

Econ 4111
Professor John Nachbar
March 24, 2010

The Law of Demand

1 Introduction

This is a survey of the Law of Demand (LOD) in static (or finite horizon) economies. It is informal but comprehensive.

Whether LOD holds is important for uniqueness and stability of competitive equilibrium (e.g., Keisler (1996); Anderson, Granat, Plott, and Shimomura (2004)) and for comparative statics (e.g., Nachbar (2002)). The overall tone of the theory presented here, culminating in the DMS theorem in Section 3.5, is negative: LOD could fail dramatically. Casual observation suggests that LOD may, nevertheless, hold, since most markets appear to be more or less stable and prices generally move as one would expect (many would argue that intertemporal markets, financial markets in particular, are an exception). Whether LOD actually holds, however, is an open question, since it is difficult to construct an empirical test that is both practical and theoretically rigorous. And if LOD does hold, it is not clear whether this is simply good luck or reflects some unknown underlying property of competitive economies.

I first consider LOD for fixed wealth demand, which is the form of demand familiar from undergraduate micro. Relative to undergraduate micro, there are two novelties here. First, I consider a generalized version of LOD, where the generalization is that more than one price can change at the same time. Second, I explicitly analyze aggregate demand, rather than implicitly assume that the analysis for individual demand carries over to aggregate demand. Both of these complications are needed for the analysis of market stability. Having analyzed LOD for fixed wealth demand, I then discuss LOD for endogenous wealth demand (for example, income depends on wages), which brings additional complications.

Much of the material contained here is covered in more detail in Mas-Colell, Whinston, and Green (1995), chapters 2, 4, and 17.

2 Fixed Wealth Demand

2.1 The Own Price LOD and Giffen Goods.

By the *own price LOD* I simply mean that if the price of good 1 goes up then the quantity of good 1 demanded goes down. In differential form,

$$\frac{d\phi_n^i}{dp_n}(p^*, m^{i*}) < 0.$$

where the i superscript indicates that this is consumer i . By the Slutsky decomposition, with $c^{i*} = v^i(p^*, m^{i*})$,

$$\frac{d\phi_n^i}{dp_n}(p^*, m^{i*}) = \frac{dh_n^i}{dp_n}(p^*, c^{i*}) - \frac{d\phi_n^i}{dm^i}(p^*, m^{i*})x_n^{i*}.$$

Since

$$\frac{dh_n^i}{dp_n}(p^*, c^{i*}) < 0,$$

a sufficient condition for LOD to hold is that the good is normal:

$$\frac{d\phi_n^i}{dm^i}(p^*, m^{i*}) > 0.$$

Conversely, a necessary condition for the own price LOD to fail is that the good is inferior. If the own-price LOD fails for some (p^*, m^{i*}) then the good is usually called a *Giffen good*. Giffen utility is utility that gives rises to a Giffen good. These terms are in honor of the British statistician Robert Giffen. Alfred Marshall credited Giffen with spotting an LOD failure in the demand for bread; Marshall (1920).

In thinking about Giffen goods, keep in mind that the budget constraint forces a crude form of compliance with LOD: as $p_n \rightarrow \infty$, demand for good n must go to zero. Theory does not provide any general result for the opposite case, in which $p_n \rightarrow 0$. But if preferences are strongly monotone then $p_n \rightarrow 0$ implies $x_n^i \rightarrow \infty$. Assuming monotonicity, then, the graph of demand for a Giffen good is *Z*-shaped (with price on the horizontal axis, as usual). In particular, for a Giffen good, LOD fails only for a proper subset of prices.

2.2 The Generalized LOD

Naively, one might guess that when more than one price changes at the same time, then the generalized LOD calls for demand to increase for any good where the price goes down and to decrease for any good where the price goes up. This version of LOD does, in fact, hold for demand generated by Cobb-Douglas utility.

The strongest version of LOD one can hope for in general is that the demand and price vectors are negatively related in the following sense. If $\Delta p = \hat{p} - p^*$ is the change in price and $\Delta x = \hat{x}^i - x^{i*}$ is the associated change in demand, then the generalized LOD requires that

$$\Delta p \cdot \Delta x < 0.$$

Geometrically, this says that the vectors Δp and Δx are at least 90° apart. This condition allows some Δx_n to be negative even if Δp_n is negative but says (I'm being loose) that on average, price drops are associated with quantity increases. The generalized LOD, although somewhat weaker than what might hope for, is sufficient to guarantee stability.

