

You have the full ninety minutes to complete this exam. Partial credit will be given for partial work, so don't panic if you get hung up on a particular question. Write neatly, and good luck.

1. Is it still Chicken if they use tractors? (15 points)

Academics, teenage boys from the '50s, and aficionados of the 1984 movie *Footloose* are all familiar with the game "Chicken." Two players each drive his car down the center of a road in opposite directions (i.e., towards each other). Each can then choose STAY or SWERVE. Staying wins adolescent admiration (big payoff) if the other player chooses SWERVE. Swerving loses face (low payoff) when the other player stays. Bad as losing face is, it is still better than the payoff when both players choose STAY, in which case they are both horribly crippled and disfigured. Formally, the game takes the following normal form.

		James Dean	
		STAY	SWERVE
Kevin Bacon	STAY	-6, -6	2, -2
	SWERVE	-2, 2	1, 1

a. Find the pure strategy Nash equilibrium(s) to this game.

There are two pure strategy equilibria to this game. If Dean plays STAY, Bacon will SWERVE, and, if Bacon plays SWERVE, Dean will STAY, so (Bacon SWERVE, Dean STAY) is an equilibrium. Likewise with (Bacon STAY, Dean SWERVE). This exhausts the set of potential pure strategy equilibria.

b. Using the principle that both James and Kevin are risk-neutral, find the unique mixed strategy Nash equilibrium to this game.

In a mixed strategy equilibrium, each player must be indifferent between his options given his opponent's probabilities of action. Let P denote Dean's probability of staying. Then a mixed strategy equilibrium requires that Bacon's expected utility (EU) from playing STAY be equal to that from playing SWERVE:

$$EU(\text{Bacon STAY}) = P*(-6) + (1 - P)*(2) = P*(-2) + (1 - P)*(1) = EU(\text{Bacon SWERVE})$$

Solving this equation for the unknown P yields $P = 1/5$, so that an indifferent Bacon requires that Dean play STAY with probability 0.2. The symmetry of the game implies that an indifferent Dean likewise requires that Bacon play STAY with probability 0.2. The unique mixed strategy equilibrium is then each player staying with 1/5 likelihood and swerving with 4/5 likelihood.

c. In the mixed strategy equilibrium, what is the chance of a collision?

A collision requires that, after randomizing, both players realize the STAY action. This will occur 4% $((1/5)*(1/5))$ of the time.

2. Yet another nightclub example (30 points)

As a nightclub manager, you realize that demand for drinks is more elastic among students, and you are trying to determine the optimal pricing schedule. Average individual demands are

$$q^K = 18 - 5P \quad \text{for those under 25}$$

$$q^A = 10 - 2P \quad \text{for those 25 and over}$$

The two age groups typically visit the nightclub in equal numbers. Assume that drinks cost \$2 each.

- a. If the nightclub can charge according to whether the customer is a student but is limited to linear pricing, what price (per drink) should be set for each group?

This question reduces to two separate monopoly pricing problems, one for each population. Given these assumptions, maximizing per-person profit is equivalent to maximizing profit over the entire population (market size is a scalar in profits). For the under-25 population,

$$\Pi^K = P^K * (18 - 5P^K) - 2 * (18 - 5P^K) = (P^K - 2)(18 - 5P^K)$$

$$\text{FOC: } (18 - 5P^K) - 5 * (P^K - 2) = 0, 28 = 10P^K, P^K = 2.80$$

For the 25 and over population,

$$\Pi^A = P^A * (10 - 2P^A) - 2 * (10 - 2P^A) = (P^A - 2)(10 - 2P^A)$$

$$\text{FOC: } (10 - 2P^A) - 2 * (P^A - 2) = 0, 14 = 4P^A, P^K = 3.50$$

If restricted to linear pricing but able to engage in third degree price discrimination, you should charge \$2.80 per drink to those under 25 and \$3.50 per drink to those 25 and over.

While you were not asked for additional information, it is useful to compare outcomes to the following options. $P^K = 2.8$, $q^K = 4$, $\Pi^K = 3.2$, so average profit per student is \$3.20. $P^A = 3.5$, $q^A = 3$, $\Pi^A = 4.5$, so average profit per geezer is \$4.50.

- b. Now suppose that it is impossible to distinguish between types. If you lowered drink prices to \$2 and still wanted to attract both types of consumers, what cover charge would you set?

If you are to attract both consumer types, your cover charge cannot exceed the lower average consumer surplus of the two populations. The marginal price per drink is fixed at \$2. Inverse demands are $P^K = 3.6 - 0.2q^K$ and $P^A = 5 - 0.5q^A$. Consumer surplus for each is then

$$CS^K = \frac{1}{2} * (3.6 - 2) * (18 - 5(2)) = 6.4$$

$$CS^A = \frac{1}{2} * (5 - 2) * (10 - 2(2)) = 9$$

As the under-25 consumer obtains less consumer surplus, you should charge a cover of \$6.40. This average profit of \$6.40 clearly dominates the average profit of part (a) of \$3.85.

