Final Answers

1. The inevitable merging of phone and PC (40 points)

Consider a mythical market for cellular phones that give PC-quality access to the Internet. On one side is the incumbent Nokia, and on the other is the potential entrant Microsoft. Both produce phones that are homogeneous, and they compete in a two-stage game. In stage 1, Nokia (but not Microsoft) can purchase up to 8 units of cost-reducing capital equipment k. In stage 2, firms compete by simultaneously choosing quantities if it is profitable for them to do so. Market (inverse) demand is described by the equation $P = 50 – 2Q$. Nokia's total cost (including the cost of the capital equipment and other fixed costs) is $TC_1 = q_1 (2 – ¼ k) + (11/4) k + 120$, where q_1 is Nokia's output. Microsoft's cost is $TC_2 = 2q_2 + 120$.

A. Find the subgame perfect equilibrium quantities and profits. On the equilibrium path, how much investment does Nokia make (i.e., how many units of k does Nokia buy in stage 1)?

Since this is a sequential game, we begin by moving backwards. Nokia's marginal revenue in stage 2 is $MR_1 = 50 – 4q_1 –2q_2$, while its marginal cost is $MC_1 = 2 – ¼ k$. Its best-response function (where $MR_1 = MC_1$) is then $q_1 = 12 + (1/16) k – ½ q_2$. By the same reasoning, Microsoft's best-response is the familiar $q_2 = 12 – ½ q_1$. The intersection of these best-responses (where both are satisfied simultaneously) is the Nash equilibrium, which yields $q_1* = 8 + (1/12)k$ and $q_2* = 8 – (1/24) k$. Market output will then be $Q* = 16 + (1/24) k$, and market price will be $P* = 18 – (1/12) k$.

All this begs the question of Nokia's initial investment. Using the above expressions, Nokia's stage 2 equilibrium profits (revenues less variable costs less investment costs less fixed costs) can be written as a function of k: $\Pi_1 = P*q_1* – q_1*(2 – ¼ k) – (11/4) k –120$. Substituting the above expressions for the equilibrium terms yields

$$\Pi_1 = (18 – (1/12) k)(8 + (1/12) k) – (8 + (1/12) k)(2 – ¼ k) – (11/4) k – 120$$

This reduces to $\Pi_1 = 8 – (1/12) k + (1/72) k^2$.

Note that this profit expression is unusual, in that profits are not concave in investment. Instead, profits are convex in investment. The implications of this will be made clear as we consider the FOC and the corner solutions.

The profit function reaches an extremum at the k such that $d\Pi_1/dk = -(1/12) + (2/72) k = 0$. This condition is satisfied at $k = 3$. Furthermore, $\Pi_1(k=3) = 8 – (1/4) + (1/72)(9) = 7\ 7/8$.

Your second clue that this is not a maximum (the first is if you thought the profit function looked convex rather than concave) would come from examining the simple case: $\Pi_1(k=0) = 8 > 7\ 7/8$.

If the FOC is not the maximum, what are the alternative candidates? We are left with the two corners, $k = 0$ and $k = 8$. We considered the $k = 0$ case above, and $\Pi_1(k=8) = 8\ 2/9 > 8$. Nokia's profit-maximizing action is to purchase the maximum amount of cost-reducing capital equipment ($k* = 8$).

What remains is to find the stage 2 equilibrium outcomes conditional upon k* = 8. Suppose that Microsoft enters. Then

