Liquidity Traps, Learning and Stagnation

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November 9, 2006; Preliminary

Abstract

We examine global economic dynamics under learning in a New Keynesian model in which the interest-rate rule is subject to the zero lower bound. Under normal monetary and fiscal policy the intended steady state is locally but not globally stable. Large pessimistic shocks to expectations can lead to deflationary spirals with falling prices and falling output. To avoid this outcome it may be necessary to suspend normal policies and replace them by aggressive monetary and fiscal policy that guarantees a lower bound on inflation.

JEL Classification: E63, E52, E58.

Keywords: Adaptive Learning, Monetary Policy, Fiscal Policy, Zero Interest Rate Lower Bound, Indeterminacy

1 Introduction

There is now widespread agreement that the zero lower bound on nominal interest rates has the potential to generate a "liquidity trap" with major implications for economic performance. There is a substantial literature that has discussed the plausibility of the economy becoming trapped in a deflationary state and what macroeconomic policies would be able to avoid or extricate the economy from a liquidity trap.[1] Our own view, reflected in the current paper as well as in the earlier paper Evans and Honkapohja (2005), is that the evolution of expectations plays a key role in the dynamics of the economy and that the tools from learning theory are needed for a realistic analysis of these issues.

The importance of expectations in the liquidity trap is now widely accepted. For example, Benhabib, Schmitt-Grohe, and Uribe (2001b), Benhabib,

[1] See Krugman (1998) for a recent seminal discussion and Adam and Billi (2005), Coenen and Wieland (2004), Eggertsson and Woodford (2003) and Eggertsson and Woodford (2004) for representative recent analyses and further references.

Schmitt-Grohe, and Uribe (2001a) show the possibility of multiple equilibria under perfect foresight, with a continuum of paths to a low-inflation steady state. Similarly, Eggertsson and Woodford (2003) emphasize the importance of policy commitment for influencing expectations under the rational expectations (RE) assumption. In Evans and Honkapohja (2005) we emphasized how the learning perspective alters both the assessment of the plausibility of particular dynamics and the impact of policy.

Under learning private agents are assumed to form expectations using an adaptive forecasting rule, which they update over time in accordance with standard statistical procedures. In many standard set-ups least-squares learning is known to converge asymptotically to rational expectations, but cases of instability can also arise. In the earlier paper we examined a flexible price model with a global Taylor-rule, which, because of the zero lower bound, generates a low-inflation steady state below the one intended by policy. We found that while the intended steady state was locally stable under learning, the lower one was not[2] and there was also the possibility of inflation slipping below the low-inflation steady state. We there showed that switching to a sufficiently aggressive monetary policy at low inflation rates could avoid these unstable trajectories. Fiscal policy in these circumstances was ineffective.

The analysis of Evans and Honkapohja (2005), however, was conducted in a flexible-price model with exogenous output. In the current paper we employ a New Keynesian model to reexamine these issues in a framework that allows for a serious analysis of monetary and fiscal policy for an economy in which recessions or slumps can arise due to failures of aggregate demand.

2 The Model

We adopt a fairly standard representative agent model along the lines of Benhabib, Schmitt-Grohe, and Uribe (2001b), Section 3, except that we allow for stochastic shocks and conduct the analysis in discrete time.[3] There is a continuum of household-firms units, which produce a differentiated consumption good under conditions of monopolistic competition and price-adjustment costs. We allow for both fiscal and monetary policy and for the government to issue debt.

[2] See also McCallum (2002) for an argument that the low-inflation steady-state is not stable under learning.

[3] We develop our analysis within a closed-economy model. For discussions of liquidity traps in open economies, see for example McCallum (2000) and Svensson (2003).

2.1 Private Sector

The objective for agent j is to maximize expected, discounted utility subject to a standard flow budget constraint:

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t U_{t,j} \left(c_{t,j}, \frac{M_{t-1,j}}{P_t}, h_{t,j}, \frac{P_{t,j}}{P_{t-1,j}} - 1 \right) \tag{1}$$

$$st.\ c_{t,j} + m_{t,j} + b_{t,j} + \tau_{t,j} = m_{t-1,j}\pi_t^{-1} + R_{t-1}\pi_t^{-1} b_{t-1,j} + \frac{P_{t,j}}{P_t} y_{t,j}, \tag{2}$$

where $c_{t,j}$ is the Dixit-Stiglitz consumption aggregator, $M_{t,j}$ and $m_{t,j}$ denote nominal and real money balances, $h_{t,j}$ is the labor input into production, $b_{t,j}$ denotes real bonds held by the agent at the end of period t, $\tau_{t,j}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest rate factor, $P_{t,j}$ is the price of consumption good j, $y_{t,j}$ is output of good j, P_t is the aggregate price level and the inflation rate is $\pi_t = P_t/P_{t-1}$. The utility function has the parametric form

$$U_{t,j} = \frac{c_{t,j}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2} \left(\frac{M_{t-1,j}}{P_t} \right)^{1-\sigma_2} - \frac{h_{t,j}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2} \left(\frac{P_{t,j}}{P_{t-1,j}} - 1 \right)^2 .$$

Note that the final term parameterizes the cost of adjusting prices in the spirit of Rotemberg (1982).

The production function for good j is

$$y_{t,j} = h_{t,j}^{\alpha}$$

where $0 < \alpha < 1$. Output is differentiated and firms operate under monopolistic competition. Each firm faces a downward sloping demand curve given by

$$P_{t,j} = \left(\frac{y_{t,j}}{Y_t} \right)^{-1/\nu} P_t.$$

Here $P_{t,j}$ is the profit maximizing price set by firm j consistent with its production $y_{t,j}$. The parameter ν is the elasticity of substitution between two goods and is assumed to be greater than one.

2.2 Fiscal and Monetary Policy

Assume that the government's budget constraint is

$$b_t + m_t + \tau_t = g_t + m_{t-1}\pi_{t-1} + R_{t-1}\pi_t^{-1} b_{t-1}, \tag{3}$$

where g_t denotes government consumption of the aggregate good and τ_t is the lump-sum tax collected. We assume that fiscal policy will follow a linear tax rule as in Leeper (1991)

$$\tau_t = \kappa_0 + \kappa b_{t-1} + \psi_t + \eta_t, \tag{4}$$

where ψ_t and η_t denote observed and unobserved shocks, respectively. We also assume that g_t is stochastic

$$g_t = g + u_t,$$

where u_t is an observable white noise shock. From market clearing we have

$$c_t = h_t^\alpha - g_t \tag{5}$$

Monetary policy is assumed to follow a global interest rate rule

$$R_t - 1 = \theta_t f(\pi_t) \tag{6}$$

The function $f(\pi)$ is taken to be non-negative and non-decreasing, while θ_t is an exogenous, *iid* and positive random shock with mean 1 representing random shifts in the behavior of the monetary policy-maker. We assume the existence of π^*, R^* such that $R^* = \beta^{-1}\pi^*$ and $f(\pi^*) = R^* - 1$. π^* can be viewed as the inflation target of the Central Bank. In the numerical analysis we will use the functional form

$$f(\pi) = (R^* - 1)\left(\frac{\pi}{\pi^*}\right)^{AR^*/(R^*-1)},$$

which implies the existence of a nonstochastic steady state at π^*. Note that $f'(\pi^*) = AR^*$, which we assume is bigger than β^{-1}.

Equations (3), (4) and (6) constitute "normal policy". In the first part of the paper we examine the system under normal policy. Later we consider modifications to these policy rules in exceptional circumstances.

2.3 Key Equations

In the Appendix it is shown that private sector optimization yields the key equations

$$-h_t^{1+\varepsilon} + \frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t + \alpha\left(1 - \frac{1}{\nu}\right)h_t^\alpha c_t^{-\sigma_1} = \beta\frac{\alpha\gamma}{\nu}E_t\left[(\pi_{t+1} - 1)\pi_{t+1}\right], \tag{7}$$

$$c_t^{-\sigma_1} = \beta R_t E_t\left(\pi_{t+1}^{-1} c_{t+1}^{-\sigma_1}\right), \tag{8}$$

$$m_t = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1 - R_t^{-1}\right)c_t^{-\sigma_1}}{E_t \pi_{t+1}^{\sigma_2 - 1}}\right)^{-1/\sigma_2}, \tag{9}$$

to which we add the equations (3), (4), (5) and (6).