For small Δp , Δx is approximately $D_p\phi^i(p^*, m^{i*})\Delta p$. So $\Delta p \cdot \Delta x$ is approximately

$$\Delta p \cdot [D_p\phi^i(p^*, m^{i*})\Delta p].$$

To simplify notation slightly, I write w rather than Δp . Then the differential version of the generalized LOD is that for any $w \in \mathbb{R}^N$, $w \neq 0$,

$$w'D_p\phi^i(p^*, m^{i*})w < 0.$$

The generalized LOD is sometimes called *monotonicity*, but I prefer LOD, first because it is descriptive (this really is the Law of Demand), and second to avoid confusion with other forms of monotonicity, such as preference monotonicity.

If only the price of good n changes then $w = (0, \dots, 0, w_n, 0, \dots, 0)$. In this case

$$w'D_p\phi^i(p^*, m^{i*})w = w_n^2 \frac{\partial \phi_n^i}{\partial p_n}(p^*, m^{i*}).$$

Since $w_n^2 > 0$, this means that the generalized LOD implies the own price LOD as a special case.

Informally, the generalized LOD requires that $D_p\phi^i(p^*, m^{i*})$ be “negative definite.” I’m using quotes because I reserve the term negative definite for symmetric matrices and $D_p\phi^i(p^*, m^{i*})$ is typically not symmetric (because the wealth effect term from the Slutsky decomposition is typically not symmetric). But the idea is the same: the generalized LOD holds if $D_p\phi^i(p^*, m^{i*})$ has a negative diagonal (the own price LOD holds) and the off diagonal terms (the cross price effects) are small in absolute value relative to the diagonal.

2.3 LODC

Although LOD may fail for some price change vectors w (e.g., Giffen Goods), LOD cannot fail for *every* w .

Let $T_{x^{i*}}$ denote the set of w such that $w \cdot x^{i*} = 0$. (In general, T_x is the set of w such that $w \cdot x = 0$.) To interpret, suppose $\hat{p} \cdot x^{i*} = p^* \cdot x^{i*}$, meaning that the price change does not alter the cost of the original x^{i*} . If $\hat{p} \neq p^*$ then some prices have to go up while others go down. Let $w = \hat{p} - p^*$. Then w , the vector of price changes, is in $T_{x^{i*}}$.

For any $w \in T_{x^{i*}}$, $w \neq 0$, from the Slutsky decomposition,

$$\begin{aligned} w'D_p\phi^i(p^*, m^{i*})w &= w'D_ph^i(p^*, c^{i*})w - w'D_{m^i}\phi^i(p^*, c^{i*})x^{i*'}w \\ &= w'D_ph^i(p^*, c^{i*})w < 0. \end{aligned}$$

The equality comes from the assumption that $w \in T_{x^{i*}}$ implies $x^{i*'}w = 0$. The inequality comes from the fact that if $w \in T_{x^{i*}}$ then w is not collinear with p^* (since $p^* \cdot x^{i*} = m^{i*}$ whereas $w \cdot x^{i*} = 0$), in which case $w'D_ph^i(p^*, c^{i*})w < 0$.

Thus $D_p\phi^i(p^*, m^{i*})$ obeys a weak version of the generalized LOD, a version that holds for all $w \in T_{x^{i*}}$, $w \neq 0$. This weak version is called the *Law of Demand for Compensated Price Changes*, or LODC. The set $T_{x^{i*}}$ is an $N - 1$ dimensional subspace of \mathbb{R}^N . By continuity, $w'D_p\phi(p^*, m^{i*})w < 0$ must hold on an open subset of \mathbb{R}^N containing $T_{x^*} \setminus \{0\}$, even if preferences are Giffen.

LODC is essentially equivalent to the Weak Axiom of Revealed Preference (WA); Kihlstrom, Mas-Colell, and Sonnenschein (1976). I won't go through the argument in detail. The difficult direction is in showing that LODC implies WA. Here is a proof that the WA implies the (discrete) LODC.