- c. Suppose that you are again restricted to linear pricing. While it is impossible to explicitly “age discriminate,” you notice that everyone remaining after midnight is a student, while only 2/7 of those who arrive before midnight are students. Pricing changes do not affect

these fractions, and there is no foresight among consumers. How should drink prices be set before and after midnight? What type of price discrimination is this? Compare your average profit per customer arriving before and after midnight with the average profit per consumer if you were to use the pricing strategy from part (b).

Given that only students remain after midnight, late-night pricing will coincide with the 3rd degree price discrimination solution of part (a): $P^{\text{late}} = 2.80$. The assumptions regarding foresight and the fixed nature of the percentages allow us to specify profits for early pricing problem as follows:

$$\Pi^{\text{early}} = (P^{\text{early}} - 2)[(2/7)(18 - 5P^{\text{early}}) + (5/7)(10 - 2P^{\text{early}})] = (P^{\text{early}} - 2)(86/7 - 20/7P^{\text{early}})$$

$$\text{FOC: } 86/7 - 20/7 P^{\text{early}} - 20/7 P^{\text{early}} + 40/7 = 0, P^{\text{early}} = 3.15$$

The optimal way to age discriminate implicitly is to charge \$3.15 per drink before midnight and offer an after-midnight discount so that late-night drink prices are \$2.80. Such a pricing scheme would be best described as 2nd degree price discrimination (consumers choosing from menu of prices). Students who arrive before midnight purchase 2.25 drinks, yielding an average profit of \$207/80 (\approx \$2.59). Geezers always arrive before midnight and purchase 3.7 drinks, yielding an average profit of \$851/200 (\approx \$4.26). Given the pre-midnight 5:2 ratio of geezers to students, the average profit per consumer arriving before midnight is \$3.78. At after midnight prices, students buy 4 drinks, yielding average profits of \$3.20. Comparing this outcome with that of the two-part tariff (\$6.40) strongly suggests that the cover charge plan of #2b is more lucrative.

3. Weekend's Only (10 points)

A local St. Louis furniture store provides "no frills" items and is open only on Friday from 12-9, Saturday from 10-8, and Sunday from 11-6. Describe the relative importance of demand, fixed costs, and variable costs for this to be a profit-maximizing strategy.

We can surmise two things from this observation. First, the furniture outlet's strategy is presumably profitable:

$$\text{Rev3Day} - \text{VarCost3Day} - \text{FixedCost} > 0 \quad \#1$$

Second, the outlet's three day strategy earns higher profits than the alternative seven day strategy:

$$\text{Rev3Day} - \text{VarCost3Day} - \text{FixedCost} > \text{Rev7Day} - \text{VarCost7Day} - \text{FixedCost} \quad \#2a$$

This latter condition can be re-expressed as

$$\text{Rev3Day} - \text{VarCost3Day} > \text{Rev7Day} - \text{VarCost7Day} \quad \#2b$$

stating that the 3-day strategy yields higher variable profits than the 7-day strategy. With the additional benign assumption that $Q_{7\text{day}} > Q_{3\text{day}}$ (Weekend's Only sacrifices some sales through their 3-day strategy), one can also state that

$$\text{AvgRev3day} - \text{AvgVarCost3day} > \text{AvgRev7day} - \text{AvgVarCost7day} \quad \#3$$

This last condition may be considered a somewhat uninformative restatement of #2b above.

4. Merger mania (5 points)

The domestic telecom industry is currently facing substantial structural change. For both proposed mergers, name the acquiring company, the company that is being acquired, and the

primary lines of business for each. Name an additional **important** fact about each of the two mergers that shows you've been paying attention.

Verizon Communications Inc. agreed to buy MCI Inc. for \$6.75 billion in cash and stock. Verizon has Domestic Telecom, Domestic Wireless, Information Services and International operations. MCI operates a communications network that is composed of approximately 100,000 route miles of network connections linking metropolitan centers and various regions across North America, Europe, Asia, Latin America, the Middle East, Africa and Australia. It owns an Internet protocol backbone and is a carrier of international voice traffic.

Most interesting fact: Qwest is/was also in the running and even offered more to acquire MCI than Verizon but Qwest's financial situation makes any offer from it a risky proposition.

SBC Communications Inc. agreed to acquire former parent AT&T Corp. in a \$16 billion deal, mostly in stock, that would create one of the world's largest telecom companies. The services and products that SBC offers vary by market, and include local exchange services, wireless communications, long-distance services, Internet services, telecommunications equipment and directory advertising and publishing. AT&T provides traditional long distance voice services, such as domestic and international dial services and calling card services with business and consumer divisions.