As first noted by Benhabib, Schmitt-Grohe, and Uribe (2001b), whenever $f(.)$ is continuous (and differentiable) and has a steady state π_H with $f'(\pi_H) > \beta^{-1}$, in accordance with the Taylor principle, non-negativity of the (net) nominal interest rate implies the existence of a second low inflation steady state π_L with $f'(\pi_L) < \beta^{-1}$. Under the parametric form for $f(.)$ we have $\pi_H = \pi^*$. For each steady state $\pi \in \{\pi^*, \pi_L\}$ the nominal interest rate factor equals

$$R = \beta^{-1}\pi. \tag{10}$$

The other steady state equations are given by

$$c = h^\alpha - g, \tag{11}$$

$$-h^{1+\varepsilon} + \frac{\alpha\gamma}{\nu}(1-\beta)(\pi-1)\pi + \alpha\left(1-\frac{1}{\nu}\right)h^\alpha c^{-\sigma_1} = 0 \tag{12}$$

and a steady state version of (9). For a given steady state $\pi \geq 1$, it is shown in the Appendix that there is a corresponding unique interior steady state $c > 0$ and $h > 0$. For steady states $\pi < 1$, there continue to be unique values for c and h provided π is close to one and $g > 0$.[4]

When there is a nonstochastic steady state, it can be shown that stochastic steady states exist when the support of the exogeneous shocks is sufficiently small. Furthermore, in this case the steady state is locally determinate, provided the corresponding linearization is determinate. Throughout the paper we assume that the shocks are small in the sense of having small support. We now consider determinacy of the linearized system.

In a neighborhood of a nonstochastic steady state (c, π) we can derive a linear approximation

$$\begin{aligned} c_t &= -\sigma_1\beta\pi^{-1}cR_t + c^e_{t+1} + \sigma_1 c\pi^{-1}\pi^e_{t+1} + k_c & (13)\\ R_t &= a\pi_t + \delta\theta_t + k_R, \text{ where } a = f'(\pi),\ \delta = f(\pi) & (14)\\ \frac{\alpha\gamma}{\nu}(2\pi-1)\pi_t &= \frac{\beta\alpha\gamma}{\nu}(2\pi-1)\pi^e_{t+1} - \frac{1+\varepsilon}{\alpha}(c+g)^{(1+\varepsilon)\alpha^{-1}-1}(c_t+u_t) & (15)\\ &\quad +\alpha(1-\nu^{-1})\left(-(c+g)\sigma_1 c^{-\sigma_1-1}c_t + c^{-\sigma_1}(c_t+u_t)\right) + k_\pi \end{aligned}$$

together with the linearized evolution of bonds

$$\begin{aligned} b_t + m_t + \kappa_0 + \psi_t + \eta_t &= g_t + \pi^{-1}m_{t-1} + (\beta^{-1}-\kappa)b_{t-1} \\ &\quad + b\pi^{-1}R_{t-1} - (m\pi^{-1} + \beta^{-1}b)\pi^{-1}\pi_t + k_b \end{aligned}$$

and the linearized money equation

$$\begin{aligned} m_t &= -\sigma_2^{-1}(\chi\beta)^{1/\sigma_2}\left(1-R^{-1}\right)^{-1/\sigma_2-1}\frac{c^{\sigma_1/\sigma_2}}{\pi^{1/\sigma_2}} \times \\ &\quad \left(-\sigma_1\left(1-R^{-1}\right)\pi c^{-1}c_t + (1-\sigma_2)\left(1-R^{-1}\right)\pi^e_{t+1} + \beta R^{-1}R_t\right) + k_m, \end{aligned}$$

where b and m are steady-state values.

The block of the first three equations determines the values of c_t, π_t, R_t solely in terms of c^e_{t+1}, π^e_{t+1} and the exogenous shocks. Determinacy of a steady state can therefore be assessed from this block plus stationarity of the bond dynamics. Substituting (14) into (13) yields a bivariate forward-looking system of the form

$$\begin{pmatrix} c_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} B_{cc} & B_{c\pi} \\ B_{\pi c} & B_{\pi\pi} \end{pmatrix}\begin{pmatrix} c^e_{t+1} \\ \pi^e_{t+1} \end{pmatrix} + \begin{pmatrix} G_{cu} & G_{c\theta} \\ G_{\pi u} & G_{\pi\theta} \end{pmatrix}\begin{pmatrix} u_t \\ \theta_t \end{pmatrix} + \begin{pmatrix} \tilde{k}_c \\ \tilde{k}_\pi \end{pmatrix}, \tag{16}$$

[4] Cases of multiple values for c and h for given $\pi < 1$ do exist. Throughout the paper we rule out these exceptional cases.

where the coefficients can be computed by solving the equations. Denoting by B the 2×2 matrix multiplying $(c^e_{t+1}, \pi^e_{t+1})'$, a necessary condition for determinacy is that both eigenvalues of B lie inside the unit circle. There is then a unique nonexplosive solution of the form

$$\begin{pmatrix} c_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} c \\ \pi \end{pmatrix} + \begin{pmatrix} G_{cu} & G_{c\theta} \\ G_{\pi u} & G_{\pi\theta} \end{pmatrix} \begin{pmatrix} u_t \\ \theta_t \end{pmatrix}. \tag{17}$$

The corresponding solution for m_t then takes the form of a constant plus white noise shocks. From the linearized bond equation it follows that the remaining condition for determinacy is that fiscal policy is "passive" in the sense of Leeper (1991), i.e. $\left|\beta^{-1} - \kappa\right| < 1$.

Determinacy needs to be assessed separately for the π^* and π_L steady states. We have the following result:

Proposition 1 *In the linearized model there are two steady states $\pi^* > \pi_L$. Provided fiscal policy is passive and $\gamma > 0$ is sufficiently small, the steady state $\pi = \pi^*$ is locally determinate and the steady state $\pi = \pi_L$ is locally indeterminate.*

Proof. As noted above, from the steady-state interest-rate equation $R - 1 = f(\pi)$ and the properties of f it follows that there are two steady state inflation rates $0 < \pi_L < \pi^*$. As $\gamma \to 0$ it is easily seen that $B_{cc}, B_{c\pi} \to 0$ and $B_{\pi\pi} \to (a\beta)^{-1}$. At π^* we have $a > \beta^{-1}$ while at π_L we have $a < \beta^{-1}$. Hence for $\gamma > 0$ sufficiently small the roots of B are inside the unit circle at π^*, while at π_L one root is larger than 1. The result follows. ■

This result generalizes the corresponding results in Evans and Honkapohja (2005) and Evans and Honkapohja (2006), which considered an endowment economy with flexible prices (i.e. $\gamma = 0$).

3 Learning and Expectational Stability

We now formally introduce learning to the model in place of the hypothesis that RE prevails in all periods. In the modeling of learning it is assumed that private agents make forecasts using a reduced form econometric model of the relevant variables and that the parameters of this model are estimated using past data. The forecasts are input to agent's decision rules and in each period the economy attains a temporary equilibrium, i.e. an equilibrium for the current period variables given the forecasts of the agents. The temporary equilibrium provides a new data point, which in the next period leads to re-estimation of the parameters and updating of the forecasts and, in turn, to a new temporary equilibrium. The sequence of temporary equilibria may generate parameter estimates that converge to a fixed point corresponding to a rational expectations equilibrium (REE) for the economy, provided the form of the econometric model that agents use for forecasts is consistent with the

REE. When the convergence takes place, we say that the REE is stable under learning.

The literature on adaptive learning has shown that there is a close connection between the possible convergence of least squares learning to an REE and a stability condition, known as E-stability, based on a mapping from the perceived law of motion (that private agents are estimating) to the implied actual law of motion generating the data under these perceptions. E-stability is defined in terms of local stability, at an REE, of a differential equation based on this map. For a general discussion of adaptive learning and the E-stability principle see Evans and Honkapohja (2001).

