Prof. Charles Moul Economics 452: Industrial Organization March 7, 2007

Midterm Exam Answers

1. Assume you're from New Zealand (35 points)

You are a rancher and use your sheep for first wool and then mutton production. The production of each good follows a Cobb-Douglas function: wool output is $Y^W = 10\ S^{½}\ T^{½}$ and mutton output is $Y^{Mutton} = 50\ S^{½}\ B^{½}$. Notation-wise, S, T, and B denote number of sheep, man-hours of trimming, and man-hours of butchering respectively. Denote the price of sheep as P and the hourly wages of trimmers and butchers as Q and R. For what follows, the algebra can be messy … if it's helpful, when you get to the appropriate place, let $D = Q(Y^W/10)^2 + R(Y^M/50)^2$.

[Note: The Cobb-Douglas specification is a ridiculous way to model this problem, in that it allows the rancher far too much leeway in substituting between sheep and labor for either product. However, when I tried to set up the problem using a more appropriate constant elasticity of substitution (CES) production, the problem quickly spiraled into intractability (solving for fourth-order polynomials and such). So you got this incredibly stylized production technology.]

A. Find the conditional factor demands for the three inputs. [Hint: Don't try to solve by setting the marginal rate of transformation equal to the ratio of factor prices.]

If we don't try to solve this by beginning with the tangency condition (which is complicated by the multiproduct production), then one begins by restating the production functions as input requirements conditional upon outputs and other inputs. For example, wool production can be reconsidered as how many trimmers are needed to produce some amount of wool using some number of sheep: $T = (Y^W/10)^2\ (1/S)$. Likewise, $B = (Y^M/50)^2\ (1/S)$. Substituting these binding constraints into the cost expression gives us the following cost minimization problem (using the suggested notation D):

$$\min_S C = P*S + Q*(Y^W/10)^2\ (1/S) + R*(Y^M/50)^2\ (1/S) = P*S + D/S$$

This yields the FOC $P – D/S^2 = 0$ and the conditional factor demand for sheep $S = (D/P)^{½}$. Substituting this expression into the prior constraints gives $T = (Y^W/10)^2\ (P/D)^{½}$ and $B = (Y^M/50)^2\ (P/D)^{½}$.

B. Calculate the simplified cost function.

The cost function is the cost expression after the conditional factor demands are substituted in. After simplification, this becomes $C(Y^W, Y^M, P, Q, R) = 2(PD)^{½}$.

C. There are two ways to show that the aggregate rancher production exhibits everywhere constant returns to scale. Show with one and how you would apply the other.

The first (and simpler) way to show constant economies of scale is to increase all inputs by some constant λ and show that all outputs also increase by that constant. For wool, $10\ (\lambda S)^{½}\ (\lambda T)^{½} = 10\ S^{½}\ T^{½}\ \lambda = \lambda Y^W$ and for mutton $50\ (\lambda S)^{½}\ (\lambda B)^{½} = 50\ S^{½}\ B^{½}\ \lambda = \lambda\ Y^M$. The alternative way to exhibit constant returns to scale in a multiproduct framework is to use the measure we discussed in class: $Scale = C/(MC^W*Y^W + MC^M*Y^M)$. Calculating the marginal costs requires a return to the expression denoted D: $MC^W = (dC/dD)(dD/dY^W)$ and $MC^M = (dC/dD)(dD/dY^M)$. We know that $dC/dD = (P/D)^{½}$, while $dD/dY^W = 2Q(1/100)\ Y^W$ and $dD/dY^M = 2R(1/2500)\ Y^M$. The denominator of "Scale" is then $(P/D)^{½}\ (2Q(1/100)\ Y^W * Y^W + 2R(1/2500)\ Y^M * Y^M) = (P/D)^{½}\ 2D = 2(PD)^{½}$. As this is also the numerator (the cost function itself), Scale = 1.

D. Using the appropriate measure, show that your ranch exhibits scope economies at all possible combinations of outputs.

The appropriate scope measure looks at the (scaled) difference of separate production to that of joint: $SC = [C(Y^W, 0, P, Q, R) + C(0, Y^M, P, Q, R) – C(Y^W, Y^M, P, Q, R)]/ C(Y^W, Y^M, P, Q, R)$. If SC>0 for certain ranges of output, then there exist scope economies over those regions.

$SC = [2(PQ(Y^W/10)^2)^{½} + 2(PR(Y^M/50)^2)^{½} – 2(PD)^{½}]/(2(PD)^{½}) > 0$ implies

$(Q(Y^W/10)^2)^{½} + (R(Y^M/50)^2)^{½} > D^{½}$ … squaring both sides yields

$Q(Y^W/10)^2 + R(Y^M/50)^2 + (QR(Y^W/10)^2\ (Y^M/50)^2)^{½} > D$ which is the same as

$D + (QR(Y^W/10)^2\ (Y^M/50)^2)^{½} > D$

As Q and R are prices and strictly positive, this inequality holds everywhere, and your ranch exhibits scope economies at all combinations of outputs.