Fix $\hat{m}^i = m^{i*} = m^i$. Suppose that WA holds and suppose that $\hat{p} \cdot x^{i*} = p^* \cdot x^{i*}$, $\hat{p} \neq p^*$. Then x^{i*} is budget feasible for (\hat{p}, m^i) , which means that \hat{x}^i is revealed (strictly) preferred to x^{i*} . Since $B(p^*, m^i) \neq B(\hat{p}, m^i)$, it must be that $x^{i*} \neq \hat{x}^i$. Hence x^{i*} cannot be revealed preferred to \hat{x}^i , which means that $\hat{x}^i \notin B(p^*, m^i)$, hence

$$p^* \cdot \hat{x}^i > m^i.$$

Then

$$\begin{aligned} \Delta p \cdot \Delta x &= (\hat{p} - p^*) \cdot (\hat{x}^i - x^{i*}) \\ &= (\hat{p} - p^*) \cdot \hat{x}^i - (\hat{p} - p^*) \cdot x^{i*} \\ &= (\hat{p} - p^*) \cdot \hat{x}^i \\ &= \hat{p} \cdot \hat{x}^i - p^* \cdot \hat{x}^i \\ &< 0 \end{aligned}$$

where the third inequality comes from the assumption that $\hat{p} \cdot x^{i*} = p^* \cdot \hat{x}^i$ and the inequality comes from the fact that, by the above WA argument, $\hat{p} \cdot \hat{x}^i \leq m < p^* \cdot \hat{x}^i$.

In addition to LODC, demand must obey one other weak form of LOD. Differentiating Walras's Law, $p \cdot \phi(p, m) = m$ reveals that

$$p^{*'} D_p\phi(p^*, m^{i*})p^* = -m^{i*} < 0.$$

In words, if prices increase, but wealth stays the same, then consumption must go down. Note that $p^* \notin T_{x^{i*}}$ (since $p^* \cdot x^{i*} = m^{i*} > 0$); thus this is a new addition to set of directions in which LOD must hold, even if preferences are Giffen. By continuity, LOD must hold on an open set containing the non-zero vectors that are collinear with p^* .

2.4 Sufficient Conditions for the Full LOD

If every good is normal then, as discussed in Section 2.1, the diagonal of $D_p\phi(p^*, m^{i*})$ is negative. This is not sufficient in general to guarantee the full LOD, for somewhat the same reason that having a negative diagonal is not sufficient to guarantee negative definiteness.

One relatively easy sufficient condition for the full LOD is homotheticity. If u is homothetic (CES, for example), then demand must be of the form

$$\phi(p, m) = \gamma(p)m$$

for some function $\gamma : \mathbb{R}_{++}^N \rightarrow \mathbb{R}^N$. The Slutsky wealth effect term is then

$$\begin{aligned} w' D_m \phi(p^*, m^{i*}) x^{i*'} w &= w' \gamma(p^*) [\gamma(p^*) m^{i*}]' w \\ &= m^{i*} [w \cdot \gamma(p^*)]^2 \\ &\geq 0. \end{aligned}$$

Moreover, $w \cdot \gamma(p^*) = 0$ implies w is not collinear with p^* (since $p^* \cdot x^{i*} = p^* \cdot \gamma(p^*) m^{i*} = m^{i*} \neq 0$). So from Slutsky and the fact that $w' D_p h(p^*, c^{i*}) w < 0$ for any w not collinear with p^* , LOD holds. Notice, incidentally, that if u is homothetic then $D_p \phi(p^*, m^{i*})$ is symmetric, since the wealth term is symmetric.

2.5 Market Demand

Given individual demand functions ϕ^i , market demand is give by,

$$\bar{\phi}(p, m) = \sum_{i=1}^I \phi^i(p, m^i).$$

where

$$m = (m^1, \dots, m^I).$$

For many applications, only market, rather than individual, demand matters. For example, prices are determined by market demand. So we need to know whether market demand satisfies LOD.

An excellent, although now somewhat old, survey of aggregation issues, for both fixed and endogenous income demand, is Shafer and Sonnenschein (1982).

