

Static profit-maximization and optimal non-strategic capacity choice

Notation

- Q = quantity produced
K = amount of capacity on hand
c = given constant marginal cost
 η = elasticity of MC with respect to output (listed on profile)

Marginal cost expression

$$MC = c + 1(Q > K)c\eta \left(\frac{Q - K}{K} \right)$$

where $1(Q > K)$ is an indicator function which equals one if quantity exceeds capacity and zero otherwise.

True total variable cost (i.e., sum of marginal costs)¹

$$TVC = cQ + 1(Q > K) \sum_{i=1}^{Q-K} c\eta \left(\frac{i}{K} \right) = cQ + 1(Q > K) \left(\frac{1}{2} \right) c\eta \left(\frac{Q - K}{K} \right) (Q - K + 1)$$

Approximate total variable cost (i.e., integral of marginal cost expression)²

$$TVC = \int MC dQ = cQ + 1(Q > K) \left(-c\eta Q + \left(\frac{1}{2} \right) c\eta \left(\frac{Q^2}{K} \right) + F \right)$$

Using knowledge that $TVC = cQ$ when $Q = K$, F can be found and substituted in to the above.

$$TVC = cQ + 1(Q > K) \left(-c\eta Q + \left(\frac{1}{2} \right) c\eta \left(\frac{Q^2}{K} \right) + \left(\frac{1}{2} \right) c\eta K \right)$$

Total cost is the sum of total variable cost and total capacity cost (entry costs omitted for brevity)

$$TC = cQ + 1(Q > K) \left(-c\eta Q + \left(\frac{1}{2} \right) c\eta \left(\frac{Q^2}{K} \right) + \left(\frac{1}{2} \right) c\eta K \right) + P_K (\delta + r) K$$

continued

¹ This is what CSG uses (but it doesn't yield a clean solution in what follows).

² This is a close approximation (especially for markets with overproduction being a small proportion of capacity), and it yields the nice cost-minimizing solution that follows.

Cost-minimizing capacity

$$\min_K TC = cQ + 1(Q > K) \left(-c\eta Q + \left(\frac{1}{2}\right) c\eta \left(\frac{Q^2}{K}\right) + \left(\frac{1}{2}\right) c\eta K \right) + P_K (\delta + r) K$$

Rearranging the first-order condition to the above yields the following cost-minimizing relationship:

$$K^* = Q \sqrt{\frac{c\eta}{c\eta + 2P_K (\delta + r)}}$$

Assuming that capacity is revised to follow this relationship for output levels, we can construct a “marginal cost” measure that appropriately incorporates capacity. Let ω denote the square root term in the above optimal capacity relationship, i.e., $K^* = Q\omega$. Then substituting K^* into the total cost expression for K (and assuming some overproduction and dropping the indicator functions) yields

$$TC = cQ - c\eta Q + \left(\frac{1}{2}\right) c\eta \left(\frac{Q^2}{\omega Q}\right) + \left(\frac{1}{2}\right) c\eta \omega Q + P_K (\delta + r) \omega Q$$

which reduces to

$$TC = Q \left(c - c\eta + \left(\frac{1}{2}\right) c\eta \left(\frac{1}{\omega}\right) + \left(\frac{1}{2}\right) c\eta \omega + P_K (\delta + r) \omega \right)$$

When making long-run decisions about capacity, this is the total cost (and the appropriate marginal cost) that should guide your thinking. To illustrate, consider Mkt B where capacity is cheap relative to the costs of overproduction because 1) the price of capacity is low relative to marginal costs (50 vs. 220) and 2) depreciation is slow. For simplicity, assume that $\omega = 1$, so that it never makes sense to overproduce. (The true value is actually $\omega \approx 0.997$.) Given this value of ω , this expression of total cost reduces to

$$TC = Q(c + P_K (\delta + r))$$

so that the relevant cost is the actual average variable cost c (about \$220) and the opportunity cost of capacity (about \$3.75), for a total of \$223.75. Ignoring the sunk entry costs, this is your break-even threshold going in. Once in, you then ignore the sunk component of the capacity cost.

Now consider Mkt A ($c = 50$, $P_K = 800$, $\delta = 0.155$, $r = 0.02$). The analogous ω is 0.884, so that when making capacity decisions conditional upon expected production, one would ideally have an amount of capacity equal to 89% of the level of production. When planning for this level of overproduction and considering capacity costs, the relevant cost is the actual average variable cost (about \$57) and the opportunity cost of capacity (\$123.76), for a total of \$180.76, which is this market’s break-even threshold. You can recalculate this threshold after entry to determine whether staying in makes sense.