2. Xirius (5 points)

The proposed satellite radio "merger of equals" between Sirius and XM has sparked concern among antitrust authorities. Make an argument about whether or not the Department of Justice should seek an injunction preventing the merger. An ideal answer will not only address points supporting your argument but also undercut points that work against your position.

FOR: The merger should not be prevented. The potential cost-side gains from eliminating redundancies are obvious. Regarding possible increases in market power, both companies (until recently) have been losing money, calling into question the viability of the existing duopoly. Furthermore, satellite radio faces many substitutes and hence competitors. The fact that they are the only two satellite radio companies is made irrelevant by all the other available listening opportunities (IPod, broadcast radio, etc.). The potential damage from such a merger is further limited by the lock-in nature of the original hardware purchase … to some extent, the two companies are hardly competing now. A merger is therefore unlikely to change substantively the prices and options facing consumers.

AGAINST: The merger should be prevented. While there exist many potential substitutes for consumers, satellite radio is differentiated enough to create significant market power in many circumstances. Going from a duopoly to a monopoly will only enhance that market power. Furthermore, economic theory on market power and product variety is ambiguous: the proposed merger may very well greatly reduce the variety of stations that is presently available, and many of these satellite radio options have no close substitutes.

3. Just because it's hype doesn't mean we can't analyze it (15 points)

The St. Louis market for bottled water is served by five firms, whose marginal costs are constant and given by the following table:

Firm	1	2	3	4	5
C in $	0.80	0.70	0.85	0.65	0.85

The market price for a bottle is $1.21. Assume that you can use the Cournot model with linear demand to describe how this market operates.

A. Which firm has the highest market share? Support your answer using the Cournot first-order condition.

Intuitively, the firm with the lowest marginal cost (Firm 4) will be the one with the highest market share. This can be shown using the Cournot FOC: $P - c_i = bq_i$. Dividing both sides by bQ (where $Q = \Sigma q_k$) leaves us with $(P - c_i)/bQ = s_i$. Lower marginal costs will by construction lead to higher market shares.

B. Find the elasticity of market demand for bottled water at the above market price.

The elasticity of market demand is $\eta = (dQ/dP)(P/Q)$. In terms of the given inverse demand, you can rewrite this as $(1/\eta) = (dP/dQ)(Q/P)$. Demand being linear tells you that $(dP/dQ) = -b$, so $/(1/\eta)/ = bQ/P$. To find bQ/P in terms of observables, return to the Cournot FOC: $P - c_i = bq_i$. Summing both sides over all firms yields $\Sigma(P - c_i) = 5P - \Sigma c_i = bQ$. Further dividing both sides by P gives $(5P - \Sigma c_i)/P = bQ/P$. Plugging the observed price and sum of marginal costs shows that $bQ/P = (6.05 - 3.85)/1.21 \approx 1.82$. Therefore, the elasticity of market demand is $/\eta/ = 0.55$.

4. You should have heard his price for bat removal (10 points)

A local exterminator addresses squirrels in the attic by plugging all holes but one to the outside and installing a spring-loaded wire noose around the remaining hole. After installation, the exterminator comes when called by the homeowner to remove squirrel carcasses. (Homeowners cannot do this on their own.) It is common for exterminators to charge a two-part tariff for squirrel removal: $125 for installation and the first squirrel, $60 for each additional squirrel. Do you think that this pricing is more indicative of price discrimination or cost? Explain. In an ideal real-world experiment, what sort of data variation might you be able to use to distinguish between these two explanations?

COST: I find it hard to believe that exterminators are able to differentiate themselves or prevent entry. The lack of either condition suggests that the squirrel-removal business will be competitive, and thus exterminators will be price-takers.

PRICE DISCRIMINATION: The consumer valuation of squirrel eradication will likely depend upon the extent of the infestation (how many squirrels are in the attic). This two-part tariff therefore allows the exterminator to extract higher payments from those consumers who have higher valuations. Aside from market power stemming from differentiation, it is also likely that municipal regulations and permits might serve as an effective entry barrier. Either condition allows the exterminators to behave as price-setters (as price discrimination requires).

Ideal Data: Suppose we have a sample of communities that are similar except for along the following dimension. Some of the communities have fairly uniform squirrel infestations (e.g., if you have squirrels in the attic, you have five varmints up there), while the other communities have widely variable squirrel infestations. If price discrimination explains the two-part tariff, then we should see higher per-squirrel prices and lower installation prices in the more variable infested communities. We should see no such difference if the two-part tariff is cost-based.