2.6 Aggregation I: LOD Aggregates

If each $\phi^i(p^*, m^{i*})$ satisfies LOD then market demand satisfies LOD, since

$$w' D_p \bar{\phi}(p^*, m^*) w = w' \left(\sum_{i=1}^I D_p \phi^i(p^*, m^{i*}) \right) w = \sum_{i=1}^I w' D_p \phi^i(p^*, m^{i*}) w < 0.$$

Thus, the individual LOD *aggregates*.

2.7 Aggregation II: LODC May Not Aggregate

For market demand, LODC holds iff

$$w' D_p \bar{\phi}(p^*, m^*) w < 0$$

for any $w \in T_{\bar{x}^*}$, $w \neq 0$, where

$$\bar{x}^* = \bar{\phi}(p^*, m^*) = \sum_{i=1}^N x^{i*},$$

(recall that $T_{\bar{x}^*} = \{w \in \mathbb{R}^N : w \cdot \bar{x}^* = 0\}$). The market LODC would follow from the individual LODC if it were true that $w \cdot \bar{x}^* = 0$ implies $w \cdot x^{i*} = 0$ for all i . Unfortunately, however, this is not true.

Explicitly, each T_{x^i} is an $N - 1$ dimensional subspace of \mathbb{R}^N . If x^1 and x^2 are independent then $T_{x^1} \cap T_{x^2}$ is $N - 2$ dimensional. For example, if $N = 3$ then T_{x^i} is 2-dimensional (a plane) and $T_{x^1} \cap T_{x^2}$ is 1-dimensional, a line in \mathbb{R}^3 . Each time I add a consumer, the dimension of $\cap_{i=1} T_{x^i}$ drops by one, assuming that the x^i are independent. Therefore, while $w \cdot \bar{x} = 0$ holds for an $N - 1$ dimensional set of w , $w \cdot x^i = 0$ can hold for every i only on an $N - I$ dimensional set, still assuming independence of the x^i . If $I \geq N$, and there are N independent x^i , then there are *no* w with the property that $w \cdot x^i = 0$ for every i . Note also that large N is precisely the case in which it is most plausible that markets are at least approximately competitive.

From Slutsky, the implication of all this is that as I add consumers to the economy, there are fewer and fewer vector price changes w for which there are zero wealth effects at the market level. This does not necessarily mean that the market wealth effects are bad. The individual, and hence market, wealth effects could actually reinforce LOD, as in the case of homothetic utility. But market wealth effects *could* be bad, meaning that LOD could easily fail for very large sets of w .

LOD cannot, however, fail for all w , even if $I \geq N$ and even if wealth effects are arbitrarily bad. Recall from Section 2.3 that

$$p^{*i} D_p \phi^i(p^*, m^{i*}) p^* < 0,$$

for every i . By continuity, LOD holds for market demand for an open set of w containing p^* . Again, the intuition is that if all prices increases, while wealth remains constant, then consumption must go down.

2.8 Aggregation III: Some Good News.

The point of section 2.7 was that market demand could be more badly behaved than individual demand. The point of this section is the reverse: market demand can be better behaved than individual demand, under the right circumstances.

One intuition, going back to Becker (1962), is that if individual demands (a) satisfy Walras's Law and (b) are sufficiently dispersed, then market demand will look

approximately Cobb-Douglas. For example, suppose that there are two goods, prices are $p_1 = 2$ and $p_2 = 1$, and everyone has the same wealth (to make the intuition more transparent). Suppose that there are a large number of consumers (a continuum) and that demands are distributed uniformly over the budget line. Then the average demand (the appropriate version of market demand if there are a continuum of consumers) will be the midpoint of the budget line, namely $(m/4, m/2)$, which is the demand of a Cobb-Douglas consumer at these prices, with wealth m and coefficients $(1/2, 1/2)$. A striking aspect of this observation is that it does not require individual demand to be “rational;” in particular, individual demand need not even satisfy WA. Modern variants on this idea are Grandmont (1992) and Quah and Giraud (2003).

