

## Economics 503 – Microeconomic Theory 1

*Homework No 5. Never due.*

1. A Von Neumann model is given by two  $n \times m$  matrices  $A$  and  $B$  that satisfy the following assumptions
  - a.1) For each  $i$  in the rows of  $B = [b_{ij}]$ , there is a  $j$  where  $b_{ij} > 0$ . We call  $B$  the output matrix and this hypothesis implies that all goods are producible.
  - a.2) For each  $j$  in the columns of  $A = [a_{ij}]$ , there is some  $i$  where  $a_{ij} > 0$ . We call  $A$  the input matrix and this hypothesis implies that there is no free lunch.
  - a.3) Let  $\bar{\sigma} = \max \sigma$  such that  $(B - \sigma A)x \geq 0$  for some  $x \geq 0, x \neq 0$ . If  $(B - \bar{\sigma}A)x \geq 0, x \geq 0, x \neq 0$  then  $Bx > 0$ .

Let  $x$  be an  $m$ -vector of activity levels for the  $m$  industries, and  $p$  an  $n$ -vector of prices for the  $n$  goods. Let  $\sigma$  be a positive real number. A triple  $(\sigma^*, x^*, p^*)$  is an equilibrium for a Von Neumann economy if ...

- Try completing the definition of equilibrium using the intuitive ideas that, in equilibrium, the economy must be able to sustain itself, profits must not be positive and value of output must be positive.
  - Prove an equilibrium exists.
  - If you cannot prove an equilibrium exists, derive as many properties of the Von Neumann equilibrium as you can. Use simple  $2 \times 3$  cases to gather intuition.
2. Solve exercises 20.C. 4-5 and 6
  3. Solve exercises 20.E. 3 and 4
  4. Use our assumptions for the standard production cone, or technology cone,  $Y$ , to show that, for every vector of inputs  $x$  the set of feasible outputs  $\Gamma(x) = \{y \in R^N, \text{ such that } (x, y) \in Y\}$  is bounded. Then show it is closed.
  5. Use the same set of assumptions to prove that “supporting prices” must be non negative. In other words, that in a production economy with a technology set such as the one we have discussed in class, the prices of all commodities that are produced in positive amount are non negative.

6. Try replicating the monotonicity of optimal paths argument for the two sector economy we studied in class, where does it fail? Can you figure out an assumption on the PPF such that the optimal path is still monotone? If you find it, can you then translate it into an assumption about the two underlying production functions?
7. Solve exercises 15.D.1 and 2, and 15.B.5-6.
8. Solve exercise 16.G.5
9. Solve exercise 19.H.6

All numbered exercises are from Mas Colell, Whinston and Green.