1. (30 points) To make Chuck's Seahorse Farm work, I invent an automatic zooplankton dispenser (the Feedbag-O-Matic), and I get a patent for it. While I have to compete with all those fishermen in Southeast Asia to sell my crop, I have a monopoly on my little gizmo, and I behave accordingly, selling it to home aquariums and other aquacultural establishments. The annual demand for this machine can be expressed as $Q^D = 4000 - 50*P$, and my costs of producing the Feedbag-O-Matic are $TC = (1/200)*q^2 + 20q + 6000$. (This yields $MC = (1/100)*q + 20$.)

   A. Find my profit-maximizing output and price. What are my annual profits?

My profit-maximizing decision depends upon my marginal revenue (MR) and my marginal cost (MC). I already know my MC. To find my MR, I invert the demand for the Feedbag-O-Matic: $P = 80 - (1/50)*Q$. My MR has the same intercept as this inverse demand and twice the slope: $MR = 80 - (1/25)*Q$. My profits are maximized at Q^M such that $80 - (1/25)*Q^M = (1/100)*Q^M + 20$, or $Q^M = 1200$. At that output, my MR and MC both equal \$32. The highest price I can charge to sell this amount is \$56(=80 – (1/50)*1200). Annual revenues are \$67200(=56*1200), and annual costs are \$37200(=$(1/200)*1200^2 + 20(1200) + 6000$). Therefore my annual profits are an even \$30,000.

Another way to calculate the annual profits (or to check your work) is to calculate the ATC at the profit-maximizing Q. Here, ATC(Q) = 31(=(1/200)*1200 + 20 + 6000/1200). The, since the average revenue is the price of \$56 and the average cost is \$31, my per-unit profit is \$25, and my total profit is \$30,000(=25*1200).

   B. Society would prefer I produce at a different level. Find the allocatively efficient output and its corresponding market price. How much does my patent cost society, i.e., what is the deadweight loss from my current level of production?

The allocatively efficient output is the Q where P = MC: $80 - (1/50)*Q = (1/100)*Q + 20$, yielding Q* = 2000, and a price of \$40. Since all relevant curves are lines, the approximation of DWL is exact: $DWL = ½*(P - MR)*(Q^* - Q^M) = ½*(56 - 32)*(2000 - 1200) = \$9600$.

   C. Why does society put up with this inefficiency?

If I expect society to force me to produce at the socially efficient output, my incentive to create the Feedbag-O-Matic in the first place is greatly reduced. My annual profits at the allocatively efficient output are \$14,000. If my development costs were such that I would be willing to invent the gizmo if my yearly profits are \$30,000 but not if they are only \$14,000, then society forcing me to produce the allocatively efficient amount will lead to no invention in the first place.

D. Suppose that I learn that the demand for the Feedbag-O-Matic is much more elastic among home aquarium owners than among aquaculturists. How might I profit from this? What difficulty must I first overcome?

This specific knowledge about the demand may allow me to raise my profits by engaging in price discrimination. The elastic demand of home aquarium enthusiasts means I could lower my monopoly price for that segment of the population and increase my revenues. Likewise, I could raise my monopoly price for the less elastic aquaculture segment of demand and see my profits rise. Unfortunately, I must first find a way to keep the aquaculturists from masquerading as home aquarium owners and buying the Feedbag-O-Matic at the lower price.

2. (10 points) We are discussing the Prisoner's Dilemma in class to illustrate the difficulties of firms colluding in the short-run. Below is the game that economists boorishly call the Battle of the Sexes. I use it here as an additional illustration of the equilibrium concept.

Cory and Pat would rather spend time together than apart. Each, however, still has preferences about his and her favorite activities. Specifically, Cory would rather watch kickboxing than go to the ballet, and Pat would rather see elegance and grace than bone-breaking violence. Their payoffs for all possible outcomes are as follows (Cory's payoff is the first number in the pair, Pat's is the second).

		Pat: Kickboxing	Pat: Ballet
Cory	Kickboxing	(5, 2)	(0, 0)
	Ballet	(0, 0)	(2, 5)

A. How many Nash equilibria are there in this game? What are they?
There are two (pure-strategy) Nash equilibria in this game. If Cory thinks Pat will choose kickboxing, then Cory will also choose kickboxing. Likewise, if Pat thinks Cory will choose kickboxing, then Pat will also choose kickboxing. Both choosing kickboxing is a Nash equilibrium. The same holds for both choosing ballet.