Although the model of this paper is fairly complex, the RE solutions for π_t and c_t described above are simply noisy steady states, i.e. *iid* processes. This greatly simplifies the analysis of learning since we can plausibly assume that private agents forecast by estimating the mean values of π_t and c_t. In the learning literature this is often called "steady-state learning."[5] Under steady state learning agents treat (17) as a Perceived Law of Motion, which they estimate according to the following simple recursive algorithm

$$\pi^e_{t+1} = \pi^e_t + \phi_t(\pi_{t-1} - \pi^e_t) \quad (18)$$
$$c^e_{t+1} = c^e_t + \phi_t(c_{t-1} - c^e_t), \quad (19)$$

where ϕ_t is known as the gain sequence. Under least-squares learning the gain-sequence is usually taken to be $\phi_t = t^{-1}$, often termed a "decreasing-gain" sequence, whereas under "discounted least-squares" or " constant gain" learning it is set to $\phi_t = \phi$, where $0 < \phi < 1$ is a small positive constant. Decreasing gains have the advantage that they can asymptotically converge to RE, while constant-gain learning rules are more robust to structural change.

In what follows, we analyze both theoretically and numerically the model under various specifications of monetary and fiscal policy. The theoretical results for learning are based E-stability analysis of the system under the learning rules (18)-(19). When we say that an equilibrium is stable (or unstable) under learning this implies that it is stable (or not) under these learning rules with decreasing gain. In the simulations we instead use a small constant gain. Under constant gain, when an equilibrium is E-stable there is local convergence of learning in a weaker sense to a random variable that is centered near the equilibrium.[6]

We next investigate our system under learning under the "normal policy" rules that we have described. In studying the economy under learning we return to the nonlinear model so that we can examine the global dynamics of the system. In doing so it is convenient to make the assumption of point expectations, e.g. replacing the expectation of $\pi^{-1}_{t+1}c^{-\sigma_1}_{t+1}$ by $(\pi^e_{t+1}(c^e_{t+1})^{\sigma_1})^{-1}$. For

[5] In some cases there can be other more complex RE solutions that would require agents to use more complicated econometric models. See Evans and Honkapohja (2006) and Evans and Honkapohja (2005) for a discussion of VAR learning for certain solutions in closely related models.

[6] For formal details see Section 7.4 of Evans and Honkapohja (2001).

stochastic shocks u_t and θ_t with small bounded support this is a reasonable approximation and it allows us to deal directly with expectations of future consumption and inflation rather than with nonlinear functions of them. Making this assumption and using also the production function to substitute out h_t leads to the system

$$\beta\frac{\alpha\gamma}{\nu}\left(\pi^e_{t+1}-1\right)\pi^e_{t+1} = -(c_t+g_t)^{(1+\varepsilon)/\alpha}+\frac{\alpha\gamma}{\nu}\left(\pi_t-1\right)\pi_t \qquad (20)$$

$$+\alpha\left(1-\frac{1}{\nu}\right)(c_t+g_t)c_t^{-\sigma_1}$$

$$c_t = c^e_{t+1}(\pi^e_{t+1}/\beta R_t)^{\sigma_1}, \qquad (21)$$

where $g_t = g+u_t$. These equations, together with the interest-rate rule (6), implicitly define the temporary equilibrium values for c_t and π_t given values for expectations c^e_{t+1}, π^e_{t+1} and given the exogenous shocks u_t, θ_t. Formally we write the temporary equilibrium map as

$$\begin{aligned}\pi_t &= F_\pi(\pi^e_{t+1}, c^e_{t+1}, u_t, \theta_t)\\ c_t &= F_c(\pi^e_{t+1}, c^e_{t+1}, u_t, \theta_t),\end{aligned}$$

where it follows from the implicit function theorem that such a map exists in a neighborhood of each steady state (the linearization was given above as (17)).[7]

The dynamic system for c_t and π_t under learning is then given by (20)-(21) and (6) together with (18)-(19). The full dynamic system under learning augments these equations with the money equation

$$m_t = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1-R_t^{-1}\right)c_t^{-\sigma_1}}{(\pi^e_{t+1})^{\sigma_2-1}}\right)^{-1/\sigma_2}$$

and the bond equation (3).

The stability of a steady-state REE under learning is determined by E-stability. The REE is said be E-stable if the differential equation (in notional time τ)

$$\begin{pmatrix} d\pi^e/d\tau \\ dc^e/d\tau\end{pmatrix} = \begin{pmatrix} T_\pi(\pi^e, c^e) \\ T_c(\pi^e, c^e)\end{pmatrix} - \begin{pmatrix} \pi^e \\ c^e\end{pmatrix}$$

is locally asymptotically stable at a steady state (π, c), where here

$$\begin{aligned}T_\pi(\pi^e, c^e) &= EF_\pi(\pi^e, c^e, u_t, \theta_t)\\ T_c(\pi^e, c^e) &= EF_\pi(\pi^e, c^e, u_t, \theta_t).\end{aligned}$$

E-stability is determined by the Jacobian matrix DT of $T = (T_\pi, T_c)'$ at the steady state, which for small noise, is approximately equal to the matrix B of (16) for the steady state in question. It follows that the E-stability conditions

[7] Numerically it appears that this function is well-defined globally.

are that both eigenvalues of $B - I$ have real parts less than zero. We have the following result for low levels of price stickiness.[8]

Proposition 2 *For $\gamma > 0$ sufficiently small, the steady state $\pi = \pi^*$ is locally stable under learning and the steady state $\pi = \pi_L$ is locally unstable under learning taking the form of a saddle point.*

Proof. In the limit $\gamma \to 0$ it is straightforward to show that the eigenvalues of $B - I$ are -1 and $(\beta f'(\pi))^{-1} - 1$. Since $f'(\pi^*) = A/\beta$ and $A > 1$ we have that $(\beta f'(\pi^*))^{-1} - 1 = A^{-1} - 1 < 0$. At the π_L steady state we instead have $f'(\pi_L) < \beta^{-1}$ implying $(\beta f'(\pi_L))^{-1} - 1 > 0$, so that the roots are real and of different signs. ■

The saddle point property of π_L creates a region in which there can be deflationary spirals. We illustrate this by numerically constructed phase diagrams. This also allows to examine larger $\gamma > 0$ and conduct a global analysis. Parameters are set at $A = 2.5$, $\pi^* = 1.05$, $\beta = 0.96$, $\sigma_1 = 0.95$, $\alpha = 0.75$, $\gamma = 5$, $\nu = 1.5$, $\varepsilon = 1$ and $g = 0.1$. Figure 1 shows the E-stability dynamics under normal monetary and fiscal policy. These indicate how expectations will adjust over time under learning when the economy is perturbed from its steady state equilibrium.

It can be seen that while π^* is locally stable the low steady state π_L is a saddle. Under learning, normal policy works satisfactorily for moderate-sized perturbations from the targeted steady state. However, there are also starting points that lead to instability. In particular, if an exogenous shock leads to a strong downward revision of expectations, relative to the normal steady state, these pessimistic expectations generate paths leading to a deflationary spiral.

These results indicate the need for more aggressive policies when expectations are pessimistic. We begin by considering changing to an aggressive monetary policy when inflation threatens to become too low. As we will see, it may be important also to alter fiscal policy in certain circumstances.

4 Adding Aggressive Monetary Policy

We first consider modifying monetary policy so that it follows the normal interest rate rule as long as $\pi_t \geq \tilde{\pi}$, but cuts interest rates to a low level floor $\hat{R}$ if inflation threatens to get below the threshold $\tilde{\pi}$. Thus

$$R_t = \begin{cases} 1 + \theta_t f(\pi_t) \text{ if } \pi_t > \tilde{\pi} \\ \hat{R} \text{ if } \pi_t < \tilde{\pi}, \end{cases}$$

and

$$\hat{R} \leq R_t \leq 1 + \theta_t f(\pi_t) \text{ if } \pi_t = \tilde{\pi},$$

[8]Some further results for general $\gamma > 0$ are available. For example local stability of π^* obtains for all $\gamma > 0$ and $g \geq 0$ if $\sigma_1 \geq 1$.

