Prof. Charles Moul | Industrial Organization | Spring 2003

Midterm Answers

1. Bananas and strained peas (45 points)

We discussed a number of variations on the Cournot and Stackelberg games in lecture. Consider the following cousin: the dominant firm-competitive fringe problem as it applies to the market for jarred baby food in the United States. The industry inverse demand curve for such baby food is $P = 100 - Q$. The dominant firm in the industry is Gerber; its total cost function is $C(q_G) = 25q_G + (1/6)q_G^2$. Heinz and Beech-Nut make up the competitive, price-taking fringe which has an inverse supply schedule of $P = 25 + 2Q$. Heinz and Beech-Nut are identical, and neither has fixed costs.

A. Determine the equilibrium price for baby food, as well as all three firms' output levels. Compute Gerber's profits. What is the value of the Herfindahl-Hirschman Index in this industry?

There are (at least) two ways to begin this question. Let Q_F denote the quantity supplied by the competitive fringe.

1) Recognize that the price that consumers pay must equal the price received by the two price-takers on the competitive fringe: $P = 100 - q_G - Q_F = 25 + 2Q_F = P$. Solving for Q_F, $Q_F = 25 - 1/3\ q_G$. Gerber's effective demand is then $P = 100 - q_G - (25 - 1/3\ q_G) = 75 - 2/3\ q_G$

2) Construct Gerber's residual demand directly: $q_G = Q - Q_F = 100 - P - ½ (P - 25)$. Solving this for P yields $P = 75 - 2/3\ q_G$

Gerber's profit expression and resulting first-order condition (MR – MC = 0) are

$\Pi_G = (75 - 2/3\ q_G)\ q_G - 25\ q_G - (1/6)q_G^2$

$75 - 4/3\ q_G - 25 - 1/3\ q_G = 0$ which is satisfied at $q_G = 30$.

One can then substitute this into the fringe-output expression of 1) or use 2) to find the price and then find the fringe's supply response. Either way, $Q_F = 15$ (so $q_H = q_{BN} = 7.5$) and $P = 55$. Substituting these values into Gerber's profit expression yields $\Pi_G = 55*30 - 25*30 - (1/6)*900$, which simplifies to $\Pi_G = 750$. As firms shares are $s_G = 2/3$, $s_H = 1/6$, and $s_{BN} = 1/6$, the sum of squared shares is $\Sigma s_i^2 = (4/9) + (1/36) + (1/36) = 18/36 = 0.5$ and the HHI is 5000.

B. The Merger Guidelines of the Department of Justice state that all mergers in concentrated industries (HHI>1800) that would raise the HHI by more than 100 points will be heavily scrutinized. (Less than 100 points places them in the "safe harbor" region.) On February 28, 2000, Heinz and Beech-Nut announced plans to merge. Would you expect a government reaction to this proposed merger?

From above, the jarred baby food industry clearly qualifies as concentrated. If Heinz and Beech-Nut merged and continued to behave as a price-taking fringe, the merged market share would be $s_{HB} = 1/3$. This new hypothetical industry's HHI would be 5555, for a change of 555, well out of the safe harbor. In fact, the FTC requested in injunction to block the merger in July 2000.

C. Suppose that the merger (if consummated) would make it possible for the two remaining firms, Gerber and HeinzBeech, to behave as Cournot duopolists. What are the new equilibrium price and quantities? What is the change in welfare arising from the merger?

This new industry is an example of the Cournot duopoly with asymmetric costs. The post-merger HeinzBeech's marginal cost expression is the same as the fringe's supply expression (this is just what supply is): $MC_{HB} = 25 + 2q_{HB}$. Since there are no fixed costs for either Heinz or Beech-Nut, this marginal cost implies that HeinzBeech's total cost function is $C(q_{HB}) = 25q_{HB} + q_{HB}^2$. (Note that the merged firm is still at a substantial cost disadvantage compared to Gerber.) One could also find equilibrium outcomes by using this marginal cost directly and calculating HeinzBeech's marginal revenue expression.

Gerber's objective function: $\Pi_G = (100 - q_G - q_{HB})q_G - 25\, q_G - (1/6)q_G^2$

FOC (as best response function): $q_G = 225/7 - 3/7\, q_{HB}$

HeinzBeech's objective function: $\Pi_{HB} = (100 - q_G - q_{HB})q_{HB} - 25q_{HB} - q_{HB}^2$

FOC (as best response function): $q_{HB} = 75/4 - ¼\, q_G$

These first-order conditions are satisfied at $q_G = 27$ and $q_{HB} = 12$, yielding P = 61. Welfare under the two scenarios is best calculated by first finding total consumer valuation (NOT consumer surplus) and then subtracting off production costs.

Post-merger: Σq = 39, P = 61

Valuation:	V = (39)(61) + ½ (39)(39) = 3139.5 (one could also use trapezoid formula)
Costs:	$C = (25)(27) + (1/6)(27)^2 + (25)(12) + (12)^2 = 1240.5$
Welfare:	W = 3139.5 – 1240.5 = 1899

Pre-merger: Σq = 45, P = 55

Valuation:	V = (45)(55) + ½ (45)(45) = 3487.5
Costs:	$C = (25)(30) + (1/6)(30)^2 + (25)(15) + (15)^2 = 1500$
Welfare:	W = 3487.5 – 1500 = 1987.5

The change in welfare is ΔW = -88.5.

D. Compute Gerber's profits in the Cournot outcome. Would Gerber have supported or opposed the Heinz-Beech-Nut merger?

Pre-merger:	$\Pi_G = 750$
Post-merger:	$\Pi_G = (61)(27) - (25)(27) - (1/6)(27)^2 = 850.5$

So, under these assumptions, Gerber would have supported the proposed merger.