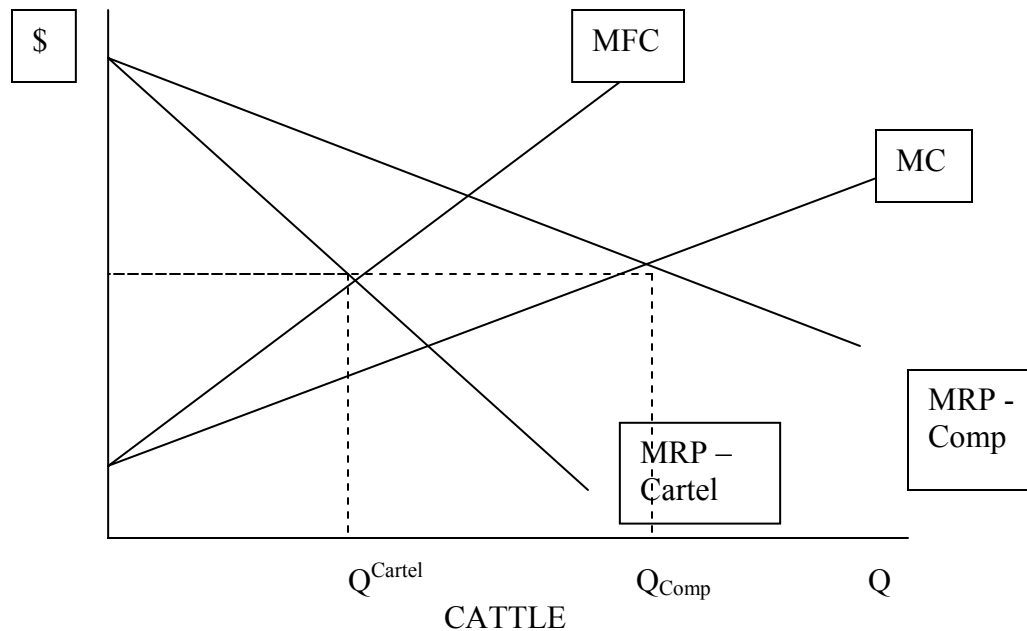
Most interesting fact: SBC is a former Baby Bell, having been split from AT&T in the mid-1980s as a result of an antitrust action, is now seeking to acquire Ma Bell.

5. Time travel & cows (40 points)

It is 1880, and you are the manager of the Chicago meat-packing cartel. Cows are brought to Chicago, and you oversee the slaughter and dressing so that these animals become beef. Your cartel has no coordination problems and exploits its market power completely. Specifically, the cartel acts as a monopsonist in its purchase of cattle and as a monopolist in its sale of beef. Given that many cow parts are inedible, 100 lbs of cattle yields only five 10 lb. packages of beef.

The (inverse) demand for beef is $P^B = 6.5 - 0.01Q^B$, and the (inverse) supply of cattle is $P^C = 1 + 0.0625Q^C$. (Cattle quantities are in 100Ms of lbs, while beef quantities are in 10Ms of lbs. All prices are in 1880 dollars.) In addition to your expenditures on cattle, your total costs include labor and non-cow input costs of \$0.30 for each 10-lb package of beef. (Final answers for quantities are integers but prices may include an additional decimal place.)

- a. Graph the market for cattle. Show supply, cartel's marginal factor cost, cartel's marginal revenue product, and marginal revenue product for the underlying demand. Highlight and label quantities and prices under the cartel solution and under the fully competitive equilibrium.



- b. Find the profit maximizing prices and quantities of cattle and beef for your cartel.

The cartel's profits are (as always) revenues less costs. Market power in selling beef implies that the cartel recognizes that it must accept a lower price to sell more beef, and thus the cartel recognizes that its revenues are $R = (6.5 - 0.01 Q^B) Q^B$. The cartel's costs have two components: purchasing cattle and dressing beef. Market power in buying cattle implies that the cartel must pay a higher price to buy more cattle, but the cartel is a price taker in the inputs for dressing beef. You thus recognize your costs are $C = (1 + 0.0625 Q^C) Q^C + 0.3 Q^B$. The profit expression is then

$$\Pi = (6.5 - 0.01 Q^B) Q^B - (1 + 0.0625 Q^C) Q^C - 0.3 Q^B$$

It is at this point that one must incorporate information about the production technology, specifically that $Q^B = 5 Q^C$ or $Q^C = 0.2 Q^B$. I arbitrarily convert all cattle quantities in the profit expression into beef quantities:

$$\begin{aligned} \Pi &= (6.5 - 0.01 Q^B) Q^B - (1 + 0.0625 * 0.2 Q^B) 0.2 Q^B - 0.3 Q^B \\ &= (6.5 - 0.01 Q^B) Q^B - (0.2 + 0.0025 Q^B) Q^B - 0.3 Q^B \end{aligned}$$

Once so simplified, this expression can be maximized over the choice of beef output.

$$\text{FOC: } 6.5 - 0.02 Q^B - 0.2 - 0.005 Q^B - 0.3 = 0, 6 = 0.025 Q^B, Q^B = 240$$

Referring back to the production technology, $Q^C = 48$. Prices are then found by using these quantities in the inverse demand and supply: $P^B = 4.1$, $P^C = 4$.

