Normal Distributions

| **Parameter** | **Data** | **Empirical Dist** | **Best Fit Normal Dist** $\mu$= 1.193, $\sigma$=0.3938 | $\mu$, $\sigma$ **Normal Dist** $\mu$=1.300, $\sigma$=1.193 |
|---|---|---|---|---|
| $\mu$ | 1-3 | 1.55 | 2.33 | $4.4 * 10^8$ |
| $\sigma$ | NA | .83 | 1.11 | $7.9 * 10^8$ |
| b% | NA | 21.5 | 30.0 | 23.4 |
| fit | NA | 0.042 | 0.040 | .045 |
| median A% | [43.1,51.9] | 48.1 | 54.7 | 38.6 |
| default % | 3.5 | 4.4 | 1.5 | 61.0 |
| cons. % | 3-5 | 3.6 | 4.9 | 3.1 |
| neg. Eq % | 15.7 | 10.6 | 13.7 | 64.4 |

[18] $\bar{\rho}$ is the highest $\rho$ for which we compute a solution. This upper bound is needed for computations; it is impossible to compute solutions for a fine grid $[\underline{\rho}, \infty]$.

## 5.5 Entrepreneur Ability

We focused on differences among agents in willingness to bear risk—a central theme entrepreneurship.

There is a large literature on differences in entrepreneurial ability. We omit this type of heterogeneity because we examined the SSBF data for it and found none.

If ability is relevant in our data, then $A/w$ should be positively correlated with $x$. We test whether a positive correlation exists

$$\begin{aligned} \text{realization} = & 1.4776 + 0.0042A + \text{error} \\ & (0.0934) \quad (0.0119) \end{aligned}$$

The absence of a positive correlation between $A$ and $x$ indicates that differences in ability is not significant in the SSBF data.

Manager ability is important for firm success, but absent from the SSBF data due to a selection bias that is acute for small firms.

1. Most SSBF firms have operated for a number of years; firms operated by less able entrepreneurs are less likely to survive.

2. Most firms with less able managers will not accumulate $50,000 in assets.

## 5.6 Conclusions:

**Are entrepreneurs different?**

1. Risk aversion in the standard range
2. Optimism: Slight
4. Credit constrained: yes
5. Irrational – no!

**Institutions matter:**

*Dynamic*: tradeoff $u(c)$ v. $\beta E[V(c')]$
*Bankruptcy*: protects from bad $x$
*Return pdf* $f(x)$: big $\sigma$ but high kurtosis (tight in center & long upper tail).

Given bankruptcy insurance, $f(x)$ explains
* willingness to take on a risky firm & poor diversification
* low default probability & bailouts with own funds
* "luck" accounts for $\frac{1}{3}$ of diversification puzzle:[19] good $x \rightarrow$ high $E$ in firm

[19] Poor diversification is partly an artifact of success: Net worth invested rises for successful firms since D is fixed.

# 6 Appendix A: Experiments

**Experiment 1: Bankruptcy Exclusion Parameter $T$**

**Table 8** Benchmark Exogenous Variables: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

| **T** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **20** |
|---|---|---|---|---|---|---|---|---|
| $\mu$ | 1.62 | 1.55 | 1.49 | 1.51 | 1.52 | 1.52 | 1.51 | 1.50 |
| $\sigma$ | 0.90 | 0.83 | 0.75 | 0.74 | 0.76 | 0.76 | 0.76 | 0.78 |
| $b$ % | 20.6 | 21.5 | 22.0 | 19.8 | 18.4 | 17.7 | 17.3 | 15.4 |
| fit | 0.046 | 0.042 | 0.037 | 0.034 | 0.034 | 0.034 | 0.035 | 0.036 |
| med $A$ % | 46.9 | 48.1 | 49.2 | 47.0 | 45.3 | 44.3 | 43.8 | 41.3 |
| default % | 4.7 | 4.4 | 4.2 | 3.8 | 3.5 | 3.3 | 3.1 | 2.5 |
| c % | 3.7 | 3.6 | 3.6 | 3.6 | 3.6 | 3.6 | 3.6 | 3.5 |
| -E % | 10.2 | 10.6 | 10.8 | 10.5 | 10.8 | 11.1 | 11.6 | 11.1 |