2. Clipping for Tony (10 points)

Consider one arm of Kellogg's cereal manufacturing operations. Frosted Flakes is purchased by two types of customers, time-crunched moms and lay-around college students. Moms have a reservation price of $4 and using a coupon costs them $1.25 (in terms of effort to cut out and remember to use the coupon). Students have a reservation price of $3 and using a coupon costs them nothing. It costs Kellogg's $2.50 to produce each box. What price and coupon discount should Kellogg's set? Calculate the profits it receives from each group. Would Kellogg's still offer coupons if production costs suddenly rose to $3?

This is as straightforward as it appears. Kellogg's should charge a base price of $4 and offer a coupon for $1 off. All surplus has now been extracted from moms (who pay their reservation price) and from students (who also pay their reservation price). Kellogg's earns profit of $1.50 for every box it sells to moms and of $0.50 for every box it sells to students. However, if costs rise to $3 per box, students yield no profit and (as making and distributing coupons must have positive cost) coupons will be discontinued.

3. Scale and Scope (10 points)

Briefly describe the concepts of economies of scale and economies of scope in a multi-product setting. Give an attribute of production that affects the likelihood of each type of economy. Your answer should also include the statistics used to measure each of these economies and how they fit into the natural monopoly concept.

The concept of scale economies depends upon the proportionate increase in output one obtains for a given proportionate increase in costs. In a multi-product setting, the most common measure of scale economies is $S = TC/(\Sigma MC_i Q_i)$, where $S>1$ implies economies of scale and $S<1$ implies diseconomies of scale. Possible sources of economies of scale include specialization of labor, economies of mass reserves, technical factors, and indivisible inputs (fixed costs).

The concept of scope economies captures if it is less costly to produce a set of goods in one firm than it is to produce the same set in more than one firm. The measure of scope economies (illustrated here for the two-product case) is $SC = (C(q1, 0) + C(0, q2) – C(q1, q2))/C(q1, q2)$. Possible sources of economies of scope include the production of distinct outputs sharing common inputs and the presence of cost complementarities.

A natural monopoly requires that production be subadditive, i.e., there is no possible arrangement of outputs using multiple firms that is less costly than production by a single firm. Scale economies are neither necessary nor sufficient for this subadditivity. Scope economies are a special case of subadditivity, and so are necessary but not sufficient.

4. Price-taking and what-not (10 points)

Evaluate the following statement: “The competitive model has little relevance in real-world applications, since it requires that market demand be perfectly elastic and that is rarely (if ever) observed.”

The statement is flat-wrong. The competitive model does not require that *market* demand be perfectly elastic. What it does require is that no firm has market power and therefore that each firm faces a perfectly elastic *residual* demand.

5. The token "Simpsons" question (25 points)

Despite the public discontent over electricity providers' perceived price-gouging, the Springfield and Shelbyville power markets are integrated as part of deregulation: what were separate price-setting monopolies are now a duopoly. C. Montgomery Burns and Aristotle Amadopolis are not pleased and vow to collude perfectly, evenly splitting the profits. To maintain this collusion, they propose the following trigger-price strategy: If either utility's price drops below the collusive price, a price war of Nash behavior that lasts for three weeks will ensue. Electricity is a homogeneous product, and either plant can provide enough power to satisfy the entire market. The total weekly demand is $Q^D = 21 - 3P$, each plant incurs total costs equal to output (TC = Q), and both Monty and Ari have the same weekly discount rate ($\beta = 0.9$).

A. Find the collusive price and the profits that each firm receives from colluding.

The price-setting cartel's objective function and first-order condition are

$$\Pi_{Cartel} = P(21 - 3P) - (21 - 3P) = (P - 1)(21 - 3P)$$

FOC: $21 - 3P - 3(P - 1) = 0$, which is satisfied at $P = 4$

Cartel profits are then $\Pi_{Cartel} = 27$. When split evenly, $\Pi_{Monty} = 27/2$ and $\Pi_{Ari} = 27/2$

B. Find Monty's immediate profits if he cheats and Ari prices collusively. What are each plant's profits during the price war?

The game as described is Bertrand, so, if Monty cheats, he will undercut Ari's price by an infinitesimal amount that will secure him the entire market. Charging virtually the same price and supplying the entire market, Monty will receive the monopolist's profit of 27. Nash behavior in a Bertrand game is for both firms to price at marginal cost and earn zero profits during the price war.

C. Show that the proposed trigger-price strategy will prevent cheating.

First note that after the price war, Monty's cheating or colluding is irrelevant … the game will effectively begin again. One need only consider the initial cheating and the price war itself. For the next four weeks, Monty's discounted profits from cheating today are

$$\Pi^{Cheat} = 27 + (0.9)(0) + (0.9)(0) + (0.9)(0) = 27$$

while his profits from maintaining the collusive agreement are

$$\Pi^{Collude} = 27/2 + (0.9)(27/2) + (0.9)^2(27/2) + (0.9)^3(27/2)$$
$$= 13.5 + 12.15 + 10.935 + 9.8415 = 46.4265$$

The trigger strategy prevents cheating if $\Pi^{Cheat} < \Pi^{Collude}$, which is the case here.

D. Is a three-week price war the shortest length that will prevent cheating? Support your answer.

A two-week price war will alter the preceding analysis by removing the last terms from PC and PC. While the payoff from cheating is still 27, the payoff from colluding is now 36.585. So the threat of a two-week price war will also deter cheating. (Note that the threat of a one-week price war will not be effective: 27 > 25.65.)