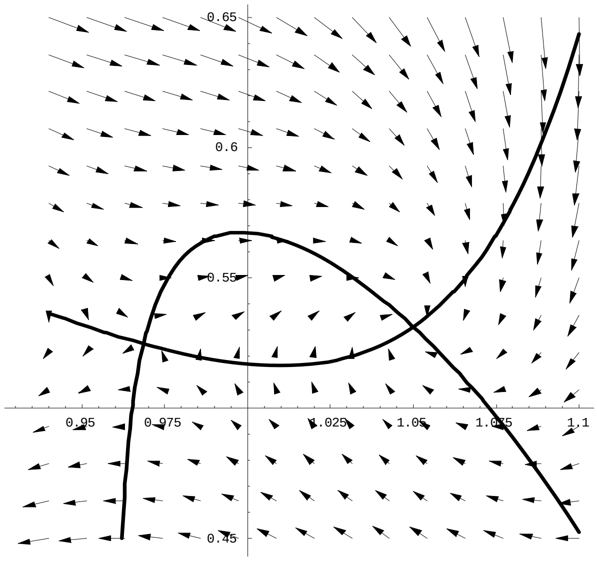

Figure 1: π^e and c^e dynamics under standard policy

where we choose

$$1 < \hat{R} < \min\left(1 + f\left(\pi_t\right), \beta^{-1}\tilde{\pi}\right).$$

Thus, if $\pi_t < \tilde{\pi}$ the temporary equilibrium is given by (20)-(21), which yields

$$\beta\frac{\alpha\gamma}{\nu}\left(\pi^e_{t+1} - 1\right)\pi^e_{t+1} = -(c_t + g_t)^{(1+\varepsilon)/\alpha} + \frac{\alpha\gamma}{\nu}\left(\pi_t - 1\right)\pi_t$$
$$+\alpha\left(1 - \frac{1}{\nu}\right)(c_t + g_t)c_t^{-\sigma_1} \quad (22)$$
$$c_t = c^e_{t+1}(\pi^e_{t+1}/\beta\hat{R})^{\sigma_1}. \quad (23)$$

We remark that real money stock is given by

$$m_t = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1 - \hat{R}^{-1}\right)c_t^{-\sigma_1}}{(\pi^e_{t+1})^{\sigma_2 - 1}}\right)^{-1/\sigma_2}$$

and M_t is endogenous. In the $\pi_t < \tilde{\pi}$ regime, expectations determine c_t through the Euler equation. Then h_t is determined by c_t and fiscal policy by $h_t^\alpha = c_t + g_t$ and the Phillips Curve gives π_t.

A policy question of major importance is whether an aggressive monetary policy of this form is sufficient to eliminate deflationary spirals from arising when expectations are pessimistic. We now show that aggressive monetary policy will not always be adequate to avoid these outcomes (see the Appendix for a proof):

Proposition 3 *There is a steady state at $\hat{\pi} = \beta\hat{R}$ and there no steady state value for π_t below $\hat{\pi}$. For all $\gamma > 0$ sufficiently small the steady state at $\hat{\pi} = \beta\hat{R}$ is a saddle point under learning.*

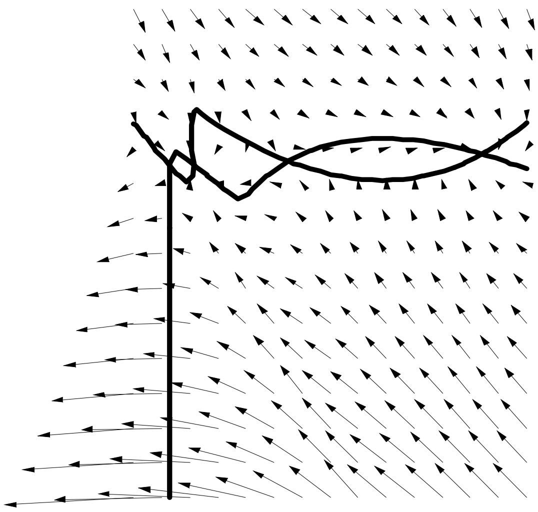

Figure 2: Four steady states with aggressive monetary policy but standard fiscal policy.

We also illustrate this point numerically using the phase diagrams showing expectational dynamics. Figure 2 gives the E-stability dynamics when the switch to aggressive monetary policy takes place only at $\tilde{\pi} = 0.97$, i.e. in the deflationary region.[9] We here set $\hat{R} = 1.0001$, so that net nominal interest rates are cut almost to zero. In the case shown case there are now four steady states and the possibility of a deflationary spiral remains.

Monetary policy can be strengthened by raising the threshold $\tilde{\pi}$ for aggressive interest-rate cuts. Figure 3 shows the impact of setting a value $\tilde{\pi} > \pi_L$.[10] Although this eliminates the unstable steady state at π_L, the deflationary spiral still exists for sufficiently pessimistic expectations. There are now two steady states: the targeted steady state at π^*, which is locally stable, and a low level steady state at $\hat{\pi} = \beta\hat{R} < \pi_L$, which is a saddle with nearby deflationary paths.

The conclusion from this analysis is that aggressive monetary policy will not always be sufficient to eliminate deflationary spirals and stagnation. We therefore now take up fiscal policy as a possible additional measure.

5 Combined Monetary and Fiscal Policy

We now introduce our recommended policy to combat liquidity traps and deflationary spirals. Normal monetary and fiscal policy is supplemented by a

[9] The parameter values are $A = 1.5$, $\pi^* = 1.1$, $\beta = 0.96$, $\sigma_1 = 0.95$, $\alpha = 0.75$, $\gamma = 1$, $\nu = 1.5$, $\varepsilon = 1$, and $\tilde{\pi} = 1.01$.

[10] Parameters are $A = 2.5$, $\pi^* = 1.05$, $\beta = 0.96$, $\sigma_1 = 0.95$, $\alpha = 0.75$, $\gamma = 5$, $\nu = 1.5$, $\varepsilon = 1, g = 0.1$ and $\tilde{\pi} = 1.01$.

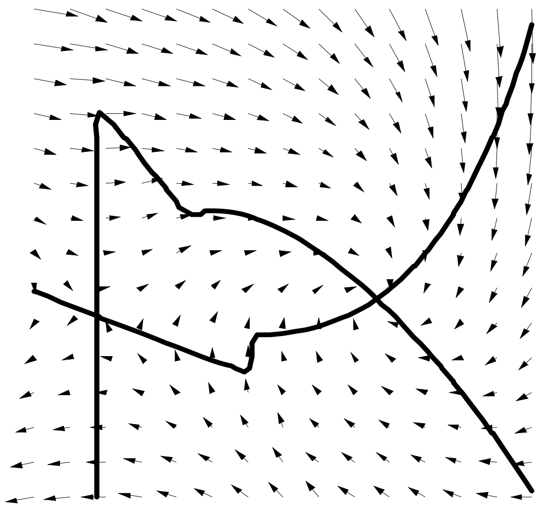

Figure 3: Two steady states with standard fiscal policy and high $\tilde{\pi}$

target floor for inflation that policy is designed to achieve:

$$\pi_t \geq \tilde{\pi}. \tag{24}$$

It is assumed that $1/2 < \tilde{\pi} < \pi^*$. If the target would not be achieved under normal policy, then monetary and/or fiscal policy is adjusted as follows.

First, monetary policy is relaxed as required to achieve these targets, subject to the requirement that the interest rate does not fall below a minimum value $\hat{R}$, which can be set just above one. It is assumed that $\hat{R} < \beta^{-1}\pi_L$. If this is not sufficient to achieve the target, then we set $R_t = \hat{R}$ and fiscal policy is used, increasing g_t as required, to meet the target. The following Lemma shows that this is indeed feasible:

Lemma 4 *Given expectations c^e_{t+1} and π^e_{t+1} and setting $R_t = \hat{R}$, any value of $\pi_t > \pi^*/2$ can be achieved by setting g_t sufficiently high.*

Proof. First, note that $c_t = c^e_{t+1}(\pi^e_{t+1}/\beta\hat{R})^{\sigma_1}$ is fixed when $R_t = \hat{R}$. Implicitly differentiating (20) we obtain

$$\frac{d\pi_t}{dg_t} = \frac{\nu}{2\alpha\gamma}\frac{(1+\varepsilon)\alpha h_t^{1+\varepsilon-\alpha} - \alpha^2(1-\nu^{-1})c_t^{-\sigma_1}}{\pi_t - 1/2}.$$

Since $\frac{dh_t}{dg_t} = \alpha h_t^{1-\alpha}$ is bounded above zero for $c_t > 0$, there exists g' such that $\frac{d\pi_t}{dg_t} > 0$ for $g_t > g'$. It follows that $\pi_t \to \infty$ as $g_t \to \infty$ since if π_t were bounded then $\frac{d\pi_t}{dg_t}$ would be unbounded. ■

The Lemma shows that policy can be designed to guarantee an inflation floor. We now specify a policy based on this result. If the inflation bound $\tilde{\pi}$ is not achieved under normal policy, then we (i) compute the interest rate $\check{R}_t$

consistent with equations (20), (21) and $\pi_t = \tilde{\pi}$, (ii) set $R_t = \max[\breve{R}_t, \hat{R}]$. If $R_t = \hat{R} > \breve{R}_t$ then g_t is adjusted upward and is set equal to the minimum value such that the bound is met. By the Lemma this is feasible.