Another strand of this literature originates with Hildenbrand (1983). Suppose that there are a large number of consumers (a continuum), with identical preferences but with wealth distributed uniformly on the interval $[0, M]$. Then market demand obeys LOD. Notice that the preferences could be Giffen. One intuition underlying this result is that the own-price LOD must hold at least weakly for the very poor. In the extreme case that income drops from $m > 0$ down to zero, demand cannot increase for any good: every good is weakly normal. Recall that the own-price demand curve is drawn for a fixed wealth level; change the wealth level and the own-price demand curve changes. In the case of a Giffen good, even if the own-price demand function is upward sloping at p_n^* for a particular wealth level, it will be downward sloping at p_n^* if wealth drops by enough. Loosely speaking, the Hildenbrand (1983) result verifies that if the wealth distribution is uniform then the good LOD properties of the very poor translate into good LOD properties of average demand. As the above intuition suggests, the result extends to wealth distributions that are decreasing (implying that the very poor occupy a large share of the population). The result does not extend, however, to distributions with a hump, which is characteristic of actual wealth distributions; see Hardle, Hildenbrand, and Jerison (1991). The Hildenbrand (1983) result thus does not appear to be directly relevant for the analysis of actual economies. It remains, however, a provocative indication of the power of aggregation to yield LOD even under conditions that superficially seem unpromising.

3 Endogenous Wealth Demand

3.1 Overview

The analysis of LOD with endogenous wealth differs from the previous analysis in two ways.

First, the fact that wealth is endogenous adds a term to the Slutsky equation and this, in turn, has some drastic implications. For example, if wealth is endogenous then own-price demand can be upward sloping even if the good is normal. You may have encountered backward bending labor supply curves in an undergraduate micro

class; this is essentially the same phenomenon.

Second, in endogenous wealth environments, LODC alone is sufficient for applications like uniqueness and stability of equilibrium. Note that LODC holds automatically if there is a single consumer, whereas the full LOD need not.

3.2 Exchange Economies

I focus on exchange economies, meaning economies with no production. The analysis for production economies is different, in ways that I discuss in Section 3.7, but it remains true that the WA/LODC is sufficient for many applications, so much of the discussion to follow extends to the production case.

In an exchange economy, each consumer i has an initial endowment of goods $e^i \in \mathbb{R}_{+\setminus 0}^N$ and has wealth equal to the value of that endowment

$$m^i = p \cdot e^i.$$

Under certain conditions, this formulation is equivalent to a more descriptively realistic formulation that explicitly allows for the passage of time and for the purchase and sale of financial assets, but I will not elaborate on this.

Define endogenous wealth demand by

$$f^{Di}(p, e^i) = \phi^i(p, p \cdot e^i).$$

Then, by the Chain Rule, Slutsky becomes

$$\begin{aligned} D_p f^{Di}(p^*, e^{i*}) &= D_p h(p^*, c^{i*}) - D_m \phi^i(p^*, m^{i*})(x^{i*} - e^{i*})' \\ &= D_p h(p^*, c^{i*}) - D_m \phi^i(p^*, m^{i*})z^{i*'}, \end{aligned}$$

where

$$z^i = f^{Di}(p, e^i) - e^i,$$

gives i 's *net trade*. If $z_n^{i*} > 0$ then i buys z_n^{i*} in good n to supplement her endowment. Conversely, if $z_n^{i*} < 0$ then she sells $|z_n^{i*}|$ of her endowment of good n and consumes whatever is left.

3.3 LODC for Endogenous Wealth Demand

By the Slutsky equation for endogenous wealth demand, for any $w \in T_{z^{i*}}$,

$$\begin{aligned} w' D_p f^{Di}(p^*, e^i) w &= w' D_p h^i(p^*, c^{i*}) w - w' D_m \phi^i(p^*, m^{i*}) z^{i*'} w \\ &= w' D_p h^i(p^*, c^{i*}) w \\ &\leq 0. \end{aligned}$$

Note that I have claimed only a weak inequality. The reason is that $p^* \in T_{z^{i*}}$ ($p^* \cdot z^{i*} = 0$ follows from Walras's Law) and hence

$$D_p f^{Di}(p^*, e^i) p^* = D_p h^i(p^*, c^{i*}) p^* = 0.$$

For example, a doubling of all prices also doubles wealth and therefore has no effect on demand. It follows that, for endogenous wealth demand,

$$w' D_p f^{Di}(p^*, e^i) w = 0.$$

for any w collinear with p^* . (In contrast, recall that if wealth is fixed then $w' D_p \phi^i(p^*, m^{i*}) w < 0$ for any w collinear with p^* .)