5. Erin go bragh (35 points)

You have a monopoly on the sale of (green) beer for the St. Louis St. Patrick's Day Parade route. You expect that 10,000 potential consumers will be spread evenly along the 5 mile path of the parade. Research shows that all paradegoers are willing to pay \$5 to consume one beer, and a new anti-drunk regulation guarantees that no more than one beer per parade-goer will be purchased. Parade-goers also dislike walking to beer stands and are willing to pay 25 cents to avoid having to walk ¼ of a mile and return. Each beer costs you 50 cents, and each beer stand that you open incurs \$400 in fixed costs.

A. If you have a single, optimally located stand, do you want to cover the entire market (i.e., is it profit-maximizing for you to sell to all parade-goers)? Explain.

You must first find the demand curve that you face when you choose not to cover the whole market (conditional upon locating at the parade-route's center). A few points on the demand curve illustrate the idea. If you charge no more than P=\$2.50, the outermost consumers (each 2.5 miles from the stand) will buy, and your quantity sold will be Q=10,000. If you charge P=\$3.50, you will sell only to consumers that are within 1.5 miles of your stand (i.e., 60% of parade-goers or Q=6000). From these two points, you can deduce $Q^D = 10000$ for $P < 2.5$, $Q^D = 20000 - 4000P$ for $2.5 \leq P \leq 5$, and $Q^D = 0$ for $P > 5$. Assuming that you will price on the downward-sloping part of demand, your profit expression is $\Pi = (P - 0.5)(20000 - 4000P)$. Solving the first-order condition (FOC) yields $P = 2.75$, so it is not profit-maximizing to cover the entire market.

B. Show that, if you choose to cover the entire market and are limited to a simple pricing regime, your profit maximizing number of beer stands is N=8. [Hint: where would you locate two stands?] Given eight stands, what price do you charge?

If you are choosing to cover the entire market, then your price is dictated by the distance from the outermost consumer to your beer stand. Using fractions of the parade-route, optimal location of beer stands follows the pattern {1/(2N), 3/(2N), …, (2N-1)/(2N)}, e.g., three beer stands will be located 1/6 of the way into the route, then 3/6 and then 5/6. Given this spacing, the price that you will charge can be characterized as $P = 5 - 5/(2N)$. Plugging this price less the average variable cost of \$0.50 into the profit expression and subtracting off beer stand costs yields

$$\Pi(N) = (5 - 5/(2N) - 0.5)*10000 - 400N = ((9N - 5)/(2N))*10000 - 400N$$

Evaluating this expression at eight stands shows that $\Pi(8) = 38675$, while $\Pi(7) \approx 38629$ and $\Pi(9) \approx 38622$. Given eight stands, you charge $P = 5 - 5/16 = 4.6875$.

C. Express cumulative travel costs as a function of the number of beer stands. [Hint: Find the average travel cost given a fixed number of beer stands and work from there.]

Consider the cases of one and two stands to see the relationship between the number of stands and the average travel cost. When N=1, the maximum travel cost faced is $2.50 and the minimum is $0. Exploiting the uniform density of consumers, this yields an average travel cost of $1.25. When N=2, the maximum travel cost falls to $1.25 (the minimum stays at $0), so the average is now $0.625 (i.e., 5/8). More generally, the average travel cost will be $T = 5/(4N)$. Cumulative travel costs are then the product of this average and the population (TravelCosts = 12500/N).

D. Now assume that you can deliver beers at the same cost that parade-goers faced. Conditional upon serving the entire parade-route, what is your profit-maximizing number of beer stands, and what are your profits? What is the efficient (welfare-maximizing) number of beer stands? Explain.

Now that you can deliver beers, you are engaging in uniform delivered pricing and (under the question's assumptions) are able to fully extract consumer surplus. You therefore charge the maximum price of $5 to all consumers and regardless of your number of stands your variable profits (revenues less variable costs) are $45000. Your problem can be reconsidered as choosing the number of stands to minimize the sum of your delivery-travel costs and your beer-stand fixed costs ($\text{Costs}(N) = 12500/N + 400N$). The FOC is $-12500/(N^2) + 400 = 0$, which yields $N^2 = 31.25$ and $N \approx 5.6$. Comparing our two possible answers, $C(5) = 4500$ and $C(6) \approx 4483$ (for completeness, $C(7) \approx 4586$), so, under uniform delivered pricing, six beer stands maximizes profits ($\Pi \approx 40517$). The efficient number of beer stands is also six. This is analogous to the result that a monopolist engaging in first-degree price discrimination maximizes welfare: when the problem is set up so that the monopolist captures all surplus, then the monopolist's problem coincides with that of the benevolent social planner.