$q_1 = 8\ 2/3$ $\qquad$ $q_2 = 7\ 2/3$

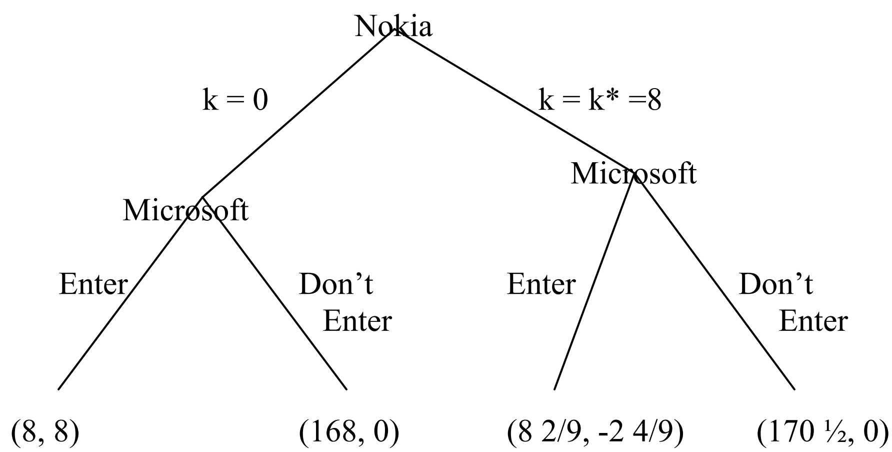
$Q = 16\ 1/3$ $\qquad$ $P = 17\ 1/3$

$\Pi_1 = 8\ 2/9$ $\qquad$ $\Pi_2 = -2\ 4/9$

But Microsoft can earn $\Pi_2 = 0$ by not entering. So Nokia has a monopoly in stage 2, and the outcomes on the equilibrium path are

$q_1 = Q = 12\ ½$ $\qquad$ $P = 25$ $\qquad$ $\Pi_1 = 170\ ½$

B. Incorporate the components of your above answer into an extensive-form game. Nokia's decision is k = 0 or k = k*, while Microsoft's decision is Enter or Not enter (with optimal decisions on quantities following).



The [Nokia: k = 0, Microsoft: Don't Enter] payoffs come from the original Nokia best-response function when $q_2 = 0$ and k = 0. All other payoffs come from the above text.

C. Does Nokia's investment in capital make Microsoft's optimal response more or less aggressive? Explain.

From above, Microsoft's equilibrium best-response function is $q_2^* = 8 - (1/24)\,k$, so higher levels of Nokia investment lead to lower Microsoft output. This arises since this investment generates the Cournot game with asymmetric costs, and lower Nokia costs lead to lower Microsoft production.

D. Is Nokia's investment decision in stage 1 predatory? Explain.

Since Nokia's investment decision leads to Microsoft deciding not to enter, it may appear to be predatory, but that appearance is incorrect. Regardless of Microsoft's entry decision, Nokia is better off making its k* = 8 investment decision. This is revealed by the fact that 8 2/9 > 8 and 170.5 > 168. In this sense, Nokia's stage 1 decision corresponds with the optimal capacity decision of a monopolist and is therefore not predatory.

Though we can be sure that Microsoft would try to sue on these grounds anyway.

2. New stuff (10 points)

Some theories of innovation in free markets predict that the market will generate too little innovation on its own, and therefore subsidies to research and development increase efficiency. Other theories (largely centered upon the patent race) predict that the market will spend too much on innovation, and therefore taxes on R&D increase efficiency. Explain each side of the argument.

The traditional view has been that, while social optimality considers both new consumer surplus and new profits from an innovation, private firms (who make the innovation decision) are concerned only with profits. Ignoring the social gains that arise in the form of consumer surplus leads firms to invest too little in research and development. Hence, the government should subsidize R&D in an attempt to bring private incentives in line with social incentives.

A more recent approach has emphasized that, while many firms will strive to innovate in the same area, there is typically only one "winner." Consequently, the resources spent by the "losers" are sunk and lost. If each firm makes its decisions based upon its likelihood of winning (rather than upon the marginal likelihood that it succeeds and all other firms fail, as optimality demands), then there will be too many entrants into the patent race. Hence, taxes on R&D will lessen this excessive entry and improve welfare.

3. What would IO be like without Chicago? (10 points)

Briefly describe the inefficiency that arises from double marginalization. Describe and contrast the Chicago School's view of vertical mergers with those who fear vertical foreclosure as an attempt to raise rivals' costs.