Thus, in endogenous wealth environments, the correct form of LODC is that

$$w' D_p f^{Di}(p^*, e^i) w \leq 0.$$

for any $w \in T_{z^{i*}}$, with

$$w' D_p f^{Di}(p^*, e^i) w < 0,$$

except if $w = 0$ or w is collinear with p^* . Again, LODC is essentially equivalent to WA.

3.4 Market Demand

For endogenous wealth demand, market demand is

$$\bar{f}^D(p, e) = \sum_i f^{Di}(p, e^i),$$

where $e = (e^1, \dots, e^I)$.

3.5 Aggregation I: LODC Could Fail

Let

$$\bar{z} = \sum_{i=1}^I z^i = \sum_{i=1}^I x^i - e^i.$$

LODC holds for market demand iff

$$w' D_p f^{Di}(p^*, e) w \leq 0.$$

for any $w \in T_{\bar{z}}$, with

$$w' D_p f^{Di}(p^*, e) w < 0,$$

except if $w = 0$ or w is collinear with p^* .

As discussed in Section 2.7, for fixed wealth demand the income effect terms of individual consumers could aggregate badly. For endogenous wealth demand,

the situation is, if anything worse. If $I \geq N$, then the standard assumptions on preferences imply only that \bar{f}^D (a) is differentiable, (b) is homogeneous of degree zero in p , and (c) satisfies the endogenous wealth version of Walras's Law (namely that $p \cdot \bar{f}^D(p, e) = p \cdot \bar{e}$). This bleak result is known as the DMS theorem, in honor of Debreu, Mantel, and Sonnenschein, who independently wrote a series of papers on this topic.¹ The Mantel (1976) version of the result is particularly striking because it holds even if preferences are homothetic.

3.6 Aggregation II: Sufficient Conditions for LODC

Since LODC is equivalent to the WA, a sufficient, but not necessary, condition for LODC is for market demand to look as if it were generated by a single consumer, a *representative consumer*, with endowment \bar{e} .

Here are four conditions, roughly in order of decreasing restrictiveness, that are sufficient to guarantee existence of a positive representative consumer.

1. There is literally just one consumer.
2. There are I identical consumers (same preferences and same endowments). The representative consumer has utility $\bar{u}(\bar{x}) = u(\bar{x}/I)$, where u is the utility function common to all consumers.
3. There are I consumers with identical homothetic preferences. The representative consumer has utility $\bar{u}(\bar{x}) = u(\bar{x})$, where u is the utility function common to all consumers.
4. There are I consumers with homothetic preferences (possibly *not* identical) and collinear endowments: for every i there is an α^i such that

$$e^i = \alpha^i \bar{e}.$$

One can show that the representative consumer has utility $\bar{u}(\bar{x})$ equal to the solution to

$$\max_x \frac{\prod_{i=1}^I [u^i(x^i)]^{\alpha^i}}{\sum_{i=1}^I x^i = \bar{x}}.$$

Remarks:

1. The Mantel (1976) version of the DMS theorem, mentioned in Section 3.5, shows that condition (4) is not robust to even small deviations from the assumption that endowments are collinear.

¹Somewhat more precisely, the Debreu (1974) version of DMS says that *any* if $I \geq N$ then function \bar{f}^D defined on a compact subset of \mathbb{R}_{++}^N can, provided it satisfies the three listed properties, be generated as the market demand function of some exchange economy, with utilities satisfying the standard assumptions. The restriction to a compact set of \mathbb{R}_{++}^N is required because standard monotonicity assumptions imply some LOD-like structure for \bar{f}^D near the boundary of \mathbb{R}_{++}^N , a generalization of a point made in Section 2.1 in connection with Giffen goods.

2. Conditions (1) and (2) have the unattractive property that there is no trade in equilibrium. In contrast, trade is possible under both (3) and (4).

3.7 Production Economies

There are two competing intuitions regarding production.

