Near-Rational Exuberance

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Judgemental forecasting

- Actual macroeconometric forecasting involves pervasive add-factoring. (Reifschneider, Stockdon, Wilcox 1997).
- Example: “financial headwinds” early to mid-1990s.
- We think of judgement as a qualitative, commonly understood, economy-wide variable.
- The conventional wisdom is that judgement is beneficial.

The dark side of judgement

- Feedback opens the possibility of self-fulfilling fluctuations.
- We mostly focus on the extreme case: judgement unrelated to fundamentals.
- Results apply even with some relationship between judgement and fundamentals.

What we do

- Standard models in which expectations play an important role.
- Replace rational expectations with recursive learning: "econometric forecasts."
- Supplement econometric forecasts with judgement.
- Structure: (1) General framework, (2) Scalar case: asset pricing (3) Multivariate case: monetary policy.

Requirements for an exuberance equilibrium

- Consistent expectations equilibrium (CEE): econometricians get all of the autocorrelations right. (Hommes and Sorger 1998).
- Individual rationality on inclusion of judgemental adjustment. (Nash).
- Learnability.

Main findings

- Conditions under which exuberance equilibria exist: “Worrisomely plausible.”
- Relationship with indeterminacy and sunspot equilibria: “Sunspot-like behavior without indeterminacy.”
- Designing policy to avoid exuberance.

Environment

- Economic structure is:

$$y_t = \beta y^e_{t+1} + u_t$$

with forecasts

$$y^e_{t+1} = E^\star_t y_{t+1} + \xi_t,$$

- $E^\star_t y_{t+1}$ = econometric forecast and ξ_t = judgemental add-factor.
- We mostly focus on the case in which the fundamentals u_t and judgement ξ_t evolve independently.

Judgement

- Let η_t represent "news" about new qualitative events judged to have significant impact on the economy.
- η_t measures its expected impact on y_{t+1}.
- The forecasted future impact is

$$\frac{\partial y_{t+1+j}}{\partial \eta_t} = \psi_{t,j}, \text{ for } j = 1, 2, 3, \ldots$$

Simplification

- We think of η_t as "unique" events and the impact $\psi_{t,j}$ is in general complex.
- For *analytical simplicity only* we assume (scalar case)

$$\psi_{t,j} = \rho^j \text{ with } 0 < \rho < 1.$$

- Then

$$\xi_t = \sum_{j=0}^{\infty} \psi_{t-j,j} \eta_{t-j} = \sum_{j=0}^{\infty} \rho^j \eta_{t-j} = (1 - \rho L)^{-1} \eta_t$$

- The total judgemental adjustment in y_{t+1}^e satisfies

$$(1 - \rho L)\, \xi_t = \eta_t.$$

Perceived law of motion

- Econometric forecasters learn using recursive algorithms so that perceptions are consistent with reality.
- With judgement, ARMA(1,1) allows this (multivariate case)

$$y_t = \theta(L) v_t$$

where

$$\theta(L) = (I - bL)^{-1} (I - aL) .$$

- Macroeconometric forecast

$$E_t^\star y_{t+1} = [b\theta(L) - a]\, v_t.$$

- Add-factored, or judgementally-adjusted, forecast

$$y_{t+1}^e = E_t^\star y_{t+1} + \xi_t$$

- Use of judgement induces actual law of motion:

$$y_t = \beta y_{t+1}^e + u_t = M(L)^{-1} \beta (I - \rho L)^{-1} \eta_t + M(L)^{-1} u_t$$

where

$$M(L) \equiv I - \beta \left[b - a\theta(L)^{-1} \right]$$

Exuberance equilibrium

- CEE: Equate autocovariance generating functions of PLM and ALM. (We also use approximate versions in which the agents' PLM is $AR(p)$.)
- Rationality: Given ALM, find conditions under which $MSE\left[y^e_{t+1} - y_{t+1}\right]_i < MSE\left[E^\star_t y_{t+1} - y_{t+1}\right]_i$ for all diagonals i.
- Also require learnability (Evans and Honkapohja 2001).
- *Exuberance equilibrium* meets CEE, individual rationality, and learnability requirements.

Lemma

- *Lemma:* There exists a CEE with $b = \rho$ and $a \in [0, \rho]$.
- Note: $R = \sigma_\eta^2 / \sigma_u^2 \to \infty \implies a \to 0$ and $R = \sigma_\eta^2 / \sigma_u^2 \to 0 \implies a \to \rho$.
- Individual rationality: should an individual agent use the judgementally-adjusted forecast if all other agents do so?
- If $R = \sigma_\eta^2 / \sigma_u^2 \to \infty$, never. If $R = \sigma_\eta^2 / \sigma_u^2 \to 0$, yes when $\rho\beta > 1/2$.
- More generally: see Figure 1.

Figure 1

Figure 1. Including Judgement.

Scalar case.