**Table 9** Comparative statics for $T$: Fix $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

| $T$ | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **20** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| fit | 0.095 | 0.085 | 0.076 | 0.066 | 0.053 | 0.042 | 0.054 | 0.065 | 0.073 | 0.079 | 0.084 | 0.107 |
| med $A$ % | 56.2 | 54.4 | 52.7 | 51.1 | 49.6 | 48.1 | 46.7 | 45.4 | 44.3 | 43.3 | 42.4 | 38.9 |
| default % | 6.1 | 5.6 | 5.3 | 5.0 | 4.7 | 4.4 | 4.2 | 4.0 | 3.8 | 3.6 | 3.5 | 2.9 |
| c % | 3.7 | 3.7 | 3.7 | 3.7 | 3.7 | 3.6 | 3.6 | 3.6 | 3.6 | 3.6 | 3.6 | 3.6 |
| -E % | 8.4 | 8.8 | 9.3 | 9.7 | 10.0 | 10.6 | 11.1 | 11.9 | 13.4 | 15.1 | 17.0 | 21.0 |

**Table 10** Welfare as $T$ Varies: % increase or decrease of net-worth over benchmark

| risk aversion $\rho$ | **0.9** | **1.2** | **1.5** | **1.8** | **2.1** | **2.5** | **3.0** | **3.5** | **4.0** |
|---|---|---|---|---|---|---|---|---|---|
| $T = 6$ | 36.9 | 11.2 | 7.7 | 6.1 | 5.0 | 3.9 | 3.1 | 2.6 | 2.2 |
| $T = 7$ | 27.5 | 8.1 | 5.6 | 4.4 | 3.6 | 2.8 | 2.2 | 1.9 | 1.6 |
| $T = 8$ | 19.8 | 5.5 | 3.9 | 3.0 | 2.4 | 1.9 | 1.5 | 1.3 | 1.1 |
| $T = 9$ | 13.5 | 3.2 | 2.4 | 1.8 | 1.5 | 1.2 | 0.9 | 0.7 | 0.6 |
| $T = 10$ | 6.3 | 1.3 | 1.1 | 0.8 | 0.7 | 0.5 | 0.4 | 0.3 | 0.3 |
| $T = 11$ | — | — | — | — | — | — | — | — | — |
| $T = 12$ | -3.6 | -0.7 | -0.9 | -0.7 | -0.5 | -0.4 | -0.3 | -0.3 | -0.2 |
| $T = 13$ | -4.4 | -3.2 | -1.5 | -1.3 | -1.0 | -0.8 | -0.6 | -0.5 | -0.4 |
| $T = 14$ | -7.6 | -4.4 | -2.1 | -1.8 | -1.4 | -1.0 | -0.8 | -0.6 | -0.5 |
| $T = 15$ | -10.5 | -5.5 | -3.2 | -2.1 | -1.7 | -1.3 | -1.0 | -0.8 | -0.6 |
| $T = 16$ | -12.7 | -6.5 | -3.8 | -2.4 | -2.0 | -1.5 | -1.1 | -0.9 | -0.7 |
| $T = 20$ | -20.4 | -9.4 | -5.7 | -4.0 | -2.8 | -1.8 | -1.5 | -1.2 | -1.0 |

**Table 11** Interest Rate as $T$ Varies

| risk aversion $\rho$ | **0.9** | **1.2** | **1.5** | **1.8** | **2.1** | **2.5** | **3.0** | **3.5** | **4.0** |
|---|---|---|---|---|---|---|---|---|---|
| $T = 6$ | 18.0 | 15.3 | 14.2 | 14.0 | 14.3 | 14.4 | 14.3 | 14.1 | 14.1 |
| $T = 7$ | 17.7 | 14.9 | 13.7 | 13.5 | 13.8 | 13.8 | 13.7 | 13.6 | 13.5 |
| $T = 8$ | 17.3 | 14.5 | 13.3 | 13.0 | 13.3 | 13.3 | 13.2 | 13.1 | 13.0 |
| $T = 9$ | 17.0 | 14.1 | 12.9 | 12.5 | 12.8 | 12.9 | 12.7 | 12.6 | 12.5 |
| $T = 10$ | 16.6 | 13.7 | 12.4 | 12.1 | 12.3 | 12.4 | 12.2 | 12.0 | 11.9 |
| $T = 11$ | 16.3 | 13.3 | 12.0 | 11.6 | 11.9 | 11.9 | 11.6 | 11.5 | 11.4 |
| $T = 12$ | 16.0 | 12.9 | 11.7 | 11.2 | 11.4 | 11.4 | 11.2 | 11.0 | 10.9 |
| $T = 13$ | 15.6 | 12.6 | 11.3 | 10.8 | 11.0 | 10.9 | 10.7 | 10.6 | 10.4 |
| $T = 14$ | 15.3 | 12.3 | 10.9 | 10.5 | 10.6 | 10.5 | 10.3 | 10.2 | 10.1 |
| $T = 15$ | 15.0 | 12.0 | 10.7 | 10.1 | 10.2 | 10.2 | 10.0 | 9.8 | 9.7 |
| $T = 16$ | 14.7 | 11.8 | 10.4 | 9.8 | 9.8 | 9.8 | 9.6 | 9.5 | 9.4 |
| $T = 20$ | 13.6 | 10.7 | 9.3 | 8.7 | 8.5 | 8.8 | 8.6 | 8.5 | 8.4 |