B. If Cory and Pat can't communicate, does game theory indicate which choice they should make?
No. Game theory has tried to address this problem by assigning probabilities to players' actions. (The third equilibrium is Pat and Cory each separately flipping a coin and going to kickboxing if it's Heads and ballet if it's Tails.) The other real-world difficulty is that, even if Pat and Cory can communicate, game theory can make no prediction regarding whether they will go to kickboxing or ballet. This problem of multiple equilibria is something game theorists are still trying to address and bargaining theory is still developing.

3. (10 points) Briefly explain why collusion is so difficult to achieve when all producers know when the "game" will end. (Consider a 17-year patent shared by two firms.)

The unique solution to the classic Prisoner's Dilemma Game (Cheat, Cheat) holds whenever there are no dynamic considerations, that is, no opportunities to punish. Therefore, each firm will rationally expect the other to cheat (i.e., produce more than the joint profit-maximizing amount) in the 17th year. But this effectively means that there is no chance to punish or reward in the 16^{th} year, either: Both firms know punishment is inevitable. Consequently, each firm expects the other to cheat in the 16^{th} year. This works backward until both firms are at the 1^{st} year, and neither firm can rationally expect any opportunity to collude without the other cheating on him.

4. (10 points) A local movie theater sells tickets to children for $4 and to adults for $8.

A. Given this, which group would you expect to have a more elastic demand for movie tickets, adults or children?

Since the theater is discounting tickets for children, we would expect that kids have a more elastic demand for movie tickets than adults do. (Recall that price discrimination arises when a firm can charge the relatively inelastic consumers a little more and the relatively elastic consumers a little less.)

B. Explain how a functioning resale market would thwart the movie theater's plans.

If kids can buy tickets and resell them to adults, then the theater's price discrimination plan backfires. Most likely, the single price that maximizes profits lies somewhere between $4 and $8, but the resale market has everyone paying $4. The monopolist is worse off (makes less profit or even takes a loss) if she sticks with the price discrimination plan when resale cannot be prevented.

C. What might keep new theaters from opening and charging a single price of $7 to steal adult customers? (Feel to free-associate on this question … professional economists themselves struggle with this one.)

The best that economists have come up with is that a large fraction of adults see movies with their children. These consumers will prefer the price-discriminating regime to the single price regime. If there are not enough adults who see movies without kids, then a single-price entrant does not face a large enough pool of potential consumers to find entry worthwhile.

5. Hershey introduces N&Ns, a new candy that is identical to Mars' M&Ms in every way except for the letter on the candy shell. The marginal cost and average variable cost of producing a pack of these candies are constant at $0.10, and (amazingly) there are no fixed costs. The demand for these candies is $Q^D = 100 – 100P$ (Q in millions)

A. If Hershey and Mars act as Cournot competitors, what is the duopoly outcome of quantities and prices? What about profits?

The inverse demand for these candies is $P = 1 – (1/100)(Q^H + Q^M)$. To Mars, this is the same as $P = 1 – (1/100)Q^H – (1/100)Q^M$, so it perceives its marginal revenues to be $MR^M = 1 – (1/100)Q^H – (2/100)Q^M$. Hershey faces the marginal revenue of $MR^H = 1 – (1/100)Q^M – (2/100)Q^H$. Each firm will maximize its profits at the quantity such that its marginal revenue equals its marginal costs. For Mars, this means $1 – (1/100)Q^H – (2/100)Q^M = 0.10$, yielding the reaction function $Q^M = 45 – ½ Q^H$. Analogously for Hershey, this means $1 – (1/100)Q^M – (2/100)Q^H = 0.10$, yielding a reaction function of $Q^H = 45 – ½ Q^M$. The Cournot-Nash solution to this problem requires that both these reaction functions hold simultaneously, so we substitute one firm's profit-maximizing quantity into the reaction function of the other firm: $Q^M = 45 – ½ (45 – ½ Q^M)$, yielding $Q^M = 30$. Using this in Hershey's reaction function gives us the symmetric outcome of $Q^H = 30$. Prices are $0.40, and profits for each are $9M.

B. Now suppose that (unlike in Cournot competition) competition forced both companies to price at marginal cost, i.e., the allocatively efficient outcome. What is the outcome in quantities and prices here? What about profits?

By assumption price equals marginal cost for both firms. Prices are then $0.10, each firm produces 45 million, and each firm gets zero profits.

C. If the candy companies perfectly collude, what is the outcome of quantities and prices? Profits?

Under perfect collusion, the firms make no distinction between who is producing what, so one need only consider total output. In this case, marginal revenue is $MR = 1 – (2/100)Q$, so joint profits are maximized at $Q = 45$. Presuming they split production equally, each firm produces 22.5M, receives a price of $0.55, and earns profits of $10.125M.