Intuitively, if the target would not be met under normal policy the first priority is to relax monetary policy to the extent required to achieve it. If the zero net interest rate lower bound renders monetary policy inadequate to the task, then aggressive fiscal policy is deployed. As we will see below, the choice of the threshold inflation rate $\tilde{\pi}$ is crucial for the success of this policy.

We now consider the possible steady states when the floor constraints are introduced. We have the result:

Proposition 5 *(i) If $\pi_L < \tilde{\pi} < \pi^*$ then π^* is the unique steady state.*
(ii) If $\beta\hat{R} < \tilde{\pi} < \pi_L$ then there are three steady states π^*, π_L and $\tilde{\pi}$.
(iii) $\tilde{\pi} < \beta\hat{R} < \pi_L$ then there are two steady states at π^* and π_L. In addition, there is a quasi-steady state at $\tilde{\pi}$ in which consumption is falling.

Proof. We first remark that the Fisher equation $\beta R = \pi$ holds in all steady states.

Case (i) follows from the assumption that $\tilde{\pi} < \pi^*$. In a steady state we must have $\pi \geq \tilde{\pi}$. For $\tilde{\pi} \leq \pi < \pi^*$ the interest rate given by $\beta^{-1}\pi > 1 + f(\pi)$, which is impossible. For $\pi > \pi^*$ the floors are necessarily met but then the only solution is π^*.

To prove (ii), suppose $\tilde{\pi} < \pi_L$. If $\hat{\pi} < \tilde{\pi}$, then clearly there are interior steady states at $\pi^*$and π_L in which normal policy is being followed. In the constrained region (where the constraint (24) is binding) we must have $\pi = \tilde{\pi}$ in a steady state. Clearly, $\pi = \tilde{\pi}$ is a steady state for $\breve{R} = \beta^{-1}\tilde{\pi}$.

To prove (iii), suppose that $\pi_t^e = \tilde{\pi}$ and c_t^e sufficiently low. Then the constraint (24) is binding, so that $\pi_t = \tilde{\pi}$ and $c_t = c_t^e(\tilde{\pi}/\beta\hat{R})^{\sigma_1}$. Since $\tilde{\pi}/\beta\hat{R} < 1$, under learning both c_t^e and c_t fall steadily over time. ■

In the last case $\tilde{\pi} < \beta\hat{R}$ there is a locally stable steady state at π^*. However, the economy can also end up in a depressionary spiral, as illustrated below. The intuition is straightforward. With pessimistic expectations and $\tilde{\pi}$ set too low, the real interest rate exceeds the subjective rate given by the discount factor, so that households want to reduce their consumption.

The following figures illustrate the different possibilities. In all three figures we set $\hat{R} = 1.0001$, so that when aggressive monetary policy is triggered the nominal interest rate is cut almost all the way to the zero lower bound. Figure 4 sets $\tilde{\pi} = 1 > \pi_L$.[11] This illustrates our recommended policy in which we set $\pi_L < \tilde{\pi} < \pi^*$. There is now a unique steady state at π^* and it is evident in the figure that it is globally stable.

In Figure 5 $\tilde{\pi}$ has been set at the intermediate value $\tilde{\pi} = 0.962$, so that we are in case (ii) of the Proposition with three steady states. The middle steady state is not E-stable but the other two are E-stable.

[11] Parameters are $A = 2.5$, $\pi^* = 1.05$, $\beta = 0.96$, $\sigma_1 = 0.95$, $\alpha = 0.75$, $\gamma = 5$, $\nu = 1.5$, $\varepsilon = 1$ and $g = 0.1$. These parameters are also used in Figures 5 and 6.

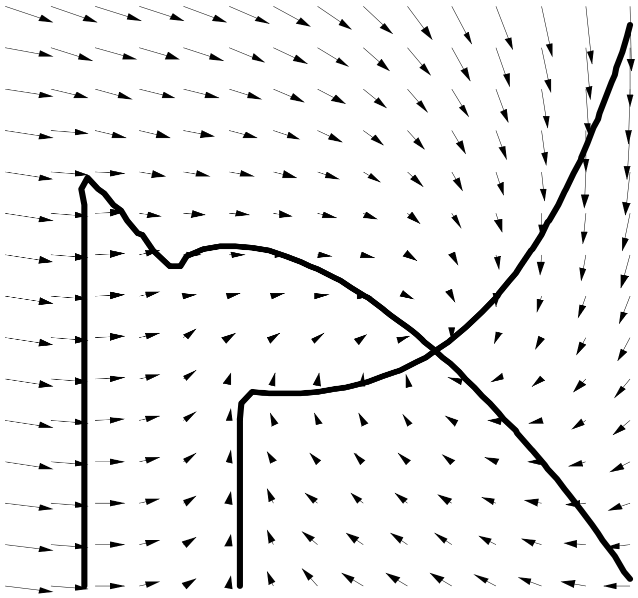

Figure 4: Unique steady state with a high value for $\tilde{\pi}$

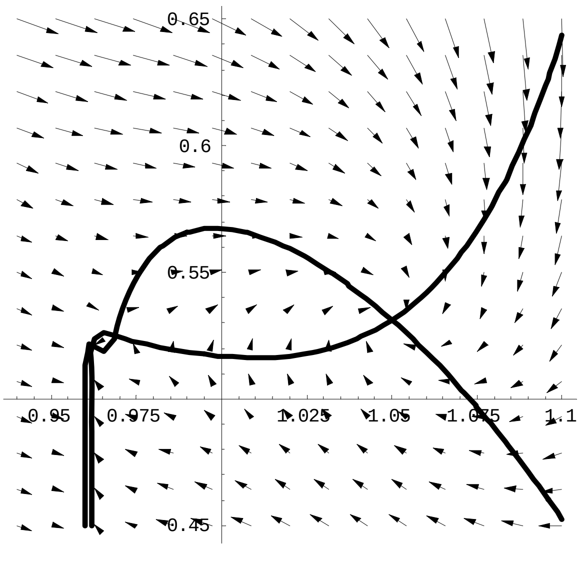

Figure 5: Case of three steady states

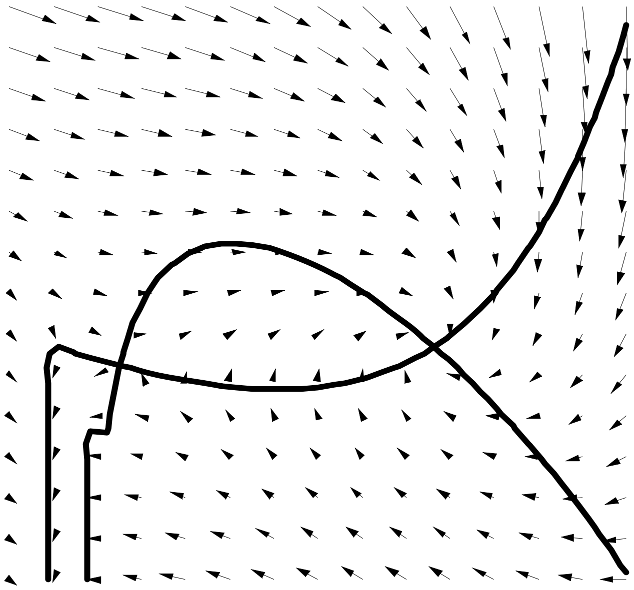

Figure 6: Two steady states and quasi-steady state

Finally Figure 6 sets $\tilde{\pi} = 0.95$. This illustrates possibility (iii) of the Proposition, in which there is a quasi-steady state and the possibility of falling c^e is present.