**Experiment 1: Bankruptcy Cost Parameter $\delta$**

**Table 12** Higher Cost $\delta$: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.30$

| **T** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **20** |
|---|---|---|---|---|---|---|---|---|
| $\mu$ | 1.79 | 1.67 | 1.55 | 1.50 | 1.52 | 1.52 | 1.51 | 1.50 |
| $\sigma$ | 1.08 | 0.95 | 0.81 | 0.74 | 0.76 | 0.76 | 0.76 | 0.78 |
| $b$ % | 14.9 | 16.9 | 19.8 | 20.1 | 18.4 | 17.6 | 17.2 | 15.4 |
| fit | 0.052 | 0.046 | 0.040 | 0.035 | 0.034 | 0.034 | 0.035 | 0.036 |
| med $A$ % | 39.8 | 42.6 | 46.3 | 47.3 | 45.3 | 44.3 | 43.6 | 41.3 |
| default % | 4.0 | 4.0 | 4.0 | 3.8 | 3.5 | 3.2 | 3.1 | 2.5 |
| c % | 3.8 | 3.7 | 3.7 | 3.6 | 3.6 | 3.6 | 3.6 | 3.5 |
| -E % | 8.7 | 9.2 | 10.2 | 10.5 | 10.7 | 11.0 | 11.4 | 11.1 |

**Table 13** Comparative Statics for $\delta$: Fix $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

| $\delta$ | **0.00** | **0.10** | **0.20** | **0.30** | **0.40** | **0.50** | **0.60** | **0.80** | **1.00** |
|---|---|---|---|---|---|---|---|---|---|
| fit | 0.042 | 0.042 | 0.046 | 0.050 | 0.054 | 0.057 | 0.060 | 0.063 | 0.065 |
| med $A$ % | 48.3 | 48.1 | 48.0 | 47.9 | 47.8 | 47.8 | 47.7 | 47.6 | 47.5 |
| default % | 4.5 | 4.4 | 4.4 | 4.4 | 4.3 | 4.3 | 4.3 | 4.2 | 4.2 |
| c % | 3.6 | 3.6 | 3.6 | 3.6 | 3.6 | 3.6 | 3.6 | 3.6 | 3.6 |
| -E % | 10.8 | 10.6 | 10.3 | 10.2 | 10.1 | 10.1 | 10.0 | 9.9 | 9.7 |

**Table 14** Welfare as $\delta$ Varies: % increase or decrease of net-worth over benchmark

| risk aversion $\rho$ | **0.9** | **1.2** | **1.5** | **1.8** | **2.1** | **2.5** | **3.0** | **3.5** | **4.0** |
|---|---|---|---|---|---|---|---|---|---|
| $\delta = 0.00$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.1 | 0.0 | 0.0 |
| $\delta = 0.10$ | — | — | — | — | — | — | — | — | — |
| $\delta = 0.20$ | 0.0 | 0.0 | 0.0 | 0.0 | -0.1 | -0.1 | 0.0 | 0.0 | 0.0 |
| $\delta = 0.30$ | 0.0 | 0.0 | 0.0 | 0.0 | -0.2 | -0.1 | -0.1 | -0.1 | 0.0 |
| $\delta = 0.40$ | 0.0 | 0.0 | 0.0 | 0.0 | -0.2 | -0.2 | -0.1 | -0.1 | -0.1 |
| $\delta = 0.50$ | 0.0 | 0.0 | 0.0 | -0.1 | -0.3 | -0.2 | -0.2 | -0.1 | -0.1 |
| $\delta = 0.60$ | 0.0 | 0.0 | 0.0 | -0.1 | -0.4 | -0.3 | -0.2 | -0.1 | -0.1 |
| $\delta = 0.80$ | 0.0 | 0.0 | 0.0 | -0.2 | -0.5 | -0.3 | -0.2 | -0.2 | -0.1 |
| $\delta = 1.00$ | 0.0 | 0.0 | 0.0 | -0.2 | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 |