So the cartel's profit-maximizing solution is to offer \$4 per 100lbs of cattle, purchase 48M 100-lbs of cattle, and sell 240M 10-lb packages of beef for \$4.1 per 10-lb package.

[TOO MUCH ATTENTION TO DETAIL ALERT: I found a time series of cattle prices (in the 100-lb denomination) from the National Bureau of Economic Research. In 1880, the nominal price per 100-lb of cattle was \$4. Alas, I could find no such data on quantities.]

- c. Find the prices and quantities of cattle and beef that arise under the fully competitive equilibrium.

As the graph above suggests, the competitive quantities should be bigger but the change in competitive prices is ambiguous. If the cartel's monopoly power dominates its monopsony power, one expects the price of cattle to rise under competition, and conversely if monopsony dominates. The competitive market for cattle will clear when $MC = MRP$, where MRP is the product of the demand for beef (transformed into cattle quantities) and the price of beef. A straightforward way of showing this result is to consider an individual competitive firm's profit expression.

$$\begin{aligned}\Pi &= P^B Q^B - P^C Q^C - 0.3 Q^B \\ &= Q^B (P^B - 0.2 P^C - 0.3)\end{aligned}$$

This price-taking firm believes that the market prices are unaffected by its quantity decisions, and such a firm maximizes profits where $P^B = 0.2 P^C + 0.3$.

If this condition holds for each firm, it must hold for the entire market. Substituting the inverse demand and supply expressions and solving yields $Q^B = 480$, $Q^C = 96$, $P^B = 1.7$, $P^C = 7$

- d. How much deadweight loss is created by the cartel?

Deadweight loss will arise in both markets and must be calculated separately for each. The derived marginal cost expression for the beef market is $MC^B = 0.2 P^C + 0.3 = 0.5 + 0.0025 Q^B$. At $Q^B = 240$ (the cartel outcome), this marginal cost is $MC^B = 1.1$. Deadweight loss in the market for beef is then $DWL^B = \frac{1}{2} (4.1 - 1.1) (480 - 240) = 360$, or \$360M. Applying a similar technique to the cattle market requires finding the derived demand for cattle: $P^C = 5 (P^B - 0.3) = 31 - 0.25 Q^C$. At $Q^C = 48$ (the cartel outcome), this derived demand is valued at \$19. Deadweight loss in the market for cattle is then $DWL^C = \frac{1}{2} (19 - 4) (96 - 48) = 360$, again \$360M. Total deadweight loss is therefore \$720M.

- e. Which side suffers more economic loss (cattle ranchers or beef consumers)? Which side would be likelier to initiate an antitrust review of the situation?

The magnitude of economic loss is not the same as the above deadweight loss, but finding it requires the same components. Begin with the market for cattle. Under competition, cattle

ranchers receive producer surplus of $PS = \frac{1}{2} (7 - 1) 96 = 288$, but under the cartel this surplus shrinks to $PS = \frac{1}{2} (4 - 1) 48 = 72$. The ranchers therefore suffer a loss of \$216M.

Now consider the market for beef. Under competition, beef consumers receive consumer surplus of $CS = \frac{1}{2} (6.5 - 1.7) 480 = 1152$, but under the cartel, this surplus shrinks to $CS = \frac{1}{2} (6.5 - 4.1) 240 = 288$. Beef consumers therefore suffer a loss of \$864M, fourfold more than the ranchers.

So long as your answer is supported by sufficient analysis, you could successfully argue either side of the antitrust question. (It is more of a political economy question, and therefore less clear-cut. Bonus point if you pointed out that the Sherman Act wasn't passed until 1890, and therefore there was no antitrust authority to which one could take the problem.) If you argued that the ranchers would be likelier to bring a review, you should emphasize that, while their losses are smaller, the smaller number of ranchers meant that this loss was not as diffused as that in the beef market. Therefore, ranchers might find it worthwhile to bear the legal costs of initiation, while the multitude of beef consumers would not. Conversely you could argue that the size of the differences in economic losses swamps this collective action problem ... the results suggest that beef prices would fall almost 60%, and that could get all carnivorous consumers excited.