Our key finding is that a combination of aggressive monetary and fiscal policy to maintain a sufficiently high lower bound on inflation will eliminate the possibility of a deflationary spiral. Choosing an appropriate switch-point $\tilde{\pi}$ from normal to aggressive policy is crucial. If $\tilde{\pi}$ is not set at sufficiently high value, the economy can converge to $\tilde{\pi}$ itself with low inflation (or even deflation) and low interest rates: a type of liquidity-trap equilibrium. (This case splits, moreover, into two subcases for consumption. Either there is a steady state value for consumption or there may be only a quasi steady state with falling consumption). On the other hand, if $\tilde{\pi}$ is set at a sufficiently high value, i.e. $\tilde{\pi} > \pi_L$, there exists a unique steady state π^*, which is globally stable.

6 An Output Target for Policy?

The preceding discussion naturally raises the question whether another type of target might be used for triggering aggressive policies. Consider in particular the possibility that the policy authorities choose a minimum output target, so that policies ensure $c_t + g_t \geq \tilde{y}$ by first dropping interest rates to $\hat{R}$ and, if necessary, by raising g_t. Surprisingly, it turns out that this form of policy does not eliminate the possibility of the economy getting stuck in a liquidity trap. This is illustrated in Figure 7.

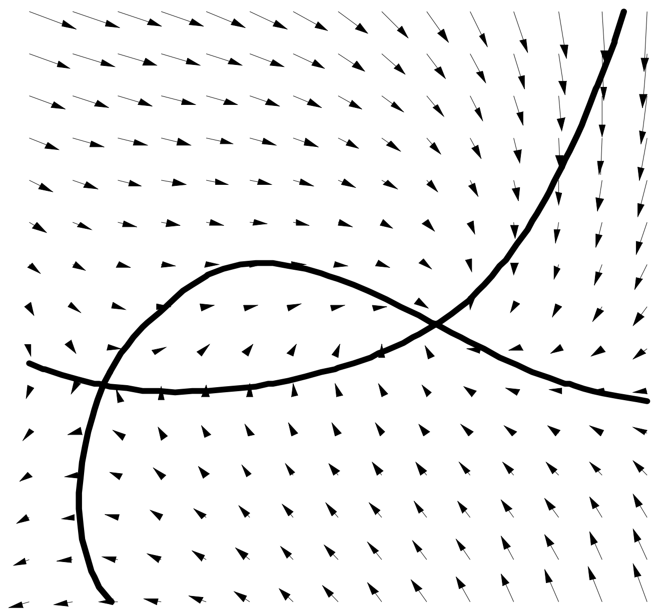

Figure 7: Learning dynamics under output targeting

Here the target $\tilde{y}$ is set at 98 percent of the high steady state level of output. It is seen that pessimistic expectations for consumption and inflation can still lead to a deflationary spiral at the bottom-left corner of the diagram. On these deflationary spiral paths consumption falls steadily after a certain point. Output is then sustained by ever increasing government spending. The intuition is that in a deflationary spiral, even at a near-zero nominal interest rate $R_t = \hat{R}$, the ex-ante real interest rate increases, which depresses private consumption. Simply maintaining output is not enough. In order to put a floor on consumption it is critical to put a floor on real interest rates, and this can only be done by stabilizing inflation.

7 Stochastic Simulations

We now illustrate our recommended policy using real-time stochastic simulations. We here assume a constant gain form of the learning rule with a small gain. Simulations confirm local convergence to the stable targeted steady state under normal policy and global convergence under our recommended policy in which when normal policy is augmented by aggressive monetary and fiscal policy if π_t threatens to fall below $\tilde{\pi} > \pi_L$.

It is beneficial to have our recommended policies in place before a collapse in expectations We illustrate how our policies work in real time, in the face of pessimistic expectations, if initially normal policies are used, and then our recommended policies are implemented after some point t_1.

Figure 8 shows consumption diverging to low values before the augmented

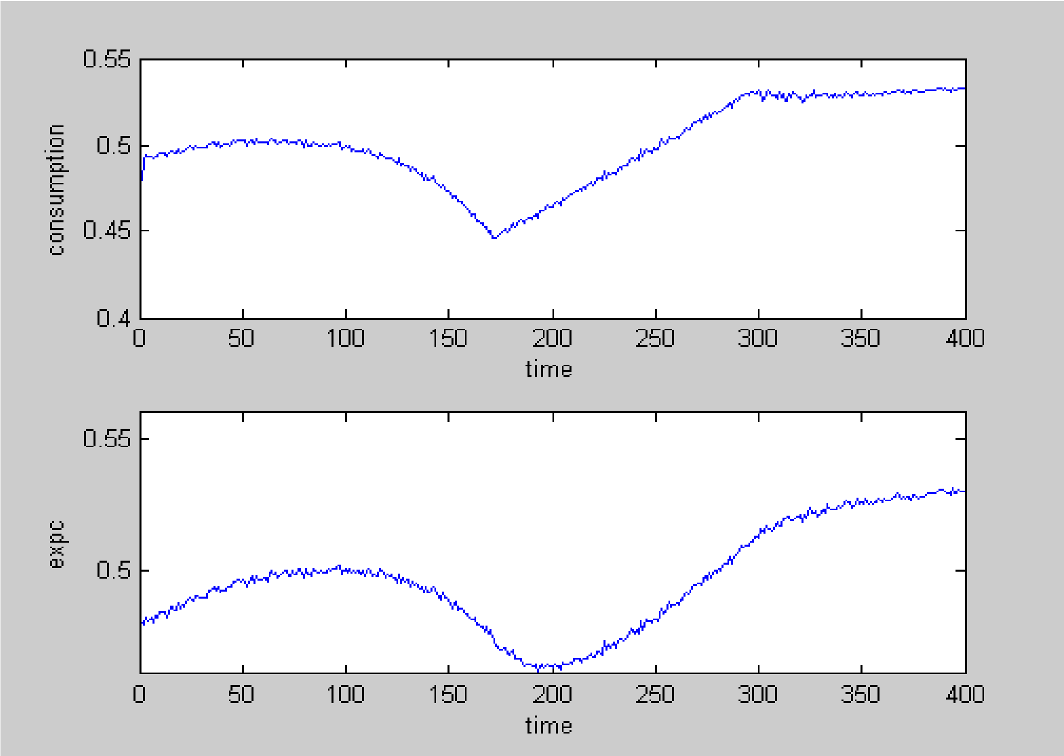

Figure 8:

policies are introduced at time $t_1 = 170$.[12] Inflation is on a steady downward trajectory when only normal policy rules are in place. Introduction of the aggressive policies at t_1 leads to a recovery of inflation and consumption to the targeted steady-state values.

Figure 9 shows the dynamics of interest rates, government spending and public debt. It is seen that interest rates falls to the floor level and debt gradually rises under the normal policy regime in which government spending is constant. At time 170, when the augmented policies are introduced, this leads to an increase in government spending and consequently a further substantial increase in debt in a short interval in time. With the new policy government spending is gradually reduced as expectations of inflation and consumption recover. This also allows debt to return gradually to the steady state. Interest rates also return to normal levels and inflation (not shown) converges towards π^*. Introduction of our policies at an earlier time avoids the worst part of stagnation. Consumption does not fall as much and returns to normal levels much earlier, and the debt level does not rise nearly as much.

[12] Parameters are $A = 1.8$, $\pi^* = 1.02$, $\beta = 0.96$, $\sigma_1 = 0.95$, $\alpha = 0.75$, $\gamma = 5$, $\nu = 1.5$, $\varepsilon = 1$, $g = 0.1$, $\hat{R} = 1.002$. Other parameters are $\phi = 1/30$, $\sigma_\theta = 0.02$, $\sigma_u = 0.000001$, $\sigma_\psi = 0.000001$, $\sigma_\eta = 0.001$, $\kappa_0 = -0.005$, $\kappa = \beta^{-1} - 1 + 0.15$, and $\chi = 0.0005$.