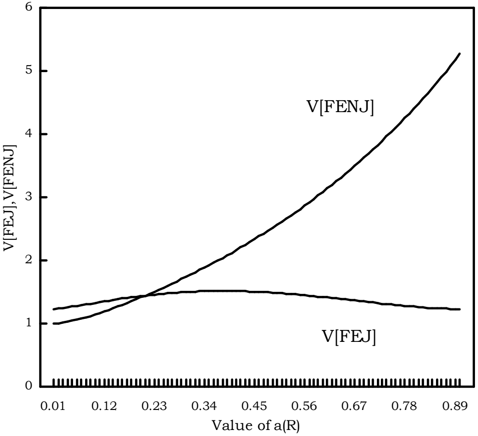

Figure drawn for ß=.9, rho=.9.

Learnability in the scalar case

- Agents use recursive maximum likelihood to estimate $ARMA(1,1)$ PLMs.
- Convergence to exact CEE.
- Approximate CEE: agents use $AR(p)$ PLM. Check E-stability condition numerically.

Theorem

- *Theorem:* Consider the univariate model with judgement and suppose that $\beta > 1/2$. Then
  - for appropriate AR(1) judgement processes there exists an exuberance equilibrium, and
  - the exuberance equilibrium has a higher asymptotic variance than the rational expectations equilibrium.

Asset pricing example

- Asset pricing example, Brock & Hommes (1998): $p^e_{t+1} + d - R_f p_t = s_t$.
- Shiller (1981): standard deviation of US stock prices relative to SD of fundamentals price lies between 5 and 13.
- Exuberance equilibria exist when ρ and β are relatively close to but less than unity. See Table 1 for excess volatilities.

Excess volatility

Table 1. Excess Volatility

	σ_y/σ_u			
	0.50	1.00	1.50	2.00
$\rho = 0.70$	1.54	2.74	–	–
$\rho = 0.80$	1.85	3.62	5.58	–
$\rho = 0.90$	2.70	5.82	9.11	12.43
$\rho = 0.95$	3.99	8.75	13.64	18.56

Table: Exuberance equilibria in the asset pricing model. A dash indicates that exuberance equilibrium does not exist. The entries in the table give one measure of the degree of excess volatility generated, namely, the ratio of the standard deviation of y to the standard deviation of u. The model can easily generate substantial excess volatility like that estimated by Shiller (1981).

Correlated judgement

- We can extend the model to allow judgement to be based on variables correlated with fundamentals:

$$\begin{aligned} y_t &= \beta y^e_{t+1} + u_t + z_t, \text{ where } y^e_{t+1} = E_t y^*_{t+1} + \xi_t \\ \xi_t &= \rho \xi_{t-1} + \eta_t \\ \eta_t &= f z_t + \varepsilon_t, \text{ where } f \geq 0. \end{aligned}$$

- The previous results go through in this model: for $\beta\rho > 1/2$ there are exuberance equilibria provided σ^2_ε, f are not too large.
- In this case exuberance equilibria can in part be interpreted as an over-reaction to fundamentals.

Discussion

- Our exuberance equilibrium is near-rational but not fully rational. There are two ways in which we impose assumptions in which deviate from full rationality.
    - The judgement process ξ_t is not directly available to econometric forecasters, who must rely on the observables y_t. This seems realistic because the ξ_t represents "unique" events.
    - The incentive condition is dichotomous. This also seems realistic and tests of whether "all" of ξ_t should have been included would (often) have low power.
    - Attempts to "downweight" judgement by averaging across past unique events do not make sense.

New Keynesian macro

- Canonical New Keynesian model (Woodford 2003):

$$\begin{aligned} x_t &= x^e_{t+1} - \sigma^{-1}\left[r_t - \pi^e_{t+1}\right] + \tilde{u}_{x,t}, & (1) \\ \pi_t &= \kappa x_t + \delta \pi^e_{t+1} + \tilde{u}_{\pi,t}, & (2) \\ r_t &= \varphi_\pi \pi_t + \varphi_x x_t. & (3) \end{aligned}$$

- Condition for determinacy and learnability is known to be

$$\kappa\left(\varphi_\pi - 1\right) + (1-\beta)\,\varphi_z > 0.$$

NK system rewritten

- Define $y_t = [x_t, \pi_t]'$ and $\tilde{u}_t = [u_{x,t}, u_{\pi,t}]'$.
- Write system as
$$y_t = \beta y^e_{t+1} + C\tilde{u}_t$$
- CEE and learnability: assume a $VAR(p)$, $p = 3$ PLM and calculate E-stability condition for approximate CEE.

Incentives to include judgement

- Calculate numerically the relevant parts of the mean-squared forecast error matrices $M(0)$ and $M(1)$ without and with judgement.
- The CEE exhibits *exuberance* if all diagonal elements of $M(0) - M(1)$ are positive. (*strong exuberance* if $M(0) - M(1)$ is a positive definite matrix).
- CEE is *indefinite* if diagonals vary in sign and exhibits *non-exuberance* if diagonals are negative.