**Experiment 2: $\mu$ and $b$**

**Table 15** Comparative Statics for $\mu$: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

| $\mu$ | **1.15** | **1.25** | **1.35** | **1.45** | **1.55** | **1.65** | **1.75** | **1.85** |
|---|---|---|---|---|---|---|---|---|
| fit | 0.224 | 0.146 | 0.109 | 0.074 | 0.042 | 0.080 | 0.117 | 0.153 |
| med $A$ % | 74.3 | 65.4 | 58.3 | 52.7 | 48.1 | 44.4 | 41.2 | 38.6 |
| default % | 4.2 | 4.2 | 4.3 | 4.4 | 4.4 | 4.5 | 4.6 | 4.7 |
| c % | 2.8 | 3.0 | 3.2 | 3.5 | 3.6 | 3.8 | 4.0 | 4.1 |
| -E % | 8.4 | 8.9 | 9.5 | 10.0 | 10.6 | 11.1 | 11.7 | 12.3 |

**Table 16** Comparative Statics for $b$: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

| $b$ | **0.10** | **0.15** | **0.20** | **0.21** | **0.25** | **0.30** | **0.35** | **0.40** | **0.50** |
|---|---|---|---|---|---|---|---|---|---|
| fit | 0.145 | 0.071 | 0.048 | 0.042 | 0.067 | 0.094 | 0.113 | 0.126 | 0.145 |
| med $A$ % | 46.9 | 47.1 | 47.8 | 48.1 | 49.0 | 50.5 | 51.8 | 52.4 | 52.5 |
| default % | 3.0 | 3.6 | 4.3 | 4.4 | 4.8 | 5.2 | 5.4 | 5.6 | 5.9 |
| c % | 3.6 | 3.6 | 3.6 | 3.6 | 3.7 | 3.7 | 3.7 | 3.6 | 3.6 |
| -E % | 5.7 | 7.7 | 10.0 | 10.6 | 11.7 | 12.9 | 13.6 | 14.2 | 15.0 |

**Table 17** Welfare as $b$ Varies: % increase or decrease in net-worth over benchmark
More risk averse agents are unaffected because the credit constraint does not bind for them

| risk aversion $\rho$ | **0.9** | **1.2** | **1.5** | **1.8** | **2.1** | **2.5** | **3.0** | **3.5** | **4.0** |
|---|---|---|---|---|---|---|---|---|---|
| $b = 0.100$ | -13.1 | -8.5 | -6.2 | -4.9 | -3.7 | -2.2 | -0.9 | -0.2 | -0.1 |
| $b = 0.150$ | -6.1 | -4.8 | -3.2 | -2.0 | -1.4 | -0.4 | 0.0 | 0.0 | 0.0 |
| $b = 0.200$ | -1.8 | -0.2 | -0.6 | -0.4 | -0.1 | 0.0 | 0.0 | 0.0 | 0.0 |
| $b = 0.215$ | — | — | — | — | — | — | — | — | — |
| $b = 0.250$ | 8.2 | 1.7 | 1.3 | 0.6 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $b = 0.300$ | 14.8 | 4.2 | 2.4 | 0.7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $b = 0.350$ | 20.9 | 6.0 | 2.7 | 0.7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $b = 0.400$ | 26.6 | 7.2 | 2.7 | 0.7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $b = 0.500$ | 35.0 | 7.5 | 2.7 | 0.7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

**Experiment 3: Slight optimism is consistent with the data**

**Table 18** 10% Optimism: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=10%

| **T** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **20** |
|---|---|---|---|---|---|---|---|---|
| $\mu$ | 1.92 | 1.89 | 1.83 | 1.79 | 1.76 | 1.73 | 1.70 | 1.61 |
| $\sigma$ | 0.83 | 0.81 | 0.77 | 0.74 | 0.72 | 0.70 | 0.69 | 0.63 |
| $b$ % | 26.6 | 26.2 | 27.0 | 27.2 | 27.3 | 27.3 | 27.3 | 27.4 |
| fit | 0.030 | 0.030 | 0.029 | 0.029 | 0.029 | 0.029 | 0.029 | 0.028 |
| med $A$ % | 54.9 | 54.1 | 54.8 | 54.8 | 54.8 | 54.8 | 54.8 | 54.7 |
| default % | 4.4 | 4.0 | 3.8 | 3.6 | 3.4 | 3.3 | 3.1 | 2.7 |
| c % | 5.2 | 5.1 | 5.1 | 5.0 | 5.0 | 4.9 | 4.9 | 4.7 |
| -E % | 15.8 | 16.7 | 17.5 | 17.8 | 17.8 | 17.8 | 17.7 | 17.6 |

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