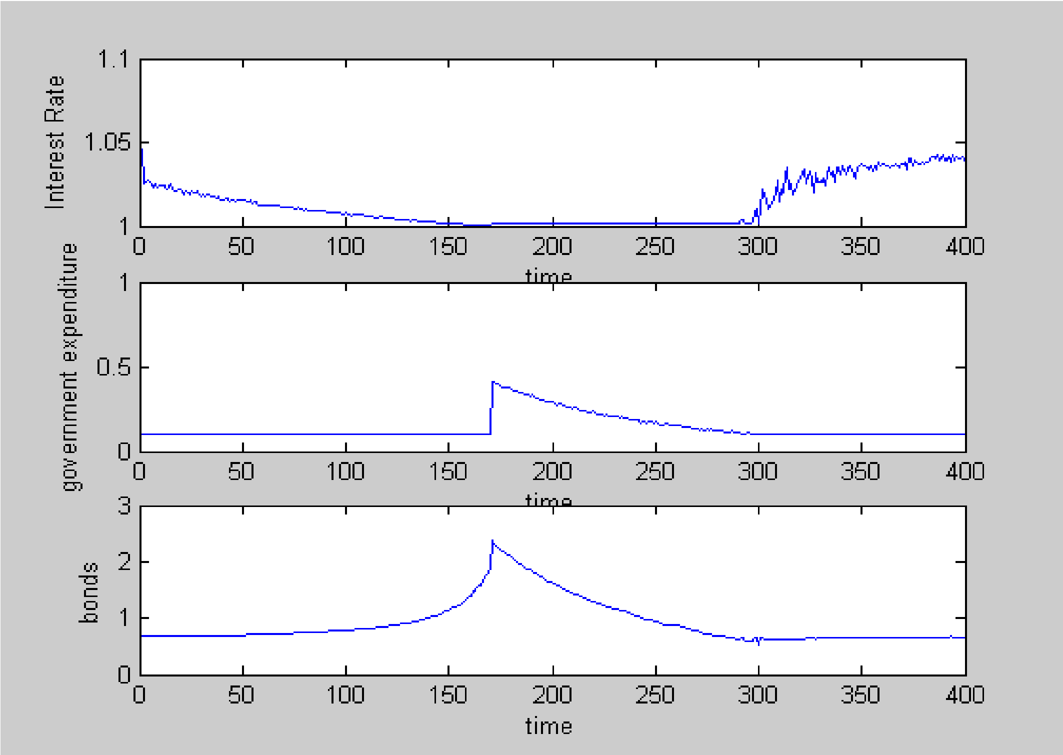

Figure 9:

8 Conclusions

The recent theoretical literature on the zero lower bound to nominal interest rates has emphasized the possibility of multiple equilibria and liquidity traps. We take these issues very seriously, but the emphasis of these models under adaptive learning is quite different and in some ways much more alarming than suggested by the rational expectations viewpoint. We have shown that under standard monetary and fiscal policy, the steady state equilibrium targeted by policymakers is locally stable. In normal times, these policies will appropriately stabilize inflation, consumption and output. However, the desired steady state is not globally stable under normal policies. A sufficiently large pessimistic shock to expectations can send the economy along a unstable deflationary spiral.

To avoid the possibility of deflation and stagnation we recommend a combination of aggressive monetary and fiscal policy triggered whenever inflation threatens to fall below an appropriate threshold. Monetary policy should immediately reduce nominal interest rates, as required, even to the zero interest floor if needed, and this should be augmented by fiscal policy if necessary. Intriguingly, targeting aggregate output in this way may not successfully reverse a deflationary spiral, but our policy combination targeting inflation at an appropriate rate will do so. When aggressive fiscal policy is necessary, this will lead to a temporary build-up of debt. However, government spending and debt will gradually return to their steady state values and early implementation of the recommended policies will mitigate the temporary run-up in debt.

A Derivations

A.1 Private sector optimization

One can show that

$$\begin{aligned}\frac{P_{t,j}}{P_t}y_{t,j} &= Y_t^{1/\nu}y_{t,j}^{1-1/\nu}\\ &= Y_t^{1/\nu}h_{t,j}^{\alpha(1-1/\nu)}\end{aligned}$$

and that firm $j's$ gross inflation can be expressed as

$$\begin{aligned}\frac{P_{t,j}}{P_{t-1,j}} &= \frac{\left(\frac{y_{t,j}}{Y_t}\right)^{-1/\nu}P_t}{\left(\frac{y_{t-1,j}}{Y_{t-1}}\right)^{-1/\nu}P_{t-1}}\\ &= \left(\frac{Y_t}{Y_{t-1}}\right)^{1/\nu}\left(\frac{h_{t,j}}{h_{t-1,j}}\right)^{-\alpha/\nu}\frac{P_t}{P_{t-1}}.\end{aligned}$$

These allow us to write the utility function in the form

$$\begin{aligned}U_{t,j} &= \frac{c_{t,j}^{1-\sigma_1}}{1-\sigma_1}+\frac{\chi}{1-\sigma_2}\left(m_{t-1,j}*\pi_t^{-1}\right)^{1-\sigma_2}-\frac{h_{t,j}^{1+\varepsilon}}{1+\varepsilon}\\ &\quad -\frac{\gamma}{2}\left(\left(\frac{Y_t}{Y_{t-1}}\right)^{1/\nu}\left(\frac{h_{t,j}}{h_{t-1,j}}\right)^{-\alpha/\nu}\frac{P_t}{P_{t-1}}-1\right)^2.\end{aligned}$$

Next, the Lagrangian can be expressed as

$$\begin{aligned}\mathcal{L} &= E_0\sum_{t=0}^{\infty}[\beta^t U_{t,j}-\beta^{t+1}\lambda_{t+1,j}[c_{t,j}+w_{t+1j}+\tau_{t,j}-ml_{t,j}\pi_t^{-1}\\ &\quad -R_{t-1}\pi_t^{-1}\left(w_{t,j}-ml_{t,j}\right)-Y_t^{1/\nu}h_{t,j}^{\alpha(1-1/\nu)}]\\ &\quad -\beta^{t+1}\mu_{t+1,j}\left(ml_{t+1,j}-m_{t,j}\right)-\beta^{t+1}\eta_{t+1,j}\left(hl_{t+1,j}-h_{t,j}\right)]\end{aligned}$$

where the notation

$$\begin{aligned}w_{t+1,j} &= m_{t,j}+b_{t,j}\\ ml_{t+1,j} &= m_{t,j}\\ hl_{t+1,j} &= h_{t,j}\end{aligned}$$

is employed. Here $w_{t,j}$, $ml_{t,j}$ and $hl_{t,j}$ are the state variables and $c_{t,j}$, $m_{t,j}$ and $h_{t,j}$ are the control variables. In addition, π_t, Y_t and Y_{t-1} are state variables, which are viewed as exogenous by each agent.

Following Chow (1996), we write the Lagrangian into the general form

$$\mathcal{L} = E_0\left[\sum_{t=0}^{\infty}\left\{\beta^t r\left(x_t,u_t\right)-\beta^{t+1}\xi_{t+1}'\left[x_{t+1}-f\left(x_t,u_t\right)-\varepsilon_{t+1}\right]\right\}\right],$$

for which the FOC's are expressed compactly as

$$\frac{\partial}{\partial u_t} r\left(x_t, u_t\right) + \beta \frac{\partial}{\partial u_t} f'\left(x_t, u_t\right) E_t \xi_{t+1} = 0$$
$$\xi_t = \frac{\partial}{\partial x_t} r\left(x_t, u_t\right) + \beta \frac{\partial}{\partial x_t} f'\left(x_t, u_t\right) E_t \xi_{t+1}.$$