Double marginalization arises when an upstream market and a downstream market are both characterized by market power. In such a case, the downstream firm effectively "marks up" both the upstream firm's cost and its mark-up. This double mark-up leads to a final downstream price that is higher than what would have been charged had the two firms been a single monopoly. Both consumer surplus and firm profits are therefore higher when the double marginalization is removed. This is the basis for the Chicago reasoning that vertical mergers are always socially beneficial and should face little regulatory scrutiny.

An alternative viewpoint is that vertical mergers can generate foreclosure. By this story, when an upstream firm and a downstream firm merge, the market power of the remaining non-merging upstream firms increases. As these upstream firms exploit their new market power, they raise the prices they charge to the downstream sector, effectively raising the costs to the merged firm's downstream rivals. This then gives the merged firm a cost advantage with which it can either gain greater profits or seek to drive the downstream rivals from the market.

4. Back before my Grandma could use a computer (10 points)

The Microsoft I case dealt at length with whether per-processor licensing constituted an anticompetitive action. Explain how Microsoft used this tactic and describe how such a tactic (or the more general market share discounts) might effectively foreclose small rivals from the market.

In 1990, Microsoft introduced two pricing strategies for licensing it operating system (OS). The first (and original) tactic was to charge Original Equipment Manufacturers (OEMs) a price for each personal computer (PC) upon which the OEM installed the Microsoft OS (e.g., $80 per OS). The second (and novel) tactic was to charge OEMs a fee based upon the number of PCs (or equivalently processors) that were sold (e.g., $40 per PC). Since Microsoft was by far the largest licensor of any OS for PCs (with market shares of about 90%), virtually all OEMs took Microsoft's second option. Having done so, they incurred zero marginal cost for installing Microsoft's OS instead of the list price of installing a rival's OS. DRI's DR-DOS was the primary rival OS. Facing this environment, it was discontinued in September 1993.

The per-processor license and market share discounts effectively take what had been a continuous payment structure and transform it into a discrete payment structure. By making payments to the upstream firm discrete, such tactics give the dominant firm a marginal cost advantage relative to its upstream rival that has nothing to do with either technological or qualitative advantages. Note that a firm's success in applying it hinges critically upon existing market dominance. With 100% market share, downstream firms are indifferent between the two tactics at the same price, so for little revenue loss the upstream firm can foreclose the upstream market to any rivals. With 90% market share, setting the per-processor fee at a 10% discount to the per-license fee will suffice and rivals will be unable to mimic the technique. With only 50% market share, however, upstream firms must sacrifice substantial revenues and face the increased likelihood that rivals will be able to strike similar deals other downstream firms.

5. Cola Wars (30 points)

Consider a duopoly of price-setting producers of differentiated beverages: Coca-Cola (firm 1) and Pepsi-Cola (firm 2). Demand for each firm's product depends upon both firms' prices, but also upon the relative advantage in advertising intensity one firm has over the other. The firms' symmetric demands are

$$q_1 = ((1 + A_1)/(1 + A_2))^{1/2} (100 - 2P_1 + P_2)$$
$$q_2 = ((1 + A_2)/(1 + A_1))^{1/2} (100 - 2P_2 + P_1)$$

Both companies simultaneously choose their price and advertising to maximize their profits each period. Each firm faces the same total cost function: $TC_i = 10q_i + 100A_i$.

A. Does this specification suggest that advertising plays more of an informative or a persuasive role?

As either firm's advertising expenditures approach infinity (while the other firm's advertising is held constant at some level), quantity demanded increases without limit. This indicates that advertising is serving a primarily persuasive role.

B. Find Coke's best-response functions for price and advertising so that the "reactions" depend only upon Pepsi's actions.