Calibration

- Woodford (2003) calibration $\sigma = 0.157, \kappa = 0.024, \delta = 0.99$.
- Exuberance: assume $\rho = \text{diag}(0.99, 0.95)$ and $\Sigma_\eta = \text{diag}(0.0035, 0.0035)$.
- Shocks: assume $\Sigma_{\tilde{u}} = \text{diag}[1.1, 0.03]$.

Results

- Initial example: $\varphi_\pi = 1.05$ and $\varphi_x = 0.05$. (Induces both determinacy and learnability.)
- The matrix $M(0) - M(1)$ is computed to be

$$\begin{pmatrix} 0.0219 & -0.0195 \\ -0.0195 & 0.0209 \end{pmatrix},$$

  and it is pd. => (strong) exuberance.
- $\varphi_\pi \rightarrow 1.5$ and $\varphi_x = 0.1 \implies$ no exuberance. More generally, see Figures 2 and 3.

Figure 2

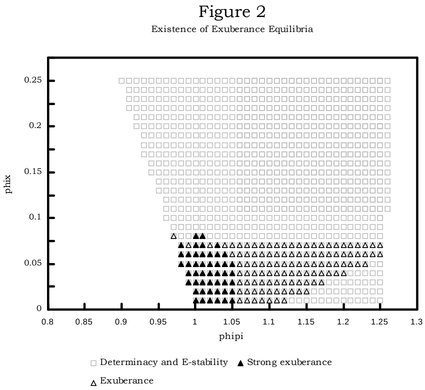

Figure: Exuberance equilibria in the New Keynesian model. Open boxes indicate points where the REE is determinate. Triangles indicate points where exuberance equilibria exist.

Figure 3

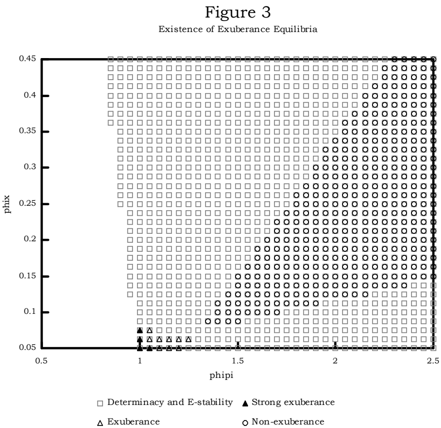

Figure: A sufficiently aggressive Taylor-type policy is associated with non-exuberance, denoted by open circles.

Forward-looking monetary policy rule

Figure 4
Exuberance with Forward-Looking Rules

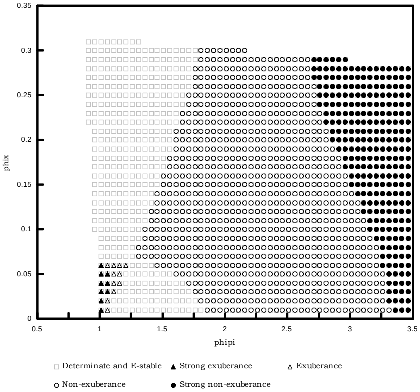

Figure: Sufficiently aggressive policy is again associated with non-exuberance when the policy rule is forward-looking.

Optimal monetary policy rules

- Optimal discretionary policy as in Evans and Honkapohja (2003):

$$r_t = \varphi_\pi^\star \pi_{t+1}^e + \varphi_x^\star x_{t+1}^e + \varphi_{u,x}^\star \tilde{u}_{x,t} + \varphi_{u,\pi}^\star \tilde{u}_{\pi,t}.$$

  where $\varphi_x^\star = \varphi_{u,x}^\star = \sigma$ and $\varphi_u^\star = \delta^{-1}\left(\varphi_\pi^\star - 1\right)$. Optimal $\varphi_\pi^\star$ depends on α, the output gap weight.
- $\alpha \to 0$ (an inflation hawk) implies $\varphi_\pi^\star = 1 + \sigma\delta\kappa^{-1} \approx 7.47$. $\alpha \to \infty$, (an inflation dove) yields $\varphi_\pi^\star \to 1$.

More on optimal rules

- For $\varphi_\pi^\star \in (1, \bar{\varphi}_\pi)$ the equilibrium is indefinite. For $\varphi_\pi^\star \in (\bar{\varphi}_\pi, 7.47)$, non-exuberance.
- The cutoff is $\bar{\varphi}_\pi \approx 1.557$. Policy must have a sufficiently small α. $\varphi_\pi^\star = 1.557$ corresponds to $\alpha \approx 0.00612$.

Conclusion

- Add-factoring is a pervasive feature of macroeconometric forecasts in industrialized economies.
- We study an "exuberance equilibrium" concept.
- Agents may be tempted to include judgemental adjustments if all others do so.
- Sunspot-like behavior in determinate economies.
- A new danger for policymakers.