Using these, the FOC's for the problem at hand are obtained as follows:
wrt $c_{t,j}$:

$$c_{t,j}^{-\sigma_1} - \beta E_t \lambda_{t+1,j} = 0, \tag{25}$$

wrt $m_{t,j}$:

$$E_t \mu_{t+1,j} = 0, \tag{26}$$

wrt $w_{t,j}$:

$$\lambda_{t,j} = \beta\left(R_{t-1}\pi_t^{-1}\right) E_t \lambda_{t+1,j}, \tag{27}$$

wrt $ml_{t,j}$:

$$\mu_{t,j} = \chi\left(ml_{t,j}\pi_t^{-1}\right)^{-\sigma_2} \pi_t^{-1} + \beta\left(\pi_t^{-1} - R_{t-1}\pi_t^{-1}\right) E_t \lambda_{t+1,j}, \tag{28}$$

wrt $h_{t,j}$:

$$\begin{aligned} -h_{t,j}^{\varepsilon} + \gamma &\left(\left(\frac{Y_t}{Y_{t-1}}\right)^{1/\nu} \frac{P_t}{P_{t-1}} \left(\frac{h_{t,j}}{hl_{t,j}}\right)^{-\alpha/\nu} - 1\right) \\ &* \frac{\alpha}{\nu}\left(\frac{h_{t,j}}{hl_{t,j}}\right)^{(-\alpha/\nu)-1} \frac{1}{hl_{t,j}} \left(\frac{Y_t}{Y_{t-1}}\right)^{1/\nu} \frac{P_t}{P_{t-1}} \\ &+ \beta Y_t^{1/\nu} \alpha\left(1 - \frac{1}{\nu}\right) h_{t,j}^{\alpha(1-1/\nu)-1} E_t \lambda_{t+1,j} + \beta E_t \eta_{t+1,j} = 0, \end{aligned} \tag{29}$$

wrt $hl_{t,j}$:

$$\begin{aligned} \eta_{t,j} &= -\gamma\left(\left(\frac{Y_t}{Y_{t-1}}\right)^{1/\nu} \frac{P_t}{P_{t-1}} \left(\frac{h_{t,j}}{hl_{t,j}}\right)^{-\alpha/\nu} - 1\right) \\ &\quad * \frac{\alpha}{\nu}\left(\frac{h_{t,j}}{hl_{t,j}}\right)^{(-\alpha/\nu)-1} \left(\frac{Y_t}{Y_{t-1}}\right)^{1/\nu} \frac{P_t}{P_{t-1}} \frac{h_{t,j}}{hl_{t,j}^2}. \end{aligned} \tag{30}$$

All firms are assumed to be identical so,

$$\frac{h_{t,j}}{hl_{t,j}} = \frac{h_{t,j}}{h_{t-1,j}} = \frac{h_t}{h_{t-1}} = \frac{Y_t^{1/\alpha}}{Y_{t-1}^{1/\alpha}}$$
$$\left(\frac{Y_t}{Y_{t-1}}\right)^{1/\nu} \left(\frac{h_{t,j}}{hl_{t,j}}\right)^{-\alpha/\nu} = 1$$
$$\left(\frac{h_{t,j}}{hl_{t,j}}\right)^{(-\alpha/\nu)-1} \left(\frac{Y_t}{Y_{t-1}}\right)^{1/\nu} \frac{h_{t,j}}{hl_{t,j}} = 1$$

(as $\eta_{t,j} = \eta_t$) this reduces equation (30) to

$$\eta_t = \frac{-\alpha\gamma}{\nu}(\pi_t - 1)\pi_t \frac{1}{h_{t-1}}. \tag{31}$$

Similarly, equation (29) can be reduced to

$$h_t^{\varepsilon} + \frac{\alpha\gamma}{\nu}(\pi_t - 1)\frac{\pi_t}{h_t} + \beta\alpha(1 - 1/\nu)\frac{Y_t}{h_t}E_t\lambda_{t+1} + \beta E_t\eta_{t+1} = 0. \tag{32}$$

Equation (25) can also be reduced to

$$c_t^{-\sigma_1} = \beta E_t \lambda_{t+1}, \tag{33}$$

so using equations (31), (32) and (33), we get a Phillips curve

$$-h_t^{1+\varepsilon} + \frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t + \alpha\left(1 - \frac{1}{\nu}\right)h_t^{\alpha}c_t^{-\sigma_1} = \beta\frac{\alpha\gamma}{\nu}E_t\left[(\pi_{t+1} - 1)\pi_{t+1}\right],$$

which is equation (7) in the main text.

As agents are homogeneous, equation (27) reduces to

$$\lambda_t = \beta\left(R_{t-1}\pi_t^{-1}\right)E_t\lambda_{t+1} \tag{34}$$

and combining equations (33) and (34) yields the consumption Euler equation

$$c_t^{-\sigma_1} = \beta R_t E_t\left(\pi_{t+1}^{-1}c_{t+1}^{-\sigma_1}\right),$$

which is equation (8) in the main text. Under the assumption that $\mu_{t,j} = \mu_t$, by combining equations (26), (28) and (33) we get the money demand equation:

$$m_t = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1 - R_t^{-1}\right)c_t^{-\sigma_1}}{E_t\pi_{t+1}^{\sigma_2 - 1}}\right)^{-1/\sigma_2},$$

which is equation (9) in the main text.

A.2 Further Properties of Steady States

We show existence of unique steady state values for c and h for a given steady state π under normal policy. Combining (11) and (12), we have the equation

$$-h^{1+\varepsilon} + (1-\beta)\frac{\alpha\gamma}{\nu}(\pi - 1)\pi + \alpha(1 - \nu^{-1})h^{\alpha}(h^{\alpha} - g)^{-\sigma_1} = 0.$$

Let $\Lambda = (1-\beta)\frac{\alpha\gamma}{\nu}(\pi - 1)\pi > 0$ and write the equation as

$$\Lambda + \alpha(1 - \nu^{-1})h^{\alpha}(h^{\alpha} - g)^{-\sigma_1} = h^{1+\varepsilon}.$$

The RHS is increasing and convex. Consider first the case $\pi \geq 1$. For $g = 0$, the LHS is increasing and concave for $\sigma_1 \leq 1$ and at $h = 0$ it is positive (or zero if $\pi = 1$), so clearly there is a unique interior solution for h. For $g = 0$, when $\sigma_1 > 1$, the LHS is decreasing with limit Λ as $h \to \infty$ so again there is a unique solution. If $g > 0$, the LHS has an asymptote at plus infinity when $h \to g^{1/\alpha}$ from above. For $h > g^{1/\alpha}$ the LHS shifts up and also

$$\frac{\partial}{\partial g}\left(\frac{\partial}{\partial h}(h^\alpha(h^\alpha - g)^{-\sigma_1})\right) = \sigma_1(h^\alpha - g)^{-\sigma_1 - 1}\left(1 - \frac{1+\sigma_1}{1 - g/h^\alpha}\right) < 0$$

for all values of σ_1, so that the preceding arguments can be extended accordingly and there is a unique interior solution.

Finally, if $\pi < 1$ (so that $\Lambda < 0$) various possibilities arise. There may be zero or two interior solutions when $g = 0$. However, for $g > 0$ and π sufficiently close to one, the argument above for the case $\pi = 1$ applies and there is a unique solution.

A.3 Proof of Proposition 3

First, $\hat{\pi}$ is clearly a steady state and by equation (8) there is no other steady state value below $\hat{\pi}$. The corresponding value $\hat{c}$ for consumption can be computed from (22).

To prove the saddle-point property, we consider the temporary equilibrium defined by the system (22) and (23) that pertains to the region where $R_t = \hat{R}$. Then we show that the determinant of the Jacobian matrix of the E-stability differential equations evaluated at that steady state is always negative for γ sufficiently small.

The temporary equilibrium equation for c_t is simply (23). The corresponding equation for π_t is obtained by solving π_t in terms of π^e_{t+1} and c_t using (22), which is a quadratic equation in π_t (the relevant solution is the larger root), and substituting (23) into the solution of the quadratic.

The determinant of the Jacobian matrix of the E-stability differential equations at the steady state $(\hat{\pi}, \hat{c})$ is a ratio of two terms, of which the denominator is always positive. The numerator is proportional to

$$-[c^{1+\sigma_1}(c+g)^{\frac{1+\varepsilon}{\alpha}}(1+\varepsilon)\nu + c^2\alpha^2(\nu-1)(\sigma_1-1) + g^2\alpha^2(\nu-1)\sigma_1 + cg\alpha^2(\nu-1)(2\sigma_1-1)].$$

This expression is increasing in σ_1, so that it maximal value obtains when $\sigma_1 = 0$. Imposing $\sigma_1 = 0$ and using (22) at the steady state the numerator can be simplified to

$$-(1+\varepsilon)[-\hat{R}(\beta-1)\beta(\beta\hat{R}-1)\gamma + (c+g)(\nu-1)] + \alpha(c+g)(\nu-1).$$

The final term $\alpha(c+g)(\nu-1)$ is dominated by the negative term $-(1+\varepsilon)(c+g)(\nu-1)$, while the first term in square brackets $-\hat{R}(\beta-1)\beta(\beta\hat{R}-1)\gamma$ can be made arbitrarily small by making γ sufficiently small. The result follows.

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