The first intuition is that, in a sense that can be formalized, it is typical for LODC to fail in economies with at least two pure factors (goods that are inputs into production but that are not consumed) and at least two consumers; see Hildenbrand (1989). The problem is that changes in factor prices can change the wealth distribution without changing the value of the aggregate endowment. Since the pure factors are not consumed, and since aggregate wealth has not changed, the WA (and hence LODC) requires that market demand remain unchanged. But the changes in individual wealth will cause changes in individual demand and these in turn will change market demand unless all of the individual $D_{m^i}\phi^i$ vectors happen to coincide, so that the individual demand changes exactly cancel.

The second intuition goes in somewhat the opposite direction: production can help with properties like uniqueness and stability of equilibrium, even if LODC fails. An extreme example of this is the “non-substitution” theorem, which applies to economies in which production is characterized by constant returns to scale and a single non-produced pure factor (meaning a pure factor that is not the output of any production process; think of raw iron ore, still in the ground). In this highly restrictive case, the equilibrium prices of all the consumption goods are determined by the technology, rather than by an interaction between preferences, technology, and endowments. The non-substitution theorem is suggestive, but it remains open whether there are reasonable restrictions, beyond the standard ones, that one can impose on production to ensure that properties like uniqueness of equilibrium hold even if LODC fails. The current state of the art is bleak: one can write down economies in which behavior is very bad (multiple equilibria, etc) even though both the consumer and producer sides of the economy satisfy standard assumptions; see Kehoe (1985) for a famous example.

References

- ANDERSON, C. M., S. GRANAT, C. R. PLOTT, AND K. SHIMOMURA (2004): “Global Instability in Experimental General Equilibrium: the Scarf Environment,” *Journal of Economic Theory*, 115, 209–49.
- BECKER, G. S. (1962): “Irrational Behavior and Economic Theory,” *Journal of Political Economy*, 70, 1–13.
- DEBREU, G. (1974): “Excess Demand Functions,” *Journal of Mathematical Economics*, 1, 15–21.

- GRANDMONT, J. (1992): “Transformations of the Commodity Space, Behavioral Heterogeneity and the Aggregation Problem,” *Journal of Economic Theory*, 57(1), 1–35.
- HARDLE, W., W. HILDENBRAND, AND M. JERISON (1991): “Empirical Evidence on the Law of Demand,” *Econometrica*, 59(6), 1525–1549.
- HILDENBRAND, W. (1983): “On the Law of Demand,” *Econometrica*, 51, 997–1018.
- (1989): “The Weak Axiom of Revealed Preference for Market Demand is Strong,” *Econometrica*, 57, 979–985.
- KEHOE, T. J. (1985): “Multiplicity of Equilibria and Comparative Statics,” *Quarterly Journal of Economics*, 100, 119–147.
- KEISLER, J. (1996): “Getting to a Competitive Equilibrium,” *Econometrica*, 62, 29–49.
- KIHLSTROM, R., A. MAS-COLELL, AND H. SONNENSCHNEIN (1976): “The Demand Theory of the Weak Axiom of Revealed Preference,” *Econometrica*, 45(5), 971–978.
- MANTEL, R. (1976): “Homothetic Preferences and Community Excess Demand,” *Journal of Economic Theory*, 12(2), 197–201.
- MARSHALL, A. (1920): *Principles of Economics*. McMillan, London, 8th edn.
- MAS-COLELL, A., M. WHINSTON, AND J. GREEN (1995): *Microeconomic Theory*. Oxford University Press, Oxford, UK.
- NACHBAR, J. H. (2002): “General Equilibrium Comparative Statics,” *Econometrica*, 70(5), 2065–2074.
- QUAH, J. K.-H., AND G. GIRAUD (2003): “Homothetic or Cobb-Douglas Behavior Through Aggregation,” *Contributions to Theoretical Economics*, 3(1).
- SHAFFER, W., AND H. SONNENSCHNEIN (1982): “Market Demand and Excess Demand Functions,” in *Handbook of Mathematical Economics, Volume II*, ed. by K. Arrow, and M. Intriligator, chap. 14, pp. 671–693. North-Holland, Amsterdam.