Coke must select its price and its advertising to maximize its profit expression, which is

$$\Pi_1 = (P_1 - 10)((1 + A_1)/(1 + A_2))^{½} (100 - 2P_1 + P_2) - 100 A_1$$

Coke should simultaneously solve its two FOCs:

P_1: $((1 + A_1)/(1 + A_2))^{½} (100 - 2P_1 + P_2) - 2 (P_1 - 10)((1 + A_1)/(1 + A_2))^{½} = 0$

A1: $(P_1 - 10)(½)(1 + A_1)^{-½} (1 + A_2)^{-½} (100 - 2P_1 + P_2) - 100 = 0$

Rearranging the pricing FOC readily yields the best-response expression of $P_1 = 30 + ¼ P_2$. Substituting this expression into the advertising FOC yields

$$((30 + ¼ P_2) - 10)(½)(1 + A_1)^{-½} (1 + A_2)^{-½} (100 - 2(30 + ¼ P_2) + P_2) - 100 = 0$$

which reduces to

$$(20 + ¼ P_2)^2(1 + A_1)^{-½} (1 + A_2)^{-½} - 100 = 0$$

Solving this for A_1 yields

$$A_1 = (((20 + ¼ P_2)^2)/100)^2/(1 + A_2) - 1$$

C. What are the equilibrium outcomes (prices, advertising, quantities, profits) to this game?

Begin by exploiting the problem's symmetry with respect to prices, so $P^* = 40$. Since $A_1 = A_2$, Coke's best-response for advertising reduces to

$$A^* = (((20 + ¼ (40))^2)/100)^2/(1 + A^*) - 1 = (9)^2/(1 + A^*) - 1$$

This readily becomes $(1 + A^*)^2 = (9)^2$, which is satisfied (among the positive, plausible solutions) at $A^* = 8$. So in equilibrium, each firm sells $q^* = 60$ and earns profits of $\Pi^* = 1000$.

D. Each firm recognizes that it would reap greater profits if both firms would reduce advertising to nothing. While pricing collusion is prevented by the antitrust authorities, such advertising collusion will land no one in legal trouble. Set up an appropriate Prisoners' Dilemma game, using the profits from the above scenario as payoffs.

The (Defect, Defect) payoffs correspond to the profits from the above equilibrium: $\Pi^{DD} = 1000$. The (Collude, Collude) payoffs are those profits without the advertising costs ($\Pi^{CC} = 1800$). To find the (Defect, Collude) payoffs, we return to Coke's best-response condition for advertising. If $P_2 = 40$ and $A_2 = 0$, Coke's optimal advertising is $A_1 = 80$ which yields quantity $q_1 = 540$ and profit $\Pi_1^{DC} = 8200$. The sucker Pepsi, on the other hand, sells quantity $q_2 = 60/9$ and earns profits $\Pi_2^{DC} = 200$. (The converse holds when Coke is the sucker.) The Prisoner's Dilemma therefore is

		Pepsi (2) Collude	Pepsi (2) Defect
Coke (1)	Collude	(1800,1800)	(200, 8200)
	Defect	(8200, 200)	(1000, 1000)

Final Answers

Note: These extra credit questions had no answer *a priori*. I only learned the answers by counting your responses.

Extra Credit: Game Theory in Action (1 point each)

A. Bruce Petersen
B. Douglas North
C. Gaetano Antinolfi
D. Chuck Moul

1. From the above list, choose the WU econ professor who will be chosen by the most students.

Petersen	1
North	4
Antinolfi	1
Moul	22

So everyone who chose Moul receives 1 point. The professor of the course is evidently the focal point. Note that another possible focal point (Nobel-prize winning economists) came in second.

2. From the above list, choose the WU econ professor who will be chosen by the fewest students.

Petersen	16
North	2
Antinolfi	5
Moul	4

So everyone who chose North receives 1 point. Like *Rock, Scissors, Paper*, this is an example of an equilibrium existing only in mixed (randomizing) strategies. Had three more students chosen North, it would no longer have been the answer. I have no explanation as to why Petersen was such a popular guess as to whom would